
Zhi-Qiang Jiang, Wei-Xing Zhou, D. Sornette, Ryan Woodard, Ken Bastiaensen, Peter Cauwels

CCSS Working Paper Series
CCSS-09-008

CCSS, the Competence Center Coping with Crises in Complex Socio-Economic Systems, was established at ETH Zurich (Switzerland) in September 2008. By means of theoretical and empirical analysis, CCSS aims at understanding the causes of and cures to crises in selected problem areas, for example in financial markets, in societal infrastructure, or crises involving political violence. More information can be found at: http://www.ccss.ethz.ch/.

Zhi-Qiang Jiang, Wei-Xing Zhou, D. Sornette, Ryan Woodard, Ken Bastiaensen, Peter Cauwels

Abstract

By combining (i) the economic theory of rational expectation bubbles, (ii) behavioral finance on imitation and herding of investors and traders and (iii) the mathematical and statistical physics of bifurcations and phase transitions, the log-periodic power law (LPPL) model has been developed as a flexible tool to detect bubbles. The LPPL model considers the faster-than-exponential (power law with finite-time singularity) increase in asset prices decorated by accelerating oscillations as the main diagnostic of bubbles. It embodies a positive feedback loop of higher return anticipations competing with negative feedback spirals of crash expectations. We use the LPPL model in one of its incarnations to analyze two bubbles and subsequent market crashes in two important indexes in the Chinese stock markets between May 2005 and July 2009. Both the Shanghai Stock Exchange Composite index (US ticker symbol SSEC) and Shenzhen Stock Exchange Component index (SZSC) exhibited such behavior in two distinct time periods: 1) from mid-2005, bursting in October 2007 and 2) from November 2008, bursting in the beginning of August 2009. We successfully predicted time windows for both crashes in advance (Sornette, 2007; Bastiaensen et al., 2009) with the same methods used to successfully predict the peak in mid-2006 of the US housing bubble (Zhou and Sornette, 2006b) and the peak in July 2008 of the global oil bubble (Sornette et al., 2009). The more recent bubble in the Chinese indexes was detected and its end or change of regime was predicted independently by two groups with similar results, showing that the model has been well-documented and can be replicated by industrial practitioners. Here we present more detailed analysis of the individual Chinese index predictions and of the methods used to make and test them. We complement the detection of log-periodic behavior with Lomb spectral analysis of detrended residuals and \((I, q)\)-derivative of logarithmic indexes for both bubbles. We perform unit-root tests on the residuals from the log-periodic power law model to confirm the Ornstein-Uhlenbeck property of bounded residuals, in agreement with the consistent model of ‘explosive’ financial bubbles (Lin et al., 2009).

Keywords: Stock market crash, financial bubble, Chinese markets, rational expectation bubble, log-periodic power law

Classifications: JEL Codes: G01, G17, O16

URL: http://web.sg.ethz.ch/wps/CCSS-09-008

Notes and Comments:

Zhi-Qiang Jiang\textsuperscript{a,c,d}, Wei-Xing Zhou\textsuperscript{a,c,d,f}, Didier Sornette\textsuperscript{b,c,*,}, Ryan Woodard\textsuperscript{b}, Ken Bastiaensen\textsuperscript{b,*}, Peter Cauwels\textsuperscript{b,*}

\textsuperscript{a}School of Business, East China University of Science and Technology, Shanghai 200237, China
\textsuperscript{b}The Financial Crisis Observatory
Department of Management, Technology and Economics, ETH Zurich, Kreuzplatz 5, CH-8032 Zurich, Switzerland
\textsuperscript{c}School of Science, East China University of Science and Technology, Shanghai 200237, China
\textsuperscript{d}Research Center for Econophysics, East China University of Science and Technology, Shanghai 200237, China
\textsuperscript{e}Swiss Finance Institute, c/o University of Geneva, 40 blvd. Du Pont d’Arve, CH 1211 Geneva 4, Switzerland
\textsuperscript{f}Research Center on Fictitious Economics & Data Science, Chinese Academy of Sciences, Beijing 100080, China
\textsuperscript{g}BNP Paribas Fortis, Warandeberg 3, 1000 Brussels, Belgium

Abstract

By combining (i) the economic theory of rational expectation bubbles, (ii) behavioral finance on imitation and herding of investors and traders and (iii) the mathematical and statistical physics of bifurcations and phase transitions, the log-periodic power law (LPPL) model has been developed as a flexible tool to detect bubbles. The LPPL model considers the faster-than-exponential (power law with finite-time singularity) increase in asset prices decorated by accelerating oscillations as the main diagnostic of bubbles. It embodies a positive feedback loop of higher return anticipations competing with negative feedback spirals of crash expectations. We use the LPPL model in one of its incarnations to analyze two bubbles and subsequent market crashes in two important indexes in the Chinese stock markets between May 2005 and July 2009. Both the Shanghai Stock Exchange Composite index (US ticker symbol SSEC) and Shenzhen Stock Exchange Component index (SZSC) exhibited such behavior in two distinct time periods: 1) from mid-2005, bursting in October 2007 and 2) from November 2008, bursting in the beginning of August 2009. We successfully predicted time windows for both crashes in advance (Sornette, 2007; Bastiaensen et al., 2009) with the same methods used to successfully predict the peak in mid-2006 of the US housing bubble (Zhou and Sornette, 2006b) and the peak in July 2008 of the global oil bubble (Sornette et al., 2009). The more recent bubble in the Chinese indexes was detected and its end or change of regime was predicted independently by two groups with similar results, showing that the model has been well-documented and can be replicated by industrial practitioners. Here we present more detailed analysis of the individual Chinese index predictions and of the methods used to make and test them.

We complement the detection of log-periodic behavior with Lomb spectral analysis of detrended residuals and \((H,q)\)-derivative of logarithmic indexes for both bubbles. We perform unit-root tests on the residuals from the log-periodic power law model to confirm the Ornstein-Uhlenbeck property of bounded residuals, in agreement with the consistent model of ‘explosive’ financial bubbles (Lin et al., 2009).

Key words: stock market crash, financial bubble, Chinese markets, rational expectation bubble, herding, log-periodic power law, Lomb spectral analysis, unit-root test

JEL: G01, G17, O16

The present paper contributes to the literature on financial bubbles by presenting two case studies and new empirical tests, in support of the proposal that (i) the presence of a bubble can be diagnosed quantitatively before its demise and (ii) the end of the bubble has a degree of predictability.

These two claims are highly contentious and collide against a large consensus both in the academic literature (Rosser, 2008) and among professionals. For instance, in his recent review of the financial economic literature on bubbles, Gurkaynak (2008) reports that “for each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble. We are still unable to distinguish bubbles from time-varying or regime-switching fundamentals, while many small sample econometrics problems of bubble tests remain unresolved.” Similarly, the following statement by former Federal Reserve chairman Alan Greenspan (2002), at a summer conference in August 2002 organized by the Fed to try to understand the cause of the ITC bubble and its subsequent crash in 2000 and 2001, summarizes well the state of the art from the point of view of practitioners: “We, at the Federal Reserve recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence. Moreover, it was far from obvious that bubbles, even if identified early, could be preempted short of the Central Bank inducing a substantial contraction in economic activity, the very outcome we would be seeking to avoid.”

To break this stalemate, one of us (DS) with Anders Johansen from 1995 to 2002, with Wei-Xing Zhou since 2002 (now Professor at ECUST in Shanghai) and, since 2008, with the FCO group at ETH Zurich (www.er.ethz.ch/fco/) have developed a series of models and techniques at the boundaries between financial economics, behavioral finance and statistical physics. Our purpose here is not to summarize the corresponding papers, which explore many different options, including rational expectation bubble models with noise traders, agent-based models of herding traders with Bayesian updates of their beliefs, models with mixtures of nonlinear trend followers and nonlinear value investors, and so on (see Sornette (2003b) and references therein until 2002 and the two recent reviews in Kaizoji and Sornette (2009); Sornette and Woordard (2009) and references therein). In a nutshell, bubbles are identified as “super-exponential” price processes, punctuated by bursts of negative feedback spirals of crash expectations. These works have been translated into an operational methodology to calibrate price time series and diagnose bubbles as they develop. Many cases are reported in Chapter 9 of the book (Sornette, 2003b) and more recently successful applications have been presented with ex-ante public announcements posted on the scientific international database arXiv.org and then published in the referred literature, which include the diagnostic and identification of the peak

*These authors express their personal views, which do not necessarily correspond to those of BNP Paribas Fortis.
**Corresponding author. Address: KPL F 38.2, Kreuzplatz 5, Chair of Entrepreneurial Risks, Department of Management, Technology and Economics, ETH Zurich, Switzerland, Phone: +41 44 632 89 17, Fax: +41 44 632 19 14.
Email addresses: dsornette@ethz.ch (Didier Sornette)
time of the bubble for the UK real-estate bubble in mid-2004 (Zhou and Sornette, 2003a), the U.S. real-estate bubble in mid-2006 (Zhou and Sornette, 2006b), and the oil price peak in July 2008 (Sornette et al., 2009).

Kindleberger (2000) and Sornette (2003b) have identified the following generic scenario developing in five acts, which is common to all historical bubbles: displacement, take-off, exuberance, critical stage and crash. For the Chinese bubble starting in 2005, the “displacement” and “take-off” can be associated with the split share structure reform of listed companies in 2005. Before the reform, only about one third of the shares of any listed company in the Chinese stock market were tradable. The other two-third shares were non-tradable (not allowed to be exchanged and to circulate between investors), and were owned by the state and by legal entities. The tradable stocks acquired therefore a significant liquidity premium, and were valued much higher than their non-tradable siblings, notwithstanding the fact that both gave the same privilege to their owners in terms of voting rights and dividends. Since 2001, the Chinese stock market entered an anti-bubble phase (Zhou and Sornette, 2004) with the Shanghai Stock Exchange Composite index falling from its then historical high 2245 on 24 June 2001 to the historical low on June 6, 2005. On 29 April 2005, the China Securities Regulatory Commission launched the pilot reform of the split share structure. The split share structure reform is defined as the process to eliminate the discrepancies in the A-share transfer system via a negotiation mechanism to balance the interests of non-tradable shareholders and tradable shareholders. On 4 September 2005, the China Securities Regulatory Commission enacted the Administrative Measures on the Split Share Structure Reform of Listed Companies1, which took effect immediately. It is widely accepted that the split share structure reform was a turning point which triggered and catalyzed the recovery of the Chinese stock market from its previous bearish regime. For the Chinese bubble starting in November 2008, the “take-off” can be associated with China’s policy reaction on the global financial crisis, with a huge RMB 4 trillion stimulus plan and aggressive loan growth by financial institutions.

Here, we present an ex-post analysis of what we identified earlier in their respective epochs as being two significant bubbles developing in the major Chinese stock markets, the first one from 2005 to 2007 and the second one from 2008 to 2009. The organized stock market in mainland China is composed of two stock exchanges, the Shanghai Stock Exchange (SHSE) and the Shenzhen Stock Exchange (SZSE). The most important indices for A-shares in SHSE and SZSE are the Shanghai Stock Exchange Composite index (SSEC) and the Shenzhen Stock Exchange Component index (SZSC). The SSEC and SZSC indexes have suffered a more than 70% drop from their historical high during the period from October 2007 to October 2008. Since November 2008 and until the end of July 2009, the Chinese stock markets had been rising dramatically. By calibrating the recent market index price time series to our LPPL model, we infer that, in both cases, a bubble had formed in the Chinese stock market and that the market prices were in an unsustainable state. We present the analysis that led us to diagnose the presence of these two bubbles respectively in September 2007 and in July 10, 2009 (Bastiaensen et al., 2009), and to issue an advance notice of the probable time of the regime shifts, from a bubble (accelerating “bullish”) phase to a (“bearish”) regime or a crash. See Fig. 1 for an overview of the two bubbles and our predictions. The figure shows the time evolution of two Chinese indexes, the

---

dates when we made our predictions and the time intervals of our predicted changes of regime.

[Figure 1 about here.]

The organization of the paper is the following. In Sec. 2, we present technical descriptions of all the methods used in this paper. Specifically, they are LPPL fitting procedure, Lomb spectral analysis, unit root tests and change-of-regime statistic. We present the results of the 2007 and 2009 bubbles in two separate subsections of Sec. 3. In Sec. 4, we document and discuss the predictions we made for both bubbles prior to their bursting and, further, describe the observations of both markets indicating that these two bubbles actually did burst. Section 5 concludes.

2. Methods

Our main method for detecting bubbles and predicting the critical time $t_c$ when the bubble will end either in a crash or change of regime is by fitting observed price time series to a log periodic power law (LPPL) model (Sornette, 2003a,b; Zhou, 2007). This is a stochastic fitting procedure that we complement with other techniques, described below. This philosophy of using multiple measures aids in filtering predictions, in that a candidate prediction must pass all tests to be considered worthy. These techniques form a toolset that has successfully been put to practice over the past years by Sornette et al. as described in the introduction. Independently a similar toolset has recently been developed within the Research Group of BNP Paribas Fortis (Global Markets) on the same methodology but with a slightly different implementation of the fitting procedure and the Lomb analysis.

2.1. General LPPL fitting technique

Consider a time series (such as share price) $p(t)$ between starting and ending dates $t_1$ and $t_2$. The LPPL model that we use is

$$\ln[p(t)] = A + Bx^m + Cx^m \cos(\omega \ln x + \phi),$$

where $x = t_c - t$ measures the time to the critical time $t_c$. For $0 < m < 1$ and $B < 0$ (or $m \leq 0$ and $B > 0$), the power law term $Bx^m$ describes the faster-than-exponential acceleration of prices due to positive feedback mechanisms. The term proportional to $\cos(\omega \ln x + \phi)$ expresses a correction to this super-exponential behavior, which has the symmetry of discrete scale invariance (Sornette, 1998). By varying $t_1$ and $t_2$, we can investigate the stability of the fitting parameters with respect to starting and ending points.

It is worthwhile pointing out that calibrating Eq. (1) to any given price (or log-price) trajectory will always provide some fit parameters. That is, any model can be fit to any data. Hence, it is necessary to establish a constraint—the LPPL condition—to filter all of the fitting results. We filter on three parameters:

$$t_c > t_2, B < 0, 0 < m < 1.$$
This filter selects regimes with faster-than-exponential acceleration of the log-price with a diverging slope at the critical future time $t_c$.

There are four nonlinear parameters ($t_c$, $m$, $\omega$, and $\phi$) and three linear parameters ($A$, $B$, and $C$) in Eq. (1). In order to reduce the fitting parameters, the linear parameters are slaved to the nonlinear parameters. By rewriting Eq. (1) as

$$\ln p(t) = A + B f(t) + C g(t)$$

and using an estimate of the nonlinear parameters, the linear parameters can be solved analytically via:

$$\begin{bmatrix}
N & \sum f_i & \sum g_i & A \\
\sum f_i & \sum f_i^2 & \sum g_i f_i & B \\
\sum g_i & \sum f_i g_i & \sum g_i^2 & C
\end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum \ln p_i \\ \sum \ln p_i f_i \\ \sum \ln p_i g_i \end{bmatrix}.$$  (3)

The implementation of the fitting proceeds in two steps. First, we adopt the Taboo search (Cvijović and Klinowski, 1995) to find 10 candidate solutions from our given search space. Second, each of these solutions is used as an initial estimate in a Levenberg-Marquardt nonlinear least squares algorithm. The solution with the minimum sum of squares between model and observations is taken as the solution.

2.2. Stability of fits vs. shrinking and expanding intervals and probabilistic forecasts

In order to test the sensitivity of variable fitting intervals $[t_1, t_2]$, we adopt the strategy of fixing one endpoint and varying the other one. For instance, if $t_2$ is fixed, the time window shrinks in terms of $t_1$ moving towards $t_2$ with a step of five days. If $t_1$ is fixed, the time window expands in terms of $t_2$ moving away from $t_1$ with a step of five days. For each such $[t_1, t_2]$ interval, the fitting procedure is implemented on the index series three times. Recall that because of the rough nonlinear parameter landscape of Eq. (1) and the stochastic nature of our initial parameter selection, it is expected that each implementation of our fit process will produce a different set of fit parameters. By repeating the process multiple times, we investigate an optimal (not necessarily the optimal) region of solution space.

By sampling many intervals as well as by using bootstrap techniques, we obtain predictions that are inherently probabilistic and reflect the intrinsic noisy nature of the underlying generating processes. This allows us to provide probabilistic estimations on the time intervals in which a given bubble may end and lead to a new market regime. In this respect, we stress that, notwithstanding the common use of the term “crash” to refer to the aftermath of a bubble, a real crash does not always occur. Rather, the end of a bubble may be the most probable time for a crash to occur, but the bubble may end without a splash and, instead, transition to a plateau or a slower decay. This point is actually crucial in rational expectation models of bubbles in that, even in the presence of investors fully informed of the presence of the bubble and with the knowledge of its end date, it remains rational to stay invested in the market to garner very large returns since the risk of a crash remains finite (Johansen et al., 1999, 2000a).

2.3. Lomb spectral analysis

Fitting the logarithm of prices to the model Eq. (1) gives strong evidence supporting log-periodicity in that stable values of the angular frequency $\omega$ are found. We test this feature further by using Lomb spectral analysis (Press et al.,
1996) for detecting the log-periodic oscillations. The Lomb method is a spectral analysis designed for irregularly sampled data and gives the same results as the standard Fourier spectral analysis for evenly spaced data. Specifically, given a time series, the Lomb analysis returns a series of frequencies $\omega$ and associated power at each frequency, $P_N(\omega)$. The frequency with the maximum power is taken as the Lomb frequency, $\omega_{\text{Lomb}}$. Following Sornette and Zhou (2002), the spectral Lomb analysis is performed on two types of signals.

**Parametric detrending approach.** The first is the series of detrended residuals, calculated as

$$r(t) = x^{-m}(\ln[p(t)] - A - Bx^m),$$

where $x \equiv t - t_c$ and $A, B, t_c$ and $m$ have been found via the method of Section 2.1 (Johansen and Sornette, 1999a; Zhou et al., 2007). As suggested in Eq. (1), the log-periodic oscillations result from the cosine part. The angular frequency $\omega_{\text{Lomb}}$ is then compared with that found in the LPPL fitting procedure, $\omega_{\text{fit}}$.

**Non-parametric, (H, q) analysis.** The second is the $(H, q)$-derivative of the logarithmic prices, which has been successfully applied to financial crashes (Zhou and Sornette, 2003b) and critical ruptures (Zhou and Sornette, 2002b) for the detection of log-periodic components. The $(H, q)$ analysis is a generalization of the $q$-analysis (Erzan, 1997; Erzan and Eckmann, 1997), which is a natural tool for the description of discrete scale invariance. The $(H, q)$-derivative is defined as,

$$D_{q}^{H}f(x) \triangleq \frac{f(x) - f(qx)}{[\ln(1-q)x]^{H}}.$$  

(5)

We vary $H$ and $q$ in the ranges $[-1, 1]$ and $[0, 1]$, respectively, and perform the Lomb analysis on the resulting series. If $H = 1$ in Eq. (5), the $(H, q)$-derivative reduces to the normal $q$-derivative, which itself reduces to the normal derivative in the limit $q \to 1^-$. Without loss of generality, $q$ is constrained in the open interval $(0, 1)$. The advantage of the $(H, q)$ analysis is that there is no need for detrending, as it is automatically accounted for by the finite difference and the normalization by the denominator. This method has been applied for detecting log-periodicity in stock market bubbles and anti-bubbles (Sornette and Zhou, 2002; Zhou and Sornette, 2003b, 2004), in the USA foreign capital inflow bubble ending in early 2001 (Sornette and Zhou, 2004) and in the ongoing UK real estate bubble (Zhou and Sornette, 2003a).

### 2.4. Ornstein-Uhlenbeck and unit root tests

Recently, Lin et al. (2009) have put forward a self-consistent model for explosive financial bubbles, in which the LPPL fitting residuals can be modeled by a mean-reversal Ornstein-Uhlenbeck (O-U) process if the logarithmic price in the bubble regime is attributed to a deterministic LPPL component. The test for the O-U property of LPPL fitting residuals can be translated into an AR(1) test for the corresponding residuals. Hence, we can verify the O-U property of fitting residuals by applying unit-root tests on the residuals. We use the Phillips-Perron and Dickey-Fuller unit-root
tests. A rejection of null hypothesis $H_0$ suggests that the residuals are stationary and therefore are compatible with the O-U process in the residuals. Our tests use the same time windows as the LPPL calibrating procedure.

2.5. Statistic of change of regime

We define a simple statistic to demonstrate the change of regime in the “post-mortem” analysis of our prediction. We calculate the difference $d_{\text{co}}(t)$ between the closing and opening prices on each trading day and track the fraction of days with negative $d_{\text{co}}(t)$ in a rolling window of $T$ days, with $T = 10, 20$ and $30$ days.

3. Results

In the following two subsections, we present our analysis of the separate 2007 and 2009 bubbles using the methods described in Section 2.


3.1.1. LPPL fitting with varying window sizes

As discussed above, we test the stability of fit parameters for the two indexes, SSEC and SZSC, by varying the size of the fit intervals. Specifically, the logarithmic index is fit by the LPPL formula, Eq. (1):

1. in shrinking windows with a fixed end date $t_2 = \text{Oct-10-2007}$ with the start time $t_1$ increasing from $\text{Oct-01-2005}$ to $\text{May-31-2007}$ in steps of five (trading) days and
2. in expanding windows with a fixed start date $t_1 = \text{Dec-01-2005}$ with the end date $t_2$ increasing from $\text{May-01-2007}$ to $\text{Oct-01-2007}$ in steps of five (trading) days.

In the above two fitting procedures, we fit the indexes 124 times in shrinking windows and 15 times in expanding windows. After filtering by the LPPL conditions, we finally observe 72 (78) results in the first step and 11 (15) results in the second step for SSEC (respectively SZSC). Figures 2 (a) and (c) illustrate six chosen fitting results of the shrinking windows for SSEC and SZSC and Figures 2 (b) and (d) illustrate six fitting results of the expanding time intervals for SSEC and SZSC. The dark and light shadow boxes in the figures indicate 20%/80% and 5%/95% quantile range of values of the crash dates for the fits that survived filtering. One can observe that the observed market peak dates (16 October 2007 for SSEC, 31 October 2007 for SZSC) lie in the quantile ranges of predicted crash dates $t_c$ using only data from before the market crash (i.e., using $t_2 < t_{c,\text{obs}}$).

[Figure 2 about here.]
3.1.2. Lomb analysis, parametric approach

Fig. 3 summarizes the results of our Lomb analysis on the detrended residuals $r(t)$. The Lomb periodograms ($P_N$ with respect to $\omega_{\text{Lomb}}$) are plotted in Fig. 3(a) for four typical examples, which are $(t_1, t_2) = (\text{Mar-13-2006, Oct-10-2007})$ and $(\text{Dec-12-2005, Sep-07-2007})$ for SSEC, and $(\text{Apr-04-2006, Oct-10-2007})$ and $(\text{Dec-01-2005, Sep-09-2007})$ for SZSC. The inset illustrates the corresponding detrended residuals $r(t)$ as a function of $\ln(t_c - t)$. We select the highest peak with its associated $\omega_{\text{Lomb}}$.

The values of $\omega_{\text{Lomb}}$ must be consistent with the values of $\omega_{\text{fit}}$ obtained from the fitting. We plot the bivariate distribution of pairs ($\omega_{\text{Lomb}}, P_{N}^{\text{max}}$) for different LPPL calibrating windows in Fig. 3(b) and find that the minimum value of $P_{N}^{\text{max}}$ is approximately 54 for SSEC and approximately 30 for SZSC. These peaks are linked to a false alarm probability, which is defined as the chance that we falsely detect log-periodicity in a signal without true log-periodicity.

To calculate this false alarm probability, a model of the distribution of the residuals must be used. We ‘bracket’ a range of models, from uncorrelated white noise to long-range correlated noise. For white noise, we find the false alarm probability to be $Pr \ll 10^{-5}$ (Press et al., 1996). If the residuals have power-law behaviors with exponent in the range $2-4$ and long-range correlations characterized by a Hurst index $H \leq 0.7$, we have $Pr < 10^{-2}$ (Zhou and Sornette, 2002c).

The inset of Fig. 3(b) plots $\omega_{\text{fit}}$ with respect to $\omega_{\text{Lomb}}$. One can see that most pairs of ($\omega_{\text{Lomb}}, \omega_{\text{fit}}$) are overlapping on the line $y = x$, which indicates the consistency between $\omega_{\text{fit}}$ and $\omega_{\text{Lomb}}$. The other pairs are located on the line $y = 2x$. We can interpret these as a fundamental log-periodic component at $\omega$ and its harmonic component at $2\omega$.

The existence of harmonics of log-periodic components can be expected generically in log-periodic signals (Sornette, 1998; Gluzman and Sornette, 2002; Zhou and Sornette, 2002a, 2009) and has been documented in earlier studies both of financial time series and for other systems (Zhou et al., 2007). When the harmonics are well defined with close-to-integer ratios to a common fundamental frequency as is the case here, this is in general a diagnostic of a very significant log-periodic component.

3.1.3. Lomb analysis, non-parametric ($H, Q$) approach

In order to non-parametrically check the existence of log-periodicity by means of ($H, q$)-analysis, we take $f(x) = \ln p(t)$ and $x = t_c - t$ with $t_c = \text{Oct-10-2007}$ or $\text{Oct-25-2007}$ (the two observed peak dates of the indexes). For each pair of $(H, q)$ values, we calculate the ($H, q$)-derivative with a given $t_c$, on which we calculate the Lomb analysis. Fig. 4(a) illustrates the Lomb periodograms for both indexes with $t_c = \text{Oct-10-2007}$, $H = 0$, $q = 0.8$ and $t_c = \text{Oct-25-2007}$, $H = 0.5$, $q = 0.7$. The corresponding $D_q^{H} \ln p(t)$, defined by formula (5), is plotted with respect to $\ln(t_c - t)$ in the inset. The highest Lomb peak of the resultant periodogram has height $P_N^{\text{max}}$ and abscissa $\omega_{\text{Lomb}}$, both $P_N^{\text{max}}$ and $\omega_{\text{Lomb}}$ being functions of $H$ and $q$.

We scan a $21 \times 9$ rectangular grid in the ($H, q$) plane, with $H \in [-1, 1]$ and $q \in [0.1, 0.9]$, both in steps of 0.1. Fig. 4(b) illustrates the bivariate distribution of pairs ($\omega_{\text{Lomb}}, P_N^{\text{max}}$). The inset shows a simple histogram of
We observe the three most prominent clusters corresponding to SSEC with $t_c = \text{Oct-10-2007 (Oct-25-2007)}$ as $\omega_{\text{Lomb}}^0 = 0.86 \pm 0.16 (1.06 \pm 0.41)$, $\omega_{\text{Lomb}}^1 = 4.04 \pm 0.29 (4.33 \pm 0.31)$ and $\omega_{\text{Lomb}}^2 = 10.05 \pm 0.56 (10.59 \pm 0.44)$. For SZSC with $t_c = \text{Oct-10-2007 (Oct-25-2007)}$, we find the three most prominent clusters to be $\omega_{\text{Lomb}}^0 = 0.80 \pm 0.27 (0.75 \pm 0.32)$, $\omega_{\text{Lomb}}^1 = 4.21 \pm 0.38 (4.33 \pm 0.31)$, and $\omega_{\text{Lomb}}^2 = 9.29 \pm 0.29 (9.65 \pm 0.27)$.

The small value of $\omega_{\text{Lomb}}^0$ corresponds to a component with less than one full period within the interval of the $\ln(t_c - t)$ variable investigated here. According to extensive tests performed in synthetic time series (Huang et al., 2000), it should be interpreted as a spurious peak associated with the most probable partial oscillations of a noisy signal and/or to a residual global trend in the $(H, q)$-derivative. Then, $\omega_{\text{Lomb}}^1$ is identified as the fundamental angular log-frequency and $\omega_{\text{Lomb}}^2 \approx 2\omega_{\text{Lomb}}^1$ is interpreted as its second harmonic.

3.2. Back test of Chinese bubble from 2008 to 2009

3.2.1. LPPL fitting with varying window sizes

The main results of our analysis for the 2008-2009 bubble is illustrated in Fig. 5. The SSEC and SZSC index series between Oct-15-2008 and Jul-31-2009 have been calibrated to the LPPL formula given by Eq. (1) in shrinking and expanding windows. The shrinking windows are obtained by increasing the starting date $t_1$ from Oct-15-2008 to Apr-31-2009 with a step of five days and keeping the last day $t_2$ fixed at Jul-31-2009. The expanding windows are obtained by fixing the starting day $t_1$ at Nov-01-2008 and moving the ending day $t_2$ away from $t_1$ from Jun-01-2009 to Jul-31-2009 in increments of five days. The results are filtered by the LPPL conditions, resulting in 38 (respectively 13) fits for SSEC and 28 (respectively 13) fits for SZSC in shrinking windows (respectively increasing windows). Figures 5(a) and (c) illustrate six chosen fitting results of the shrinking windows for SSEC and SZSC and Figures 5(b) and (d) illustrate six fitting results of the increasing time intervals for SSEC and SZSC. The dark and light shadow boxes in the figures indicate 20%/80% and 5%/95% quantile range of values of the crash dates for the fits that survived filtering. Our calibration confirms the faster-than-exponential growth of the market index over this time interval, a clear diagnostic of the presence of a bubble. It also diagnoses that the critical time $t_c$ for the end of the bubble and the change of market regime lies in the interval August, 1-26, 2009 for SSEC and August, 3-9, 2009 for SZSC (20%/80% quantile confidence interval).

3.2.2. Lomb analysis, parametric approach

We use the Lomb spectral analysis technique to further investigate the log-periodic oscillations of Eq. (1) in both indexes from Oct-15-2008 to Jul-31-2009. First, we calculate the detrended residuals $r(t)$ in all the surviving LPPL windows and calculate the Lomb periodogram. The highest peak $P_N^{\text{max}}$ and its abscissa $\omega_{\text{Lomb}}$ are extracted from the residual Lomb periodograms to plot as points in Fig. 6(a). The inset plots $\omega_{\text{fit}}$ with respect to $\omega_{\text{Lomb}}$. One can see

[Figure 5 about here.]
that most pairs of \( (\omega_{\text{Lomb}}, \omega_{\text{fit}}) \) are overlapping on the line \( y = x \) and the other pairs are located on the line \( y = 2x \), confirming the existence of a strong log-periodic component. Fig. 6(a) shows four clusters of values for \( \omega_{\text{Lomb}} \) around 

\[
\omega^1_{\text{Lomb}} = 5 \pm 1, \quad \omega^2_{\text{Lomb}} = 10 \pm 1, \quad \omega^{3-4}_{\text{Lomb}} = 17.5 \pm 2 \quad \text{and} \quad \omega^5_{\text{Lomb}} = 27 \pm 2.
\]

The first value \( \omega^1_{\text{Lomb}} \) can be interpreted as the main angular log-frequency, while the others are the harmonics of order 2 to 5, with a rather large noise on \( \omega^{3-4}_{\text{Lomb}} \) and \( \omega^5_{\text{Lomb}} \).

### 3.2.3. Lomb analysis, non-parametric \((H, Q)\) approach

We next scan a \( 21 \times 9 \) rectangular grid in the \((H, q)\) plane with \( H \in [-1, 1] \) and \( q \in [0, 1] \), both in steps of 0.1, using \( t_C = \text{Jul-31-2009} \), a date four days before the peak of the SSEC index. We calculate the corresponding \((H, q)\)-derivatives \( D^H_q \ln p(t) \), defined by formula (5), for this set of \( H \) and \( q \) values. For each obtained \( D^H_q \ln p(t) \), we estimate the Lomb periodogram and plot the highest Lomb peak \( P^\text{max}_N \) as a function of its abscissa \( \omega_{\text{Lomb}} \) in Fig. 6(b). The inset shows the simple histogram of \( \omega_{\text{Lomb}} \).

For the Shanghai index (SSEC), the three most prominent clusters correspond to \( \omega^0_{\text{Lomb}} = 1.12 \pm 0.66, \omega^1_{\text{Lomb}} = 4.0 \pm 1 \) and \( \omega^3_{\text{Lomb}} = 17.6 \pm 2.7 \). We interpret the first cluster, \( \omega^0_{\text{Lomb}} \), as due to the noise decorating the power-law. This is because it corresponds to a component with less than one full period within the interval of the \( \ln(t_c-t) \) variable investigated here. According to extensive tests performed in synthetic time series (Huang et al., 2000), it is a spurious peak associated the most probable partial oscillations of a noisy signal. We identify the second cluster, \( \omega^1_{\text{Lomb}} \), as the fundamental angular log-frequency for SSEC. The second cluster around \( \omega^3_{\text{Lomb}} \) is compatible with interpreting it as being the third harmonics. It is notable that the second harmonic is not visible in this distribution, a phenomenon which has been reported for other systems (Johansen et al., 2000b) and can be rationalized from a renormalization group analysis (Gluzman and Sornette, 2002).

![Figure 6 about here.]

### 3.3. Unit root tests of the 2005-2007 and 2008-2009 Chinese bubbles

We apply unit root tests to the series of residuals for each \([t_1, t_2]\) interval, where the residuals are calculated by subtracting the model from the observations. The goal is to investigate the stationarity of the residuals to determine if a mean-reversal Ornstein-Uhlenbeck process is a good model for them. The null hypothesis of the unit-root test is that the series being tested is non-stationary. Rejection of the null hypothesis, then, implies that the series (or residuals in our case) is stationary. Refer to Section 2.4 for further details.

We calibrate the LPPL model Eq. (1) to the SSEC and SZSC indexes in two representative intervals. The first interval is from Dec-01-2005 to Oct-10-2007 and the other is from Oct-15-2008 to Jul-31-2009. We scan each interval with growing and shrinking windows, as described above, and report the fraction \( P_{\text{LPPL}} \) of these different windows that meet the LPPL conditions, Eq. (2). We then calculate the conditional probability that, out of the fraction \( P_{\text{LPPL}} \) of windows that satisfy the LPPL condition, the null hypothesis of non-stationarity is rejected for the residuals. Results of these tests are shown in Table 1.
For the time interval from Dec-01-2005 to Oct-10-2007 for the SSEC index (respectively the SZSC index), the fraction $P_{LPPL} = 56.9\%$ (respectively $67.6\%$) of the windows satisfy the LPPL conditions. All of the fitting residuals of both indexes reject the null hypothesis at significance level 0.01 based on the two tests, implying that the residuals are stationary. For the fraction of windows which satisfy the LPPL conditions, the fitting residuals of the SSEC (respectively SZSC) index can be regarded as generated by a stationary process at the 99.9\% (respectively 99\%) confidence level.

For the time interval from Oct-15-2008 to Jul-31-2009, we find that a fraction $P_{LPPL} = 94.4\%$ (respectively $74.7\%$) of the windows satisfy the LPPL conditions for SSEC (respectively SZSC). All of the fitting residuals of both indexes reject the null hypothesis at significance level 0.001 based on the two tests, implying that the residuals are stationary. For the fraction of windows which satisfy the LPPL conditions, the fitting residuals of both indexes can be considered as a stationary process at the 99.9\% confidence level.

4. Prior prediction of both crashes

We make the title of this section pleonastic to emphasize that we predicted both crashes with our techniques before the actual dates of the observed peaks in the two indexes. The previous sections have presented more thorough ‘post-mortem’ analyses performed after the observed crashes. This section documents the specific but simpler predictions that we announced in advance.

4.1. 2005-2007 bubble

Two of us (WXZ and DS) performed a LPPL analysis in early September 2007, which led to (i) a diagnostic of an on-going bubble and (ii) the prediction of the end of the bubble in early 2008. One of us (DS) communicated this prediction on October 18, 2007 at a prominent hedge-fund conference in Stockholm. The participants, managers of top global macro hedge-funds, constitute arguably the best proxy for the academic idealization of “rational investors” with access to almost unlimited resources and with the largest existing incentives to motivate themselves to acquire all possible relevant information and trade accordingly. These participants responded that the predicted change of regime was impossible because, in their opinion, the Chinese government would prevent any turmoil on the Chinese stock market until at least the end of the Olympic Games in Beijing (August 2008). After the communication of October 18, 2007, the Hang Seng China Enterprises Index (HSCEI) reached the historical high 20609.10 on 2 November 2007. Afterwards, the first valley $HSCEI=15460.72$ (-25\% from historical high) was reached on 22 Nov 2007 and the bottom $HSCEI=4792.37$ (-77\% from historical high) was on 29 Oct 2008. On 19 March 2008, $HSCEI=11379.91$ was another deep valley. These drops occurred after a six-fold appreciation of the Chinese market from mid-2005 to October 2007.
4.2. 2008-2009 bubble

4.2.1. The announcement of the prediction

On 10 July 2009, we submitted our prediction online to the arXiv.org (Bastiaensen et al., 2009), in which we gave the 20%/80% (respectively 10%/90%) quantiles of the projected crash dates to be July 17-27, 2009 (respectively July 10 - August 10, 2009). This corresponds to a 60% (respectively 80%) probability that the end of the bubble occurs and that the change of regime starts in the interval July 17-27, 2009 (respectively July 10 - August 10, 2009). Redoing the analysis 5 days later with $t_2 = July 14, 2009$, the predictions tightened up with a 80% probability for the change of regime to start between July 19 and August 3, 2009 (unpublished).

The following paragraph and figure 7 are reproduced from the online (and un-refereed) prediction, which is available in its initial form at the corresponding URL (Bastiaensen et al., 2009).

The result of the analysis is summarized below in the Figure. We analyzed the Shanghai SSE Composite Index time series between October 15, 2008 and July 9, 2009. We increased the starting date of the LPPL analysis in steps of 15 days while keeping the ending date fixed, resulting in 10 fits. The figure shows observations of the SSEC Index as black dots (joined by straight lines) and the LPPL fits as smooth lines until the last day of analysis. The y-axis is logarithmically scaled, so that an exponential function would appear as a straight line and a power law function with a finite-time singularity would appear with a slightly upward curvature. Note that the LPPL fits to the observations exhibit this slightly upward curvature. The vertical and horizontal dashed lines indicate the date and price of the highest price observed, July 6, 2009. Extrapolations of the fits to 100 days beyond July 9, 2009 are shown as lighter dashed lines. The darker shaded box with diagonal hatching indicates the 20%/80% quantiles of the projected crash dates, July 17-27, 2009. The lighter shaded box with horizontal hatching indicates the range of all 10 projected crash dates, July 10 - August 10, 2009. These two shaded boxes indicate the most probable times (with the associated confidence levels) to expect peak and possible subsequent crash of the Index. The parameters of the fit confirm the faster-than-exponential growth of the Shanghai SSE Composite Index over this time interval, a clear diagnostic of the presence of a bubble.

4.2.2. What actually happened

On July 29, 2009, Chinese stocks suffered their steepest drop since November 2008, with an intraday bottom of more than 8% and an open-to-close loss of more than 5%. The market rebounded with a peak on August 4, 2009 before plummeting the following weeks. The SSEC slumped 22 percent in August, the biggest decline among 89 benchmark indices tracked world wide by Bloomberg, in stark contrast with being the no. 1 performing index during the first half of this year. These striking facts show the detachment of the Chinese equity market from other markets.
This bubble was probably nucleated by China’s central government’s reaction to the global financial crisis. Besides announcing the huge stimulus plan on 9 November 2008, a loose monetary policy and regulations caused massive new loan issuance as shown in Fig. 8. With overproduction and lower global demands, analysts estimate that up to 50% of the increase in credit was used to speculate in equities, property and commodities. Rumours of asset bubbles were widely heard in the market, but when or if they might crash was unknown as usual.

Note that the change of regime in the SSEC occurred while the total loan of financial institutions was still growing at close to its peak YoY 35% monthly rate. This illustrates that the change of regime has occurred in absence of any significant modification of the economic and financial conditions or any visible driving force. This observation, which should be surprising to most economists and analysts, is fully expected from the mathematical and statistical physics of bifurcations and phase transitions on which our LPPL methodology is based: a possibly vanishingly small change of some control parameter may lead to a macroscopic bifurcation or phase transition. Rather than leading to an absence of predictability, the accelerating susceptibility of the system associated with the approach towards the critical point can be diagnosed, as we have shown. The very clear change of regime documented here provides a case-in-point demonstrating this concept of an emergent rupture point characterizing the end of the bubble.

Figure 9 presents the evolution with time of the close-open statistic introduced in subsection 2.5 over the period from Jan. 2007 to August 25, 2009. The low (respectively high) values of the index correlate well with the ascending (respectively descending) trend of the market. One can also observe the recent remarkably abrupt jump upward of the close-open statistic at the time scale $T = 10$ days, confirming the existence of a sudden change of regime.

These events unfolded in a rather bullish atmosphere for the Chinese stock markets. For instance, Bloomberg reported on July 30, 2009 that billionaire investor Kenneth Fisher emphasized the great success of China’s economy compared to the rest of the World and that speculation that the “Chinese government will limit bank loans is unfounded.” Anecdotal sampling of comments on Chinese online forums suggests a majority of doubters until August 12, after which a majority endorsed the notion of a change of regime. Several commentators stressed again that predictions of Chinese stock markets cannot be correct since China’s stock markets are heavily influenced by policies (known as a policy market). These comments are similar to the disbelief of hedge-fund managers mentioned in subsection 4.1 concerning the prediction of the change of regime at the end of 2007, before the 2008 Beijing Olympics.

---

$^2$see, e.g., BNP Paribas FX Weekly Strategist: China Lending Support (31 July 2009); RBS, Local Markets Asia, Alert China: A savings glut is causing problems (3 July 2009)
5. Discussion

We have performed a detailed analysis of two financial bubbles in the Chinese stock markets by calibrating the LPPL formula (1) to two important Chinese stock indexes, Shanghai (SSEC) and Shenzhen (SZSC) from May 2005 to July 2009. Bubbles with the property of faster-than-exponential price increase decorated by logarithmic oscillations are observed in two distinct time intervals within the period of investigation for both indexes. The first bubble formed in the middle of 2005 and burst in October 2007. The other bubble began in November 2008 and reached a peak in early August 2009.

Our back tests of both bubbles find that the LPPL model describes well the behavior of faster-than-exponential increase corrected by logarithmic oscillations in both market indexes. The evidence for the presence of log-periodicity is provided by applying Lomb spectral analysis on the detrended residuals and \((H,q)\)-derivative of market indexes. Unit-root tests, including the Phillips-Perron test and the Dickey-Fuller test, on the LPPL fitting residuals confirm the O-U property and, thus, stationarity in the residuals, which is in good agreement with the consistent model of ‘explosive’ financial bubbles (Lin et al., 2009).

While the present paper presents post-mortem analyses, we emphasize that we predicted the presence and expected critical date \(t_c\) of both bubbles in advance of their demise (Sornette, 2007; Bastiaansen et al., 2009). These two successes prolong the series of favorable outcomes following the prediction of the peak in mid-2004 of the real-estate bubble in the UK by two of us (Zhou and Sornette, 2003a), of the peak in mid-2006 of the US housing bubble by two of us (Zhou and Sornette, 2006b) and of the peak in July 2008 of the global oil bubble by three of us (Sornette et al., 2009).

But not all predictions based on the present methodology have fared so well. In particular, Lux (2009) and Rosser (2008) have raised severe objections, following the failure of the well-publicized prediction published in 2002 that the U.S. stock market would follow a downward log-periodic pattern (Sornette and Zhou, 2002). How can one make sense of these contradictory claims? We summarize the present state of the art as follows.

1. **Prediction of the end of bubbles should not be confused with predictions based on extrapolation, such as those associated with antibubbles.** There is a confusion between predicting crashes, on the one hand, and predicting the continuation of an “antibubble” bearish regime, on the other hand. It seems that both Lux (2009) and Rosser (2008) amalgamate these two issues, when they focus on the failure of the antibubble prediction in Sornette and Zhou (2002) and conclude that “Sornette and his collaborators failed to forecast future crashes.” There is indeed a fundamental difference between, on the one hand, (i) the prediction of the end of a bubble analyzed here, which is characterized by its critical time \(t_c\) and, on the other hand, (ii) the extrapolation of an “antibubble” pattern. This difference is similar to that between (i) the prediction of the approximate parturition time of a foetus on the basis of the recording of key variables obtained during its maturation in the uterus of his mother and (ii) the prediction of the death of this individual later in old age from an extrapolation of medical variables recorded during his adult life. The former (i) is associated with the maturation phase (the
financial bubble versus the uterus-foetus development). It has a rather well-defined critical time which signals the transition to a new regime (crash/stagnation/bearish versus birth). In contrast, the latter (ii) may be influenced by a variety of factors, particularly exogenous, which may shorten or lengthen the life of the antibubble or of the individual. One should thus separate the statistics of successes and failures in the prediction of bubbles on the one hand and in the prediction of continuation of LPPL antibubble patterns on the other hand. This was the spirit of the experiment proposed by Sornette and Zhou (2002) to test the distinct hypothesis that bearish regimes following a market peak could be predicted when associated with LPPL patterns.

2. **Intrinsic limits of the prediction of the end of an antibubble.** For the reasons just mentioned, it is still an open problem to determine when an antibubble ends. Our methods show that one cannot avoid a delay of about 6 months before identifying the end of an antibubble (Zhou and Sornette, 2005). This is a partial explanation for the failure of the 2002 antibubble prediction (Sornette and Zhou, 2002).

3. **Track record of the antibubble method.** However, one should not forget that, taken as a distinct class separated from that of diagnosing bubbles and their ends, the predictions based on the antibubble method can count several past successes: (a) on the Nikkei antibubble (Johansen and Sornette, 1999b, 2000) and (b) on the Chinese stock market antibubble (Zhou and Sornette, 2004), in addition to the failure mentioned above. This track record is insufficient to conclude. More tests in real time should be performed and rigorous methods developed to assess the statistical significance of short catalogs of success/failure predictions can be applied, based on “roulette” approaches (see Chapter 9 in Sornette (2003b), Bayes’ theorem (Johansen and Sornette, 2000) and Neyman-Pearson or error diagrams (Molchan, 1990, 1997).

4. **What we learned from the antibubble prediction failure.** Lux (2009) and Rosser (2008) are right to stress that the 2002 antibubble prediction of Sornette and Zhou (2002) failed. However, a post-mortem analysis in Zhou and Sornette (2005) has revealed an interesting fact. While the prediction failed when the S&P500 is valued in U.S. dollars, it becomes quite accurate when expressed in euro or British pounds (Zhou and Sornette, 2005). A plausible interpretation would be that the energetic Fed monetary policy of decreasing its lead rate from 6.5% in 2000 to 1% in 2003 has boosted the stock market in local currency from 2003 on, but has degraded the dollar, so that the net effect was that the value of the US stock market from an international reference point was unfolding as expected from the analysis of Sornette and Zhou (2002). We do not claim that this changed the failure into a success. Instead, it illustrates the effect of monetary feedbacks that have to be included in improved models incorporating fundamental factors, for instance in the spirit of Zhou and Sornette (2006a).

5. **Track record for diagnosing bubbles and their ends.** Our group has announced advanced prediction (not just in retrospect) of bubbles and their end (often a crash). The status of these predictions as of 2002 has been discussed in details in Chapter 9 of Sornette (2003b)’s book. As mentioned above, subsequent successes include the predictions of the peak in mid-2004 of the real-estate bubble in the UK by Zhou and Sornette (2003a), of the peak in mid-2006 of the US housing bubble by Zhou and Sornette (2006b) and of the peak in July 2008 of the global oil bubble by Sornette et al. (2009). The present analysis on two bubbles in the Chinese market provide
additional evidence for the relevance of LPPL patterns in the diagnostic of bubbles.

6. **Recent improvements in methodology.** While the core model (or forecasting system) has not changed much since the late 1990s, several new developments used here allows us to quantify more accurately both the reliability and the uncertainties. These improvements include multi-window analysis, probability estimates, and a consistent LPPL rational expectation model with mean-reverting residuals. Note that the more recent bubble in the Chinese indexes was detected and its end or change of regime was predicted independently by two groups (the first four authors from academia on the one hand and the last two authors from industry on the other hand) with similar results, showing that the model has been well-documented and can be replicated by industrial practitioners. In addition, we stress that the method relies essentially on the competition between the positive feedback loop of higher return anticipations competing with negative feedback spirals of crash expectations (Ide and Sornette, 2002), which is at the origin of the acceleration oscillations. In the spirit of Lux and Marchesi (1999) and of Gallegati et al. (2008), this is accounted for in heterogeneous agent models by including nonlinear fundamental investment styles competing with nonlinear momentum trading styles (Ide and Sornette, 2002). The initial JLS model of Johansen et al. (1999, 2000a) was based on a conventional neoclassical model assuming a homogeneous rational agent, but it also enriched this set-up by introducing heterogeneous noise traders driving a crash hazard rate. More recently, Lin et al. (2009) have considered an alternative framework which extends the JSL model to account for behavioral herding by using a behavioral stochastic discount factor approach, with self-consistent mean-reversal residuals.

In conclusion, given all the above, we feel this technique is the basis of a prediction platform, which we are actively developing, motivated by the conviction that this is the only way to make scientific progress in this delicate and crucial domain of great societal importance, as illustrated by the 2007-2009 financial and economic crisis (Sornette and Woodard, 2009).

**Acknowledgments:** The authors would like to thank Li Lin and Liang Guo for useful discussions. This work was partially supported by the Shanghai Educational Development Foundation (2008SG29) and the Chinese Program for New Century Excellent Talents in University (NCET-07-0288). We also acknowledge financial support from the ETH Competence Center Coping with Crises in Complex Socio-Economic Systems (CCSS) through ETH Research Grant CH1-01-08-2. Much appreciation goes to Prof. Jan Ryckebusch of the University of Ghent, Subatomic Physics Department, for useful discussions.

**References**


Figure 1: Evolution of the price trajectories of the SSEC index and the SZSC index over the time interval of this analysis. The solid red lines indicate the dates of the respective public announcement of our predictions for the two bubbles (October 18, 2007 and July 10, 2009) while the grey zones indicate the 20%/80% confidence intervals for which we forecasted the change of regime. Final closing prices shown in these plots are 10,614.3 (SZSC) and 2683.72 (SSEC) from September 1, 2009.
Figure 2: Daily trajectory of the logarithmic SSEC (a,b) and SZSC (c,d) index from May-01-2005 to Oct-18-2008 (dots) and fits to the LPPL formula (1). The dark and light shadow box indicate 20/80% and 5/95% quantile range of values of the crash dates for the fits, respectively. The two dashed lines correspond to the minimum date of $t_1$ and the maximum date of $t_2$. (a) Examples of fitting to shrinking windows with varied $t_1$ and fixed $t_2 = Oct-10-2007$ for SSEC. The six fitting illustrations are corresponding to $t_1 = Sep-30-2005$, Dec-05-2005, Feb-13-2006, Apr-24-2006, Jan-15-2007, and Mar-12-2007. (b) Examples of fitting to expanding windows with fixed $t_1 = Dec-01-2005$ and varied $t_2$ for SSEC. The six fitting illustrations are associated with $t_2 = Aug-20-2007$, Aug-29-2007, Sep-07-2007, Sep-17-2007, Sep-26-2007, Oct-05-2007. (c) Examples of fitting to shrinking windows with varied $t_1$ and fixed $t_2 = Oct-10-2007$ for SZSC. The six fitting illustrations are corresponding to $t_1 = Sep-30-2005$, Dec-12-2006, Feb-24-2006, May-12-2006, Jan-09-2007, and Apr-13-2007. (d) Examples of fitting to expanding windows with fixed $t_1 = Dec-01-2005$ and varied $t_2$ for SZSC. The six fitting illustrations are associated with $t_2 = Aug-01-2007$, Aug-10-2007, Aug-24-2007, Sep-07-2007, Sep-21-2007, Oct-08-2007.
Figure 3: Lomb tests of the detrending residuals $r(t)$ for SSEC and SZSC. The residuals are obtained from Eq. (4) by substituting different survival LPPL calibrating windows with the corresponding fitting results including $t_c$, $m$, and $A$. (a) Lomb periodograms for four typical examples, which are presented in the legend. The time periods followed the index names represent the LPPL calibrating windows. The inset illustrates the corresponding residuals $r(t)$ as a function of $\ln(t_c - t)$. (b) Bivariate distribution of pairs $(\omega_{\text{Lomb}}, P_{\text{max}})$ for different LPPL calibrating intervals. Each point in the figure stands for the highest peak and its associated angular log-frequency in the Lomb periodogram of a given detrended residual series. The inset shows $\omega_{\text{fit}}$ as a function of $\omega_{\text{Lomb}}$. 
Figure 4: Lomb tests of \((H,q)\)-derivative of logarithmic indexes. (a) Lomb periodograms of \(D^q_H \ln p(t)\) for four typical examples, which are \(t_c = \text{Oct-10-2007} \) with \(H = 0\) and \(q = 0.8\) for SSEC, \(t_c = \text{Oct-25-2007} \) with \(H = 0.5\) and \(q = 0.7\) for SSEC, \(t_c = \text{Oct-10-2007} \) with \(H = 0\) and \(q = 0.8\) for SZSC, and \(t_c = \text{Oct-25-2007} \) with \(H = 0.5\) and \(q = 0.7\) for SZSC, respectively. The inset shows the corresponding plots of \(D^q_H \ln p(t)\) as a function of \(\ln(t_c - t)\). (b) Bivariate distribution of pairs \((\omega_{\text{Lomb}}, P_{\text{max}})\) for different pairs of \((H,q)\). Each point corresponds the highest Lomb peak and its associated angular log-frequency in the Lomb periodogram of the \((H,q)\)-derivative of logarithmically indexes for a given pair \((H,q)\). The inset shows the empirical frequency distribution of \(\omega_{\text{Lomb}}\).
Figure 5: Daily trajectory of the logarithmic SSEC (a,b) and SZSC (c,d) index from Sep-01-2008 to Jul-31-2009 (dots) and fits to the LPPL formula (1). The dark and light shadow box indicate 20/80% and 5/95% quantile range of values of the crash dates for the fits, respectively. The two dashed lines correspond to the minimum date of $t_1$ and the fixed date of $t_2$.  


(c) Examples of fitting to shrinking windows with varied $t_1$ and fixed $t_2 = \text{Jul-31-2009}$ for SZSC. The six fitting illustrations are corresponding to $t_1 = \text{Oct-15-2008}$, Nov-03-2008, Nov-26-2008, Dec-19-2008, Jan-14-2008, and Jan-23-2008.  

Figure 6: Detection of log-periodicity in the Chinese bubble from 2008 to 2009. (a) Plots of $P_{\text{max}}^N$ with respect to $\omega_{\text{Lomb}}$ for different LPPL calibrating windows. The inset illustrates the dependence of $\omega_{\text{fit}}$ on $\omega_{\text{Lomb}}$. (b) Bivariate distribution of pairs ($\omega_{\text{Lomb}}, P_{\text{max}}^N$) for different pairs of $(H, q)$ of $(H, q)$-derivatives $D_q^H \ln p(t)$, defined by formula (5). The inset depicts the empirical frequency distribution of $\omega_{\text{Lomb}}$. 

24
Figure 7: Shanghai Composite Index with LPPL result, as presented in the July 10, 2009 arXiv.org submission of Bastiaensen et al. (2009).
Figure 8: (left axis, dots) SSEC compared with Total Loans of Financial Institutions as reported by The People’s Bank of China (“Summary of Sources & Uses of Funds of Financial Institutions” http://www.pbc.gov.cn/english) (right axis, solid line) YoY % monthly change. This shows graphically the widespread belief that the credit growth has fueled the last Chinese equity bubble.
Figure 9: Left scale: SSE Composite index from Jan. 2007 to August 25, 2009 (closing price 2980.10). Right scale: fraction of days with negative (close-open) in moving windows of length $T = 10$ days (continuous blue line), $T = 20$ days (dashed green line) and $T = 30$ days (dotted-dashed red line).
Table 1: Unit-root tests on the LPPL fitting residuals for SSEC and SZSC index in our two calibrating ranges. $p_{LPPL}$ denotes the fraction of windows that satisfy the LPPL condition. $p_{StationaryResi|LPPL}$ denotes the conditional probability that, out of the fraction $p_{LPPL}$ of windows that satisfy the LPPL condition, the null unit test for non-stationarity is rejected for the residuals.

<table>
<thead>
<tr>
<th>Index</th>
<th>Calibrating Range</th>
<th>Number of Windows</th>
<th>$p_{LPPL}$</th>
<th>Signif. Level</th>
<th>Percentage of Rejecting $H_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha = 0.01$</td>
<td>Phillips-Perron</td>
</tr>
<tr>
<td>SSEC</td>
<td>2005/12/01-2007/10/10</td>
<td>146</td>
<td>56.9%</td>
<td>$\alpha = 0.01$</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha = 0.001$</td>
<td>95.2%</td>
</tr>
<tr>
<td>SZSC</td>
<td>2005/12/01-2007/10/10</td>
<td>139</td>
<td>67.6%</td>
<td>$\alpha = 0.01$</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha = 0.001$</td>
<td>81.3%</td>
</tr>
<tr>
<td>SSEC</td>
<td>2008/10/15-2009/07/31</td>
<td>54</td>
<td>94.4%</td>
<td>$\alpha = 0.01$</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha = 0.001$</td>
<td>100.0%</td>
</tr>
<tr>
<td>SZSC</td>
<td>2008/10/15-2009/07/31</td>
<td>54</td>
<td>74.7%</td>
<td>$\alpha = 0.01$</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha = 0.001$</td>
<td>100.0%</td>
</tr>
</tbody>
</table>