Frank Schweitzer, Jörg Zimmermann:
Communication and Self-Organization in Complex Systems: A Basic Approach
in: *Knowledge, Complexity and Innovation Systems* (Eds. M. M. Fischer, J. Fröhlich)
(Advances in Spatial Sciences) Springer, Berlin 2001, pp. 275–296

Communication and Self-Organization in Complex Systems: A Basic Approach

Frank Schweitzer, Jörg Zimmermann

Real World Computing Partnership - Theoretical Foundation GMD Laboratory
Schloss Birlinghoven, D-53754 Sankt Augustin, Germany

e-mail: schweitzer@gmd.de

Abstract

Complex Systems usually consist of a large number of interacting components; therefore multi-agent models can play a valuable role in exploring and simulating their dynamic behavior. In the first part of this paper we address some conceptual issues associated with designing agent-based models for complex systems. In the second part we investigate the dynamics of a "minimalistic" multi-agent system, where spatially distributed agents communicate via a spatio-temporal multicomponent "communication field". For the example of a binary choice problem we find the emergence of a majority/minority relation among the agents. Moreover, the minority and the majority concentrate in particular spatial domains; thus we observe a spatial coordination of decisions as the result of a self-organization process based on the exchange of information. We also show by means of computer simulations that the subpopulation with the more efficient communication will have a better chance to become the majority in the system.

1 Introduction

The emergence of complex behavior in a system consisting of interacting elements is among the most fascinating phenomena of our world. Examples can be found in almost every field of today's scientific interest, ranging from coherent pattern formation in physical and chemical systems (Feistel and Ebeling, 1989; Cladis and Palffy-Muhoray, 1995), to the motion of swarms of animals in biology (DeAngelis and Gross, 1992) and the behavior of social groups (Weidlich, 1991; Vallacher and Nowak, 1994).

In the life and social sciences, one is usually convinced that the evolution of social systems is determined by numerous factors, such as cultural, sociological, economic, political, ecological etc.

However, in recent years, the development of the interdisciplinary field "science of complexity" has lead to the insight, that complex dynamic processes may also result from simple interactions. Moreover, at a certain level of abstraction, one can also find many common features between complex structures in very different fields (Schweitzer, 1997a).

The recent progress in the understanding of non-equilibrium phenomena in complex systems has initiated a lot of activities in analyzing, modelling and simulating "living" systems by means of methods from *statistical physics*. This does not hold only for biological systems (Parisi *et al.*, 1998), but also for social and economic systems (Dendrinos and Sonis, 1990; Weidlich, 1991; Levenstein *et al.*, 1992, Helbing, 1995; Kacperski and Hołyst, 1996; Allen, 1998). Already in the early seventies, physicists have realized that these methods can help to understand social phenomena, such as opinion formation, migration, and settlement formation (Weidlich, 1972, Weidlich and Haag, 1983).

Very recently, physicists have also focussed their interdisciplinary interests to particular economic processes, such as trading, market dynamics, decision processes, economic agglomeration, or company growth (Bruckner *et al.*, 1994; Levy *et al.*, 1995, Galam, 1997, Schweitzer, 1998, Lee *et al.*, 1998).The joint efforts of many research groups spread over the world eventually lead to the establishment of *econophysics* – a young and fast growing field, the potential importance of which can be hardly overestimated (Mantegna and Stanley, 2000, Schweitzer and Helbing, 2000). Even with the analysis of financial time series as its current focus, econophysics is meant to be a more comprehensive enterprise. Basically, it focusses on the question how and to what extent methods from statistical physics can be used for the analysis, modeling, simulation, and optimization of economic systems.

In order to make this enterprise a successful one, a broad and openminded dialog is needed between physics, economics and the social sciences. This dialog should help to overcome the gap between these different disciplines (i) by providing methods from the natural sciences, which could be adapted to solving problems in social or economic fields, and (ii) by increasing among natural scientists the sensitivity for problems in the fields of economics and the social sciences.

This paper wants to contribute to this discussion in a twofold way: In the first part, we will address general problems in defining and simulating complex systems which are also of relevance in an economic context. The second part deals with a basic model to simulate communication and self-organization in an agent system via the exchange of information. "Basic" means here that we want to focus only on particular interactions among the agents, with no attempt to model a specific socio-economic system most realistically.

Instead, we will concentrate on the spatial coordination of decisions among the agents dependent on the various information received. We will consider certain important features, namely the exchange of information with a finite velocity, the existence of a memory, or the local heterogeneity of available information, while other features within this basic approach cannot be and will be not considered.

2 Complex systems and self-organization

Despite many efforts, there is no commonly accepted definition of a complex system (Ebeling *et al.*, 1998). Heuristic approaches basically focus on the interaction between ("microscopic") subsystems and the emergence of new qualities at the ("macroscopic") system level, e.g.

- "Complex systems are systems with multiple interacting components whose behavior cannot be simply inferred from the behavior of the components."[1]
- "By complex system, it is meant a system comprised of a (usually large) number of (usually strongly) interacting entities, processes, or agents, the understanding of which requires the development, or the use of, new scientific tools, nonlinear models, out-of equilibrium descriptions and computer simulations."[2]

The latter description already raises the question which kind of scientific methodologies or tools would be sufficient to investigate complex systems. It is now commonly accepted that *computer simulations* will play a major role in this enterprise, as a third methodology in addition to formal theories and empirical studies (experiments).

Among the different simulation approaches developed within the last twenty years, the *multi-agent* approach seems to be most promising and versatile. While agent models have originally been developed in the *Artificial Life community* (Maes, 1991; Meyer and Wilson, 1991)they recently turned out to be a suitable tool in various scientific fields, ranging from ecology to engineering (DeAngelis and Gross, 1992; Lam and Naroditsky, 1992), and especially in economics and the social sciences (Andersen *et al.*, 1988; Troitzsch *et al.*, 1996; Hegselmann *et al.*, 1996; Arthur *et al.*, 1997; Silverberg, 1997; Schweitzer and Silverberg, 1998). However, agent-based models are not restricted to the social and life sciences, they are also useful in natural sciences in cases where continuous approximations are less appropriate. Here, the discrete approaches to structure formation range from lattice gas models in hydrodynamics to stochastic cellular automata and models of active walkers or active Brownian particles (Boon, 1991; Crutchfield and Hanson, 1993; Lam, 1995; Schimansky-Geier *et al.*, 1995, Schweitzer, 1997d)

The advantage of an agent-based approach is given by the fact that it is applicable also in cases where only a small number of actors (particles, agents) govern the further evolution. Here deterministic approaches or mean-field equations are not sufficient to describe the behavior of the complex system. Instead, the influence of history, i.e. irreversibility, path dependence, the occurence of random events/stochastic fluctuations play a considerable role.

In general, agents are regarded as relatively autonomous entities which may represent local processes, individuals, species, agglomerates, chemical components, firms, etc. These entities have a

[1] New England Complex Systems Institute
[2] Journal *Advances in Complex Systems*

set of different rules to interact with each other. Which of the rules applies for a specific case, may also depend on local variables, which in turn can be influenced by the (inter)action of the agents. In this paper, an *agent* is meant to be a subunit with an "intermediate" complexity. This means on one hand that he/she is not assumed as a "physical" particle only reacting to external forces, but on the other hand should not already have the same complex capabilities as the whole system. Instead the agent should be characterized by some "activity", which of course may depend on the system the model is applied to – examples are treated in more detail in this paper.

A *multi-agent system* (MAS) then may consist of a *large number* of agents, which can be also of *different types*. The complex behavior of the multi-agent system as a whole basically depends (i) on the complexity of the agent (i.e. the range of possible actions), (ii) on the complexity of the interaction. The latter one is considered even more important, since it has been shown e.g. in physical systems that already from the interaction of *simple* entities a rich variety of structures can emerge. The interactions between the agents may usually occur on *different spatial and temporal scales*. That means, in addition to local or spatially restricted interactions which may occur only at specific locations or if agents are in closer distance, we also have to consider global interactions, where all agents are involved. Further, the time scale of interactions is of significant importance. Whereas some interactions occur rather frequently, i.e. on a shorter time scale, others become effective only over a long time. A third distinction to be mentioned is between direct and indirect interactions. The latter one occurs e.g. if agents use a common resource that can be exhausted in the course of time. This way, the actions of all agents are indirectly coupled via the resource and its current availability further provides some information about the cumulative activity of others.

As the result of the different interactions, we may observe different kinds of collective dynamics and the *emergence of new system properties* not readily predicted from the basic equations. This process is often denoted as *self-organization*, i.e. "the process by which individual subunits achieve, through their cooperative interactions, states characterized by new, emergent properties transcending the properties of their constitutive parts." (Biebricher *et al.*, 1995)

However, whether these emergent properties occur or not depends of course not only on the properties of the agents and their interactions, but also on suitable external conditions, such as global boundary conditions, the in/outflux of resources (matter, energy, information). A description which tries to include these conditions is given by the following heuristic definition: "Self-organization is defined as spontaneous formation, evolution and differentiation of complex order structures forming in non-linear dynamic systems by way of feedback mechanisms involving the elements of the systems, when these systems have passed a critical distance from the statical equilibrium as a result of the influx of unspecific energy, matter or information." (SFB 230, 1994)

In this sense, the self-organized structure formation can be considered as the opposite of a hierarchical design of structures which basically proceeds *from top down to bottom*: here, structures are *originated* bottom up, leading to an emerging hierarchy, where the structure of the "higher" level appears as a new quality of the system (Haken, 1978; Darley, 1994). For the prediction of these global qualities from local interactions fundamental limitations exist which are discussed e.g. in chaos theory. Moreover, stochastic fluctuations also give unlikely events a certain chance to occur, which in turn affects the real history of the system. This means, the properties of complex systems cannot be determined by a hierarchy of conditions, the system creates its complexity in the course of evolution with respect to its global constraints. Considering, that also the boundary conditions may evolve and new degrees of freedom appear, co–evolutionary processes become important, and the evolution may occur on a qualitatively new level.

3 Complex versus minimalistic agents

In order to gain insight into the interplay between microscopic interactions and macroscopic features in complex systems, it is important to find a level of description, which on one hand considers specific features of the system and is suitable to reflect the origination of new qualities, but on the other hand is not flooded with microscopic details. As pointed out above, the multi-agent approach may provide a suitable tool for describing and simulating complex systems – however, this raises the question how to design the agent's features appropriately. There is no general answer to this, since the agent design strongly depends on the system under consideration. Nevertheless, some general remarks can be applied.

Let us take the example of agent-based computational economics, which is meant to describe and to simulate the economic interaction between "agents" which could be either individuals, or firms (Föllmer, 1974; Holland and Miller, 1991; Lane, 1992; Arthur, 1993; Epstein and Axtell, 1996; Kirman, 1993). One of the standard paradigms of neoclassical economic theory, the *rational agent* model is based on the assumption of the agent's *complete knowledge* of all possible actions and their outcomes or a known probability distribution over outcomes, and the *common knowledge assumption*, i.e. that the agent knows that all other agents know exactly what he/she knows and are equally rational (Silverberg and Verspagen, 1994). In this particular form, the rational agent is just one example of a *complex agent* with either knowledge based or behavior based rules (Maes, 1991), performing complex actions, such as rational choices or BDI (belief-desire-intention) (Müller *et al.*, 1997). Moreover, in many cases the complex agent is capable of specialization, learning, genetic evolution, etc.

Different problems are involved in such a complex agent design; two of them shall be shortly mentioned here. A problem blinded out quite often is concerned with the *information flow* in the system. The common knowledge assumption implecitely demands an infinitely fast, loss-free and error-free distribution of information in the whole system, but does not give a hint how this shall be realized. A more realistic assumption would be based on the heterogeneous, time-delayed, incomplete and noise-affected information distribution, but in consequence, this would demand to model the information flow between the agents explicitely.

A second problem is concerned with the combinatoric explosion of the state space. Commonly, the freedom to define rules and interactions for the agents, is much appreciated. However, each of these additional rules blows up the state space of possible solutions for the agent system. Already for 1000 agents with 10 rules, the state space contains about 10^{13} possibilities. Hence almost every desirable result could be produced from such a simulation model and the freedom could very soon turn out to be a pitfall.

In fact, due to their rather complex simulation facilities many of the currently available multi-agent tools, for example SWARM, lack the possibility to investigate systematically and in depth the influence of specific interactions and parameters. Instead of incorporating only as much detail as is *necessary* to produce a certain emergent behavior, they put in as much detail *as possible*, and thus reduce the chance to understand *how* emergent behavior occurs and *what* it depends on.

A quite different approach is given by the *minimalistic agent design*, which is used in this paper. A minimalistic agent acts on the possible *simplest set of rules*, without deliberative actions. Instead of specialization, the minimalistic agent model is based on a large number of “identical” agents, and the focus is mainly on *cooperative interaction* instead of autonomous action. Of course, also this approach is based on a certain trade-off: some features which might be considered important for a specific system, are dropped here, in order to investigate a particular kind of interaction more carefully. Instead of describing a whole system most realistically, the minimalistic approach only focuses on particular dynamic effects within the system dynamics - but, as an advantage, also provides numerous quantitative methods to investigate the influence e.g. of certain parameters or quantities. For example, bifurcations of the dynamics, the structure of attractors, conditions for stable non-equilibrium states, etc. can be investigated by means of advanced methods borrowed from statistical physics, this way providing a clear-cut idea about the role of particular interaction features.

Like any other agent-based approach, also minimalistic agent models are based on a specific kind of reductionism which should be addressed from a philosophy of science view. Compared to the reductionistic approaches especially in natural sciences, self-organization theory is often interpreted as a holistic approach which conquers the classical reductionism. However, self-organization itself is a phenomenon which is only realized from a certain perspective, i.e. it’s observation depends on the specific level of description, or on the focus of the “observer”, respectively (Niedersen and Schweitzer, 1993). In this particular sense, self-organization theory is an *aithetical* theory (cf. the Greek meaning of “aisthetos”, perceptable) (Schweitzer, 1994). The particular level of perception for self-organization has been denoted as *mesoscopy* (Schweitzer, 1997b). It is different from the microscopic perception level which focuses primarily on the smallest entities or elements, as well as from the macroscopic perception level, which rather focuses on the system as a whole. Instead, mesoscopy focuses on elements complex enough to allow an interaction which eventually results into emergent properties or complexity on the macroscopic scale. These elements are the “agents” in the sense denoted above: they provide an “intermediate complexity” and are capable of a certain level of activity, i.e. they do not just passively respond to external forces, but are actively involved e.g. in nonlinear feedback processes.

Thus, for the further discussion we have to bear in mind to what degree agent-based models are based on certain reductions regarding the system elements and their interactions. On the way toward a generalized self-organization theory, we have to understand carefully the nature of these reductions, especially when turning to the social and life sciences (Hegselmann *et al.*, 1996). Self-organization in social systems is confronted with the mental reflexions and purposeful actions of their elements, creating their own reality. While we are on one hand convinced that the basic dynamics of self-organization originates analogies between structure formation processes in very different fields regardless of the elements involved, we should on the other hand not forget about the differences between these elements, especially between humans and physical particles. Thus, a deeper understanding of self-organization, complex dynamics and emergence in socio-economic systems has to include also a better insight into these reductions.

4 An information-theoretic approach

In the following, we want to characterize the minimalistic agent approach in terms of an information-theoretic description. This shall allow us to describe the interaction of agents as a generalized form of "communication", based on the exchange of information. To this end, we need to distinguish between three different kinds of information: functional, structural and pragmatic information (Ebeling *et al.*, 1998; Schweitzer, 1997c).

Functional information denotes the capability of the agent to process external information (data) received. It can be regarded as an *algorithm* specific to the agent. This algorithm is applied to "data" in a very general sense, which can be also denoted as *structural information* because it is closely related to the (physical) structure of the system. The DNA is an example for structural information in a biological context. As a complex *structure*, it contains a mass of (structural) information in a coded form, which can be selectively activated in dependence on different circumstances. Another example for structural information would be a book or a user manual, written in letters of a particular alphabet.

Structural information is meaningless, it does not contain *semantic* aspects but only *syntactic* aspects - hence, the content of structural information can be analyzed for instance by means of different physical measures (e.g. conditional or dynamic entropies, transinformation etc.) (Ebeling *et al.*, 1998). Functional information on the other hand is related to the semantic aspects of information; it reflects the contextual relations of the agent. It is the purpose of functional information to *activate* and to *interpret* the existing structural information with respect to the agent, this way creating a new form of information denoted as *pragmatic information.* This means a type of operation relevant information which allows the agent to *act*. In the examples above, cells are able by means of specific "functional" equipment to extract different (pragmatic) information from the genetic code, which then allows to evolve differently e.g. in morphogenesis. In the example of the user manual, the reader (agent) is able by means of a specific functional information, i.e. an algorithm to process the letters, to extract useful, "pragmatic" information from the text, which only allows him/her to act accordingly. If the functional information (algorithm) does not match the structural information (data), i.e. if the manual is written in Chinese and the reader can only process latin letters, then pragmatic information will not emerge from this process – even though the structural and the functional information is still there. With respect to the term of pragmatic information, we can express this relation as follows: *It is the purpose of functional information to transfer structural into pragmatic information.*

In order to characterize the minimalistic agent model in terms of a generalized communication approach based on the exchange of information, we have to identify the kind of functional and structural information used in the system, and then have to investigate which kind of pragmatic information may emerge from this. As denoted above, the functional information shall be a (simple) algorithm which can be steadily repeated by each agent. For example, we may assume that during every time step the agent is able (i) to read data, (ii) to write data, and (iii) to process the data currently read (e.g. to compare their value). The data read and written are *structural* information which is stored on a *blackboard* external to the agent. Like a usual blackboard, it plays the role of a communication medium (Veit and Richter, 2000). Because of this, the communication among the agents may be regarded as *indirect communication*; however, the involvement of a medium seems to be always the case, even in "direct" oral communication.

The emergence of *pragmatic* information for a specific agent will of course depend (i) on the functional information, i.e. the "algorithm" to process a specific structural information, and (ii) on the availability of this structural information, i.e. the access to the respective blackboard at a particular time or place. For different applications, we may consider various possibilities to restrict the access to the blackboard both in space and time. This way, the communication between the agents can be modeled as a *local* or a *global* one. On the other hand, we may also assume that there are different *spatially distributed* blackboards in the system, modeling a spatially heterogeneous distribution of (structural) information.

Additionally, we may consider *exchange processes* between different blackboards, for instance for the reconciliation of data. The data originally stored on a particular blackboard may then also propagate to other blackboards in the course of time. This way, we observe a rather complex interaction dynamics determined by two quite different space- and time-dependent processes: (i) changes of blackboards caused by the *agents*, (ii) changes of blackboards caused by an *eigendynamics* of the structural information. The latter one may also involve dynamic elements such as (i) a finite life time of the data stored which models the existence of a *memory*, (ii) an *exchange of data* in the system with a *finite* velocity, (iii) the spatial heterogeneity of structural information available.

As one possibility to consider these features within a minimalistic agent model, we have introduced the concept of a *spatio-temporal communication field* (Schweitzer and Hołyst, 2000), which models the eigendynamics of an array of spatially distributed blackboards. This communication field is external to the agents, but is created by the various types of data (structural information) consecutively produced by them. For example, agents contribute to the communication field by making choices, by consuming resources, by producing some output – e.g. in economic production or simply as waste or thermal radiation which heats up the atmosphere, etc. The distribution of the different kinds of structural information is spatially heterogeneous and time dependent. However, it may affect other agents in different regions of the system, provided they notice this information and are capable to extract some meaning out of it, which then may influence their own decisions, output, etc.

This concept has proved its applicability in a variety of models. We want to mention only two examples here. In a model of economic agglomeration, the communication field has been regarded as a spatially heterogeneous, time dependent *wage field*, from which agents can extract meaningful information for their migration decisions. The model then describes the emergence of economic centers out of a homogeneous distribution of productivity (Schweitzer, 1998). Another example deals with spatial self-organization in urban growth (Schweitzer and Steinbrink, 1997). In order to find suitable places for aggregation, "growth units" (specific urban agents) may use the information of an urban attraction field, which has been created by the existing urban aggregation and this way provides an indirect communication between different types of urban agents. To be more specific, we want to discuss in the following a simple model of agents communicating by means of a two-component communication field, which contains information for their local decisions.

5 Basic model of communicating agents

Let us consider a rather simple toy model of agent interaction. Suppose, we have a 2-dimensional spatial system with the total area A, where a community of N agents exists. In general, N can be changed by birth and death processes but A is assumed fixed. Each agent i is assigned two individual parameters: its position in space, r_i, which should be a continuous variable, and its current "opinion", θ_i (with respect to a definite aspect or problem). The latter one is a discrete valued parameter representing an *internal degree of freedom* (which is a rather general view of "opinion").

To be specific, let us discuss the following example. Imagine a certain problem, for instance the separate disposal of recycling material. Each agent in the system needs to decide whether he/she will collaborate in the recycling campaign or deny to do so. Then, there are only two (opposite) opinions, i.e. $\theta_i \in \{+1, -1\}$. From the classical economic perspective, the agents' decision about his/her opinion may depend on an estimate of his/her *utility*, i.e. what he/she may gain compared to her own effort, if he/she decides to collaborate or not. Here, we neglect any question of utility and may simply assume that the agent will more likely do what others do with respect to the specific problem, i.e. he/she will decide to collaborate in the recycling campaign if most of his/her neighbors will do so, and refuse to collaborate in the campaign if most of their neighbors have the same opinion in this case. This kind of contagious behavior in decision processes is well known from different fields, i.e. fashion demand or selection of movies (Weidlich, 1991; Helbing, 1995; Lane and Vescovini, 1996; Solomon *et al.*, 2000).

This problem raises the question about the interaction between agents at different locations, i.e. how is agent i at position r_i affected by the decisions of other agents at closer or far distant locations? In a checkerboard world, commonly denoted as cellular automaton, a common assumption is to consider only the influence of agents which are at the (four or eight) nearest neigbour sites or also at the second-nearest neighbor sites etc. (Schelling, 1969; Sakoda, 1971, Hegselmann and Flache, 1998). Contrary, in a *mean-field approximation*, all agents are considered as influencial via a mean field which affects each agent at the same time in the same manner.

Our approach will be different from these ones in that we will consider a continuous space and a gradual, time delayed interaction between all agents. We assume that agent i at position r_i is not directly affected by the decisions of other agents, but only receives information about their decisions via a *communication field* generated by the agents with the different opinions. This field is assumed a scalar *multi-component spatio-temporal field* $h_\theta(\boldsymbol{r}, t)$, which obeys the following equation:

$$\frac{\partial}{\partial t} h_\theta(\boldsymbol{r}, t) = \sum_{i=1}^{N} s_i\ \delta_{\theta,\theta_i}\ \delta(\boldsymbol{r} - \boldsymbol{r}_i)\ -\ k_\theta h_\theta(\boldsymbol{r}, t)\ +\ D_\theta \Delta h_\theta(\boldsymbol{r}, t). \tag{1}$$

Every agent contributes permanently to this field with its personal “strength” or influence, s_i. Here, δ_{θ,θ_i} is the Kronecker Delta indicating that the agents contribute only to the field component which matches their opinion θ_i. $\delta(\boldsymbol{r} - \boldsymbol{r}_i)$ means Dirac’s Delta function used for continuous variables, which indicates that the agents contribute to the field only at their current position, $\boldsymbol{r}_i$.

The *structural information* generated this way has a certain life time $1/k_\theta$ [s], further it can spread throughout the system by a diffusion-like process, where D_θ [m^2/s] represents the diffusion constant for information exchange. We have to take into account that there are two different opinions in the system, hence the communication field should also consist of two components, $\theta = \{-1, +1\}$, each representing one opinion. Note, that the parameters describing the communication field, s_i, k_θ, D_θ do not necessarily have to be the same for the two opinions.

Eq. (1) for the communication field $h_\theta(\boldsymbol{r}, t)$ is a partial differential equation continuous in space and time. In a *discretized* version, it describes a spatial array of two different kinds of blackboards, each storing the contributions s_i (“data”, structural information) produced by the agents of a particular opinion in a particular spatial domain $x + \Delta x, y + \Delta y$. These blackboards are updated in time intervals Δt and have an eigendynamics determined by the exchange and the life time of the “data” stored. This eigendynamics can be used to reflect some important features of communication in social systems:

(i) the existence of a *memory*, which reflects the past history of actions. In our model, this memory exist as an external memory, the lifetime of which is determined by the decay rate of the structural information, k_θ.

(ii) an *exchange of information* in the community with a *finite* velocity. In our model, this exchange is described by a diffusion-like process with the exchange constant D_θ. This implies that the structural information will eventually reach every agent in the whole system, but of course at different times.

(iii) the influence of *spatial distances* between agents. Thus, the information generated by a specific agent at position r_i will affect agents at a closer spatial distance earlier and thus with larger weight, compared to far distant agents.

The communication field $h_\theta(\boldsymbol{r}, t)$ influences the agent's decisions as follows: At a certain location $\boldsymbol{r}_i$, the agent i with opinion θ_i is affected by two kinds of information: the information $h_\theta(\boldsymbol{r}_i, t)$ resulting from agents who share his/her opinion, and the information $h_{-\theta}(\boldsymbol{r}_i, t)$ resulting from the opponents. The diffusion constants D_θ determine how fast he/she will receive any information, and the decay rate k_θ determines, how long a generated information will exist. Dependent on the information received locally, the agent has two opportunities to act: he/she can *change his/her opinion* or he/she can keep it. A possible ansatz for the transition rate to change the opinion reads (Schweitzer and Hołyst, 2000):

$$w(-\theta_i|\theta_i) = \eta \ \exp\left\{-\frac{h_\theta(\boldsymbol{r}_i, t) - h_{-\theta}(\boldsymbol{r}_i, t)}{T}\right\} \tag{2}$$

The probability to change opinion θ_i is rather small, if the local field $h_\theta(\boldsymbol{r}_i, t)$, which is related to the support of opinion θ_i, overcomes the local influence of the opposite opinion. Here, η [1/s] defines the time scale of the transitions. The scaling parameter T may be interpreted as a "social temperature" (Kacperski and Hołyst, 1996) describing a degree of *randomness* in the behavior of the agents, but also their average volatility (Bahr and Passerini, 1998).

In order to summarize our model, we note the non-linear feedback between the agents and the communication field as shown in Fig. 1. The agents generate the field, which in turn influences their further decisions. In terms of synergetics, the field plays the role of an order parameter, which couples the individual actions, and this way initiates coherent behavior within the agent community.

For $N =$ const., the community of agents may be described by the multivariate distribution function $P(\underline{\theta}, \underline{r}, t) = P(\theta_1, \boldsymbol{r}_1, ..., \theta_N, \boldsymbol{r}_N, t)$ which gives the probability to find the N agents with the opinions $\theta_1, ..., \theta_N$ at positions $\boldsymbol{r}_1, ..., \boldsymbol{r}_N$ on the surface A at time t. The time depentent change of $P(\underline{\theta}, \underline{r}, t)$ can then be described by a *master equation* which considers any possible transition within the opinion distribution $\underline{\theta}$ (the formal details are skipped here, cf. Schweitzer and Zimmermann, 2000). The master equation, together with eqs. (1), (2) forms a complete description of our system,

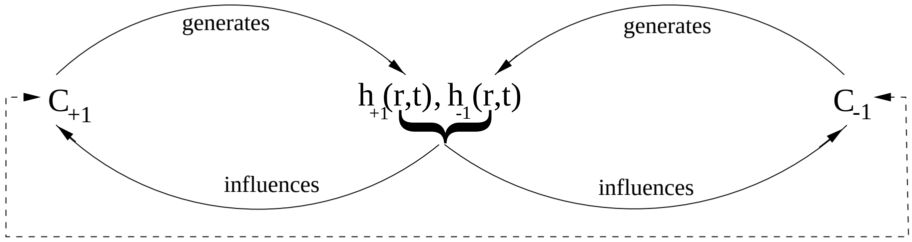

Figure 1: Circular causation between the agents, C_{-1}, C_{+1}, and the two-component communication field, $h_\theta(\boldsymbol{r}, t)$.

which depends on the parameters describing the agent density (N, A) and the components of the communication field (s_i, k_θ, D_θ). In order to find possible solutions of the master equation, we will use computer simulations, and in particular apply the stochastic simulation technique. The results are presented in the following sections.

6 Mean-field approach

Before investigating the spatially distributed system, we first discuss a mean-field approximation, in order to get some insight into the complex dynamics of the agent system. This case, which has been discussed in more detail also by Schweitzer and Hołyst (2000), may have some practical relevance for communities existing in small systems with small distances between different agents. In particular, in such small communities a very fast exchange of information may hold, i.e. spatial heterogenities in the communication field are equalized immediately. In terms of the blackboard interpretation, this means consequently that all agents have access to the same (two) blackboards independent of their loactions. Thus, in this section, the discussion can be restricted to subpopulations with a certain opinion rather than to agents at particular locations.

Let us define the share x_θ of a subpopulation θ and the respective mean density $\bar{n}_\theta$ in a system of size A consisting of N agents:

$$x_\theta(t) = \frac{N_\theta(t)}{N} \; ; \quad \bar{n}_\theta(t) = \frac{N_\theta}{A} \tag{3}$$

where the total number of agents sharing opinion θ at time t fulfills the condition

$$\sum\nolimits_\theta N_\theta(t) = N_{+1}(t) + N_{-1}(t) = N = \text{const.} \; ; \quad x_{+1}(t) = 1 - x_{-1}(t) \tag{4}$$

The dynamics of the system is then determined by the equations for the subpopulation $x_\theta(t)$, which are coupled via the equations for the two-component communication field $h_\theta(\boldsymbol{r}, t)$. The stationary states of the dynamics follow from the conditions $\dot{x}_\theta = 0$, $\dot{h}_\theta = 0$. For the two field components we find with the assumption that agents with the same opinion θ will have the same influence $s_i \to s_\theta$

and with $\bar{n} = N/A$ (Schweitzer and Hołyst, 2000):

$$\bar{h}_{+1}^{stat} = \frac{s_{+1}}{k_{+1}}\,\bar{n}x_{+1}\ ; \quad \bar{h}_{-1}^{stat} = \frac{s_{-1}}{k_{-1}}\,\bar{n}(1 - x_{+1}) \tag{5}$$

Let us for the moment assume that the parameters of both field components are identical, i.e. $s_{+1} = s_{-1} \equiv s$, $k_{+1} = k_{-1} \equiv k$, a more complex case will be discussed below. Then, we find for the stationary values of x_θ in the case $\theta = +1$ (Schweitzer and Hołyst, 2000):

$$(1 - x_{+1})\,\exp\left[\kappa\, x_{+1}\right] = x_{+1}\,\exp\left[\kappa\,(1 - x_{+1})\right] \tag{6}$$

Here, the *bifurcation parameter*

$$\kappa = \frac{2s\,\bar{n}}{k\,T} \tag{7}$$

includes the specific *internal conditions* within the community, such as the population density, the social temperature, the individual strength of the opinions, or the life time of the information generated.

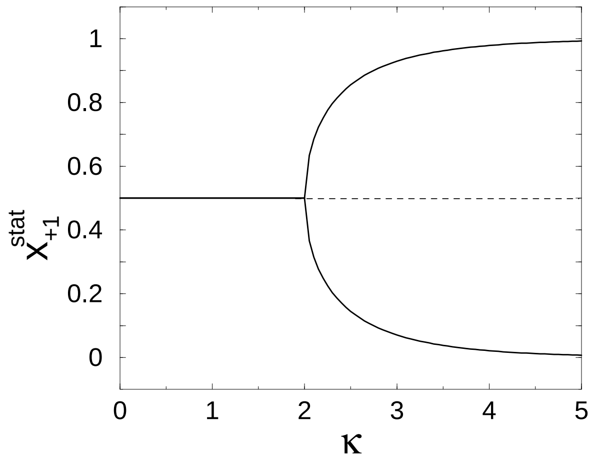

Figure 2: Stationary solutions for x_{+1} (eq. 6) for different values of κ. The bifurcation at the critical value $\kappa^c = 2$ is clearly visible.

Schweitzer and Hołyst (2000) found that depending on κ different stationary values for the fraction of the subpopulations exist (cf. also Fig. 2). For $\kappa < 2$, $x_{+1} = 0.5$ is the only stationary solution, which means a stable community where both opposite opinions have the same influence. However, for $\kappa > 2$, the equal distribution of opinions becomes unstable, and a separation process towards a preferred opinion is obtained, where $x_{\pm 1} = 0.5$ plays the role of a separation line. Then two stable solutions are found where both opinions coexist with different shares in the community, as shown in Fig. 2. Hence, each subpopulation can exist either as a *majority* or as a *minority* within the community. Which of these two possible situations is realized depends in a deterministic approach on the initial fraction of the subpopulation. For initial values of x_{+1} below the separatrix, 0.5, the minority status will be most likely the stable situation (Schweitzer and Hołyst, 2000). In the stochastic approach considered here, the realization of a possible minority/majority relation will also depend on the fluctuation during the early stage of the evolution of the agent system as shown below.

From the condition $\kappa = 2$ we can derive a *critical population size*,

$$N^c = k\,A\,T/s, \tag{8}$$

where for larger populations an equal fraction of opposite opinions is certainly unstable. If we consider e.g. a *growing community* with fast communication, then both contradicting opinions are balanced, as long as the population number is small. However, for $N > N^c$, i.e. after a certain population growth, the community tends towards one of these opinions, thus necessarily separating into a majority and a minority. Which of these opinions would be dominating, depends on small fluctuations in the bifurcation point. Fig. 3 shows a particular realization obtained from computer simulation of 400 agents who at $t = 0$ are randomly assigned one of the opinions $\{+1, -1\}$.

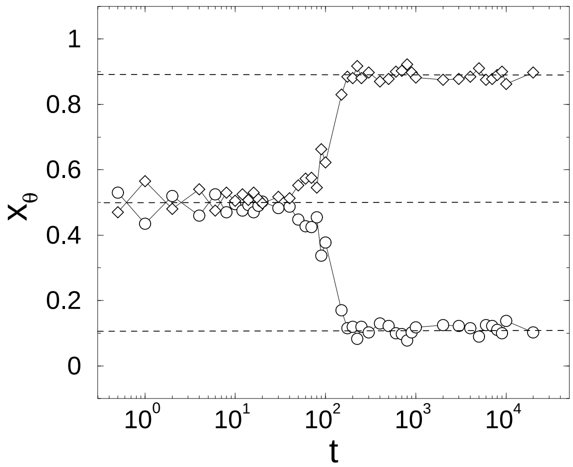

Figure 3: Computer simulation of the relative subpopulation sizes x_{+1} ($\circ$) and x_{-1} ($\diamond$) vs. time t for a community of $N = 400$ agents. Parameters: $A = 400$, $s = 0.1$, $k = 0.1$, $T = 0.75$, i.e. $\kappa = 2.66$. Initially, each agent has been randomly assigned opinion $+1$ or -1. The dashed lines indicate the inital equal distribution ($x_\theta = 0.5$) and the minority and majority sizes ($x_\theta = \{0.115; 0.885\}$) which follow from eq. (6).

As indicated in Fig. 3, there is a latent period in the beginning *before* the minority/majority relation emerges, i.e. during this period it is not clear which one of the two subpopulations will gain the majority status. This initial time lag $t^\star$ is needed to establish the communication field which plays the role of an *order parameter* known from synergetics (Haken, 1978). Consequently, for $t \geq t^\star$, a transition from the unstable equal distribution between both opinions toward a majority/minority relation is clearly visible in Fig. 3. The time period to eventually establish this relation is then rather short, since the case discussed in this section is related to a very fast exchange of information.

Eventually, we want to note that the symmetry between the two opinions can be broken due to external influences on the agents. Schweitzer and Hołyst (2000) have considered two similar cases: (i) the existence of a *strong leader* in the community, who possesses a strength s_l which is much larger than the usual strength s of the other individuals, (ii) the existence of an external field, which may result from government policy, mass media, etc. which support a certain opinion with a strength s_m. The additional influence $s_{ext} := \{s_l/A, s_m/A\}$ mainly effects the mean communication field due to an extra contribution, normalized by the system size A. It was found within the mean-field approach that at a critical value of s_{ext}, the possibility of a minority status completely vanishes. Hence, for a certain supercritical external support, the supported subpopulation will grow towards a majority, regardless of its initial population size, with no chance for the opposite opinion to be established. This situation is quite often realized in communities with one strong political or religious leader ("fundamentalistic dictatorships"), or in communities driven by external forces, such as financial or military power ("banana republics").

7 Spatial information distribution

The previons section has shown within a mean-field approach the emergence of a minority/majority relation in the agents community. With respect to the example of the recycling campaign adressed previously, it means that *either* most of the agents decide to collaborate *or* most of them decide to refuse to collaborate. If we start from an unbiased initial distribution, i.e. an equal distribution between both opinions, then there is no easy way to break the symmetry towards a preferred opinion, except an external bias is taken into account.

In this section, we will investigate a possibility to break the symmetry by means of different information distribution. That requires now to consider the spatial dimension of the system explicitely. Let us start with the previous example of $N = 400$ agents randomly distributed in a system of size A (cf. Fig. 4), with random initial opinions. They get information about the opinions of other agents by means of the two-component communication field $h_\theta(\boldsymbol{r}, t)$, eq. (1), which now explicitely considers space and therefore "diffusion" of information. The two-dimensional system is treated here as a torus, i.e. we assume periodic boundary conditions.

As a first example, we assume that the parameters decribing the communication field, are again the same for both components, i.e. $s_{+1} = s_{-1} \equiv s$, $k_{+1} = k_{-1} \equiv k$, $D_{+1} = D_{-1} \equiv D$. Fig. 4 shows three snapshots of the spatial distribution of the agent's opinion, while Fig. 5 shows the respective evolution of the subpopulation shares. Evidently, we find again the emergence of a majority/minority relation – this time however, on a larger time scale compared to Fig. 3, which is basically detemined by the information diffusion, expressed in terms of D. But the initial latent time lag $t^\star$ for the emergence of the majority/minority relation is about the same, which is needed again to establish the communication field.

As the different snapshots of Fig. 4 show, the minority and majority organizes itself in space in such a way that both are *separated*. Thus, besides the existence of a global majority, we find regions in the system which are dominated by the minority. From this we can conclude a *spatial coordination of decisions*, i.e. agents which share the same opinion are spatially concentrated in particular regions. With respect to the example of the recycling campaign this means that those agents who refuse to collaborate (or collaborate in the opposite case), are mostly found in a spatial domain of a like-minded neighborhood. This result might remind on the famous simulations of segregation in social systems (Schelling, 1969; Sakoda, 1971; Hegselmann and Flache, 1998) - however, we would like to note that in our case the agents do *not migrate* toward supportive places; they rather *adapt* to the opinion of their neighborhood.

The spatial distribution of the majority and the minority is also reflected in the different components of the communication field, as shown in Fig. 6. We find that the maxima of both components are of about equal value, however, the information generated by the majority, is roughly spread over the whole system, whereas the information generated by the minority eventually concentrates only in specific regions dominated by them.

So far we have noticed the importance of fluctuations during the initial time lag $t^\star$, which decide which of the two possible opinions will appear as the majority opinion. For the spatial coordination of decisions we may now exploit the different properties of the information exchange in the system, as expressed in terms of the parameters s_θ, k_θ, D_θ of the communication field. For instance, we may assume that the information generated by one of the subpopulations is distributed *faster* in the system than the information generated by the other one. Alternatively, we may also consider different life times of the different components of the communication field. However, in order to model a faster exchange of information, it is not sufficient to simply increase the value of D_θ, we need to consider its effect on the local values of the communication field in more detail. A closer inspection of eq. (1) shows (Schweitzer and Zimmermann, 2000) that a faster communication in the system via a faster diffusion of the generated information, also lowers the information available at the agent's position. This might be considered as a drawback in modeling information exchange by means of reaction-diffusion equations. Obviously, the field $h_\theta(\boldsymbol{r}, t)$ obeys certain boundary conditions and conservation laws, which do not hold for "information *per se*". In particular, the local value of available information is not lowered if this information spreads out faster, but the local value of the "communication field" obeying eq. (1) does.

In order to compensate the unwanted effect of a local decrease of $h_\theta(\boldsymbol{r}, t)$, we have to choose the

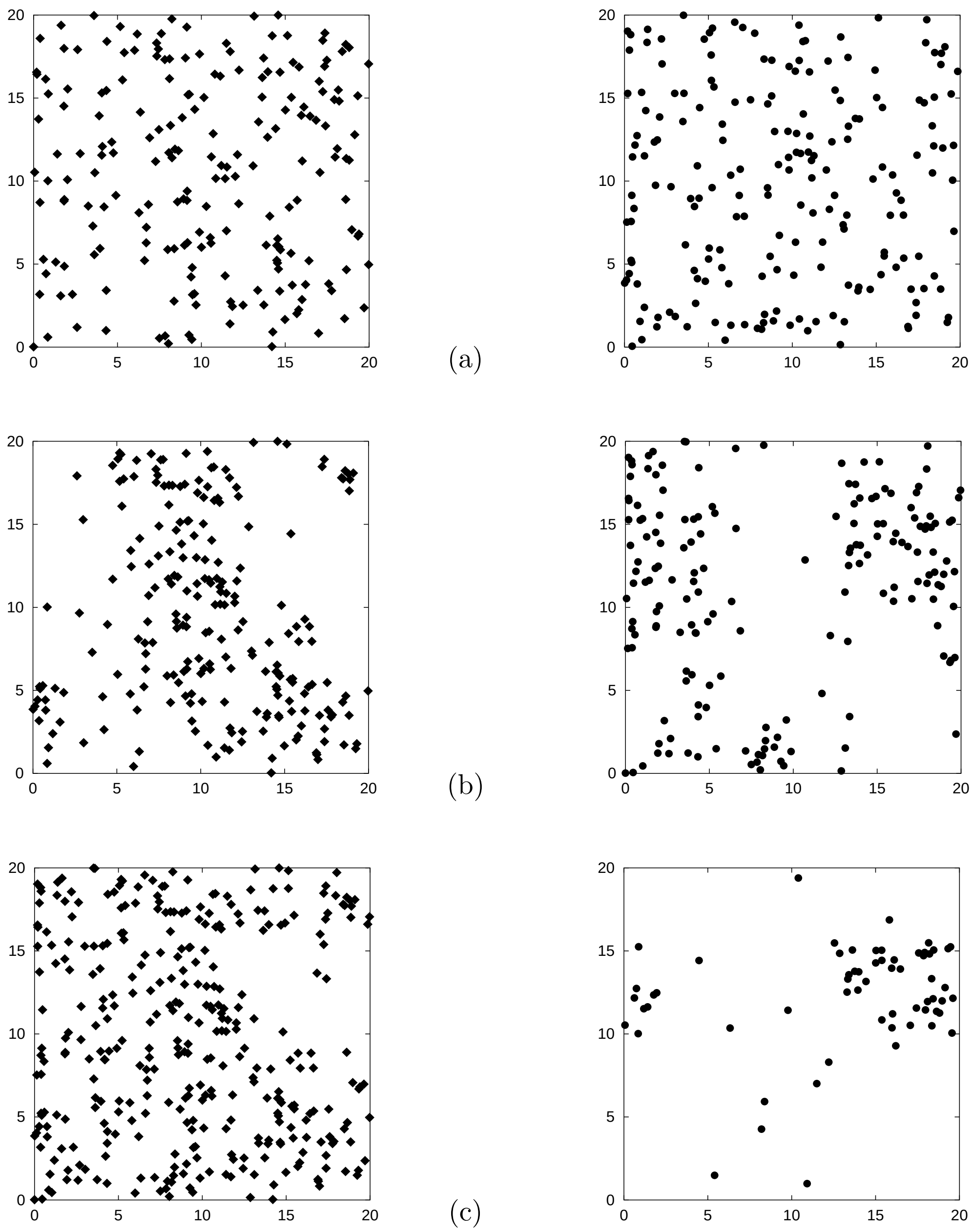

Figure 4: Computer simulations of the spatial distribution of agents with opinion +1 ($\diamond$) and -1 ($\circ$). The snapshots are taken at three different times: (a) $t = 10^0$, (b) $t = 10^2$, (c) $t = 10^4$. For the parameters and initial conditions see Fig. 3, additionally $D = 0.06$.

parameters s_θ, k_θ, D_θ in such a way that both the ratios

$$\frac{k_\theta}{s_\theta} = \beta\,; \quad \frac{D_\theta}{s_\theta} = \gamma \tag{9}$$

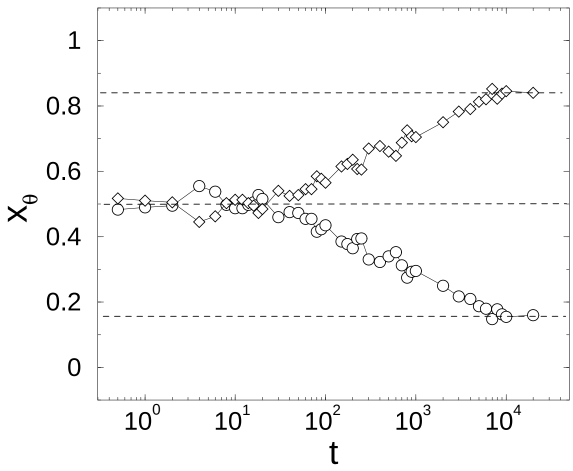

Figure 5: Relative subpopulation sizes x_{+1} ($\diamond$) and x_{-1} ($\circ$) vs. time t for the computer simulation shown in Fig. 4.

need to be constant for both components $\theta = \{+1, -1\}$. In this case, eq. (1) for the dynamics of the multi-component communication field can be rewritten as:

$$\frac{\partial}{\partial \tau} h_\theta(\boldsymbol{r}, \tau) = \sum_{i=1}^{N} \delta_{\theta,\theta_i} \, \delta(\boldsymbol{r} - \boldsymbol{r}_i) \; - \; \beta \, h_\theta(\boldsymbol{r}, \tau) \; + \; \gamma \, \Delta h_\theta(\boldsymbol{r}, \tau). \tag{10}$$

where the time scale τ is now defined as $\tau = t \, (D_\theta / \gamma)$. If both parameters β and γ are kept constant, eq. (10) means that the dynamics of the respective component of the communication field occurs on a different time scale τ, dependent on the value of D_θ. In terms of the blackboard interpretation this means that the array of blackboards containing the information about a particular opinion θ will be updated more (or less) frequently than the blackboard array representing the opposite opinion. An increase in the diffusion constant D_θ then models indeed the information exchange on a faster time scale, as expected, without affecting the stationary distribution resulting from eq. (10).

Computer simulations of the evolution of the subpopulations for the case of different information diffusion are shown in Fig. 7. We find again the emergence of a majority/minority relation - but this time the subpopulation (-1) with the faster diffusing communication field becomes more likely the majority in the system. We have also found that the minority is no longer concentrated in particular regions, but only randomly distributed, so there is no longer a coordination of decisions on the side of the minority.

From various runs of computer simulations we can deduce the following general conclusions regarding the influence of the ratio $d = D_{+1}/D_{-1}$, under the presumption that β and γ are kept constant (cf. also Schweitzer and Zimmermann, 2000):

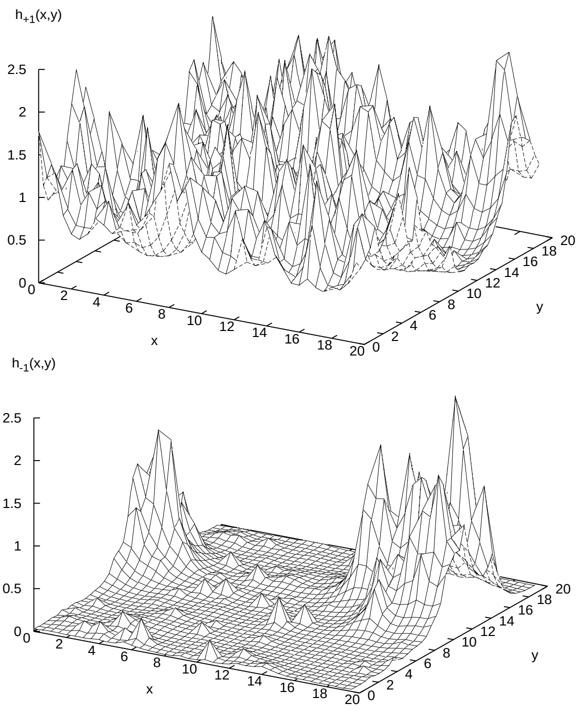

Figure 6: Spatial distribution of the two-component communication field, (top) $h_{+1}(\boldsymbol{r}, t)$, (bottom) $h_{-1}(\boldsymbol{r}, t)$ at time $t = 10^4$, which refers to the spatial agent distribution of Fig. 4c.

- For $d = 1$, both subpopulations have an equal chance to become the majority in the system. With an increasing difference in the values of D_{+1} and D_{-1}, the subpopulation with the faster (more "efficient") communication more likely becomes the majority.

- With an increasing difference between D_{+1} and D_{-1}, the initial time lag, when the decision about which subpopulation becomes the majority is yet pending, decreases (cf. Figs. 3, 7). This reduces the influence of early fluctuations to break the symmetry toward one of the subpopulations.

- With an increasing difference between D_{+1} and D_{-1}, the size of the respective minority size will be decreasing (which can be also seen by comparison of Figs. 3, 7). A smaller minority will also have a smaller chance to organize itself in space, to form regions of coordinated decisions. Further, due to the shorter initial time lag, they will also do not have the time to establish their own communication field.

In order to summarize our simulations, we want to link the discussion to the different kind of information introduced in Sect. 4. At the individual or microscopic level, we have the genuine *local* decisions of each agent which result from an interplay between the functional and the structural

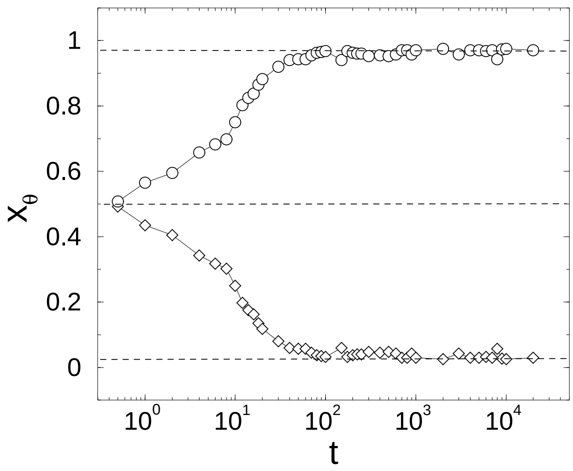

Figure 7: Relative subpopulation sizes x_{+1} ($\diamond$) and x_{-1} ($\circ$) vs. time t for the computer simulation with different information diffusion. Parameters: D_{+1}=0.02, D_{-1}=0.06, $\beta = 1$, $\gamma = 0.6$.

information. The latter one describes the data stored on the different arrays of blackboards, which are modeled here as a two-component communication field $h_\theta(\boldsymbol{r}, t)$. Functional information means here the ability of the agent to read data from and to write data to the blackboards or the communication field, respectively. The functional information also considers the contextual situation of the agent, i.e. whether he/she is able to get access to a particular blackboard dependent on his/her current position etc. It further describes how these data are processed by the agent - in this particular example, just their values are compared.

As the result of this interplay *pragmatic* information emerges that allows the agent to make a decision whether to collaborate or not. This pragmatic information is individual information, it exists only for a particular agent at a particular position and a particular time. Already the next time step may change the whole situation: dependent on the structural information read, the agent may make a quite different decision; thus pragmatic information is not an invariant of the dynamics, it has to be consecutively generated by each agent. The local and independent decisions of the different agents are coupled via the communication field, which is commonly generated by the agents but also feeds back to their decisions. This kind of non-linear feedback between local actions and non-local coupling suddenly results in global repercussions: the random distribution of agents with a particular opinion changes toward an ordered state on the macro scale. Thus, the emergence of a spatial coordination of decisions can be regarded as a transition from the locally independent decision to the globally coordinated decision of the agents.

8 Conclusions

Self-organization and the emergence of new properties at the collective level play an important role in socio-economic dynamics. Despite this commonly accepted conclusion, a number of conceptual problems associated with defining and simulating complex socio-economic systems still exist. In the first part of this paper, we have addressed some of these issues. Since complex systems usually consist of a large number of interacting components, multi-agent models can play a valuable role in exploring and simulating their dynamic behavior. However, the dependence of emergent system properties on specific agent's interactions is sometimes hard to investigate systematically, because of the rather complex design of multi-agent systems.

Therefore, we have proposed a "minimalistic" agent approach which focuses only on particular interactions, with no attempt to model a socio-economic system most realistically. The minimalistic agent design can be seen as a stepping stone strategy where more sophistication can be added gradually on the path to a deeper understanding of complex phenomena. At first, the restriction to a possibly simple set of interaction rules certainly involves reductions, but on the other hand it opens the door to apply quantitative methods developed within physics for the analysis of interacting systems. Different promising examples for this transfer of methods can be found in the fields of quantitative sociology (Weidlich, 1991, 2000; Helbing, 1995) or econophysics (Mantegna and Stanley, 2000; Schweitzer and Helbing, 2000).

In our approach, the basic interaction between the agents can be described as a generalized form of *communication*. Each agent produces/releases structural information (generalized form of "data") in the course of time which is stored externally on blackboards. On the other hand, each agent is also able to "read" the structural information stored, provided he/she possesses an algorithm denoted as functional information for processing these data. This way, pragmatic or action-relevant information can emerge from the interplay between functional and structural information. That means the agent is enabled to perform a specific task, to make a decision etc.

In order to give an example of the minimalistic agent design described above, in the second part of the paper we have investigated the *spatial coordination of decisions* within a multi-agent system. Each agent at his/her current location has to decide whether he/she wants to collaborate in a campaign or not. Different from classical economic approaches, this decision is not based on the calculation of utilities, but simply on the decision of other agents. This raises the problem of communication, i.e. how an agent at a particular location gets the information about the decision of other agents. The spatio-temporal distribution of information in the system is described by means of a two-component communication field, which couples the actions of the different agents and this way plays the role of an order parameter. We find both analytically and by means of stochastic computer simulations, that for some critical parameters such as the population density a majority/minority relation appears, i.e. a majority of the agents either decides to collaborate or not to collaborate. Additionally, considering the spatial extension of the system, we find that both the majority and the minority organizes itself in particular spatial domains. That means we clearly find a spatial coordination of particular decisions mediated by the communication among the agents.

Both the appearence of the majority/minority and the spatial concentration of these like-minded subpopulations are emergent properties of the multi-agent system; in addition to the spatial self-organization (emergence of spatial domains) we also observe the self-organization in the state space of possible decisions (majority/minority relation). This dynamical process can be influenced by means of the communication among the different subpopulations. We find from our computer simulations, that the subpopulation with the more efficient communication (i.e. the structural information is distributed faster, the blackboard arrays are updated more frequently) will have a much better chance to become the majority in the system. This also allows an interpretation in the socio-economic context: if the decision between competiting opinions about a given subject is yet pending and not particular determined by the private utilities of the agents, the faster distribution about the relevant information may decide about the success of a given opinion.

Our papers has focused on the communication between the agents from a rather "minimalistic" point of view - but it is worth to notice that in contrast to other approaches widely used in economics we have not embarked on common knowledge assumptions or rational decisions. Contrary, we have emphasized important questions such as the heterogeneous distribution of information, effects of local decisions, or the "effectivity" of communication among the different subpopulations. Besides the possibility to obtain some quantitative results (such as the critical population size), our minimalistic agent model also allows to understand the process of self-organization in more detail; simply because in this case the complex dynamics emerges from readily understandable interactions between relatively unsophisticated agents.

References

Allen, P. (1998): Modelling Complex Economic Evolution. In: Schweitzer, F. and Silverberg, G. (eds.): *Evolution and Self-Organization in Economics*, Duncker & Humblot, Berlin, pp. 47–75.

Andersen, P. W.; Arrow, K. J.; Pines D. (eds.) (1988): *The Economy as an Evolving Complex System*. Addison Wesley, Reading, MA.

Arthur, W.. B. (1993): On designing economic agents that behave like human agents. *Journal of Evolutionary Economics* 3, 1–22.

Arthur, W.. B.; Durlauf, S. N., Lane, D. (eds.) (1997): *The Economy as an Evolving Complex System II*. Addison Wesley, Reading, MA.

Bahr, D. B.; Passerini, E. (1998): Statistical mechanics of opinion formation and collective behavior. *Journal of Mathematical Sociology* 23, 1-27.

Biebricher, C. K.; Nicolis, G., Schuster, P. (1995): *Self-Organization in the Physico-Chemical and Life Sciences*. EU Report 16546.

Boon, J. P. (ed.) (1991): Lattice gas automata: Theory, simulation, implementation. *Journal of Statistical Physics* 68, no. 3-4.

Bruckner, E.; Ebeling, W.; Jimenez Montano, M. A.; Scharnhorst, A. (1994): Hyperselection and innovation described by a stochastic model of technological change. In: Leydesdorff, L; van den Besselaar, P. (eds.): *Evolutionary Economics and Chaos Theory: New Directions in Technology Studies*, Pinter, London, pp. 79–90.

Cladis, P. E.; Palffy-Muhoray, P. (eds.) (1995): *Spatio-Temporal Patterns in Nonequilibrium Complex Systems*. Addison-Wesley, Reading, MA.

Crutchfield, J. P.; Hanson, J. E. (1993): Turbulent pattern bases for cellular automata. *Physica D*, 69, 279-301.

Darley, V. (1994): Emergent Phenomena and Complexity, in: Brooks, R. A.; Maes. P. (eds.): *Artificial Life IV*, MIT Press, Cambridge, MA, pp. 411–416.

DeAngelis, D. L.; Gross, L. J. (eds.) (1992): *Individual-based Models and Approaches in Ecology: Populations, Communities, and Ecosystems*. Chapman and Hall, New York.

Dendrinos, D. S.; Sonis, M. (1990): *Chaos and Socio-spatial Dynamics*, Springer, Berlin.

Ebeling, W.; Freund, J.; Schweitzer, F. (1998): *Komplexe Strukturen: Entropie und Information*. Teubner, Stuttgart.

Epstein; J. M.; Axtell, R. (1996): *Growing Artificial Societies: Social Science from the Bottom Up*. MIT Press/Brookings, Cambridge, MA.

Feistel, R.; Ebeling, W. (1989): *Evolution of Complex Systems. Self-Organization, Entropy and Development*. Kluwer, Dordrecht.

Föllmer, H. (1974): Random economies with many interacting agents. *Journal of Mathematical Economics* 1, 51–62.

Galam, S. (1997): Rational group decision making. *Physica A* 238, 66-80.

Haken, H. (1978): *Synergetics. An Introduction. Nonequilibrium Phase Transitions in Physics, Chemistry and Biology*. Springer, Berlin, 2nd edition.

Hegselmann, R. H.; Flache, A. (1998): Understanding Complex Social Dynamics: A Plea For Cellular Automata Based Modelling. *Journal of Artificial Societies and Social Simulation* 1, 3

Hegselmann, R. H.; Mueller, U.; Troitzsch, K. G. (eds.) (1996): *Modeling and simulation in the social sciences from the philosophy of science point of view*, Kluwer, Dordrecht.

Helbing, D. (1995): *Quantitative Sociodynamics. Stochastic Methods and Models of Social Interaction Processes*. Kluwer Academic, Dordrecht.

Holland, J.; Miller, J. (1991): Adaptive agents in economic theory. *American Economic Review Papers and Proceedings* 81, 365–370.

Kacperski, K.; Hołyst, J. A. (1996): Phase transitions and hysteresis in a cellular automata-based model of opinion formation. *J. Statistical Physics* 84, 169–189.

Kirman, A. (1993): Ants, rationality, and recruitment. *The Quarterly Journal of Economics* 108, 37–155.

Lam, L.; Naroditsky, V. (eds.) (1992): *Modeling Complex Phenomena*. Springer, New York.

Lam, L. (1995): Active Walker Models for Complex Systems. *Chaos, Solitons & Fractals* 6, 267-285

Lane, D. (1992): Artificial worlds and economics. *Journal of Evolutionary Economics* 3:89–107.

Lane, D.; Vescovini, R. (1996): Decision Rules and Market Share: Aggregation in an Information Contagion Model. *Industrial and Corporate Change* 5, 127-146.

Lee, Y.; Amaral, L. A. N.; Canning, D.; Meyer, M.; Stanley, H. E. (1998): Universal features in the growth dynamics of complex organizations. *Physical Review Letters* 81, 3275–3278

Levy, M.; Levy, H.; Solomon, S. (1995): Microscopic simulation of the stock market: The effect of microscopic diversity, *J. Physique I (France)* 5, 1087-1107.

Lewenstein, M.; Nowak, A.; Latané, B. (1992): Statistical mechanics of social impact, *Physical Review A* 45, 763-776.

Maes, P. (ed.) (1991): *Designing Autonomous Agents. Theory and Practice. From Biology to Engineering and Back*. MIT Press, Cambridge, MA.

Mantegna, R. N.; Stanley, H. E. (2000): *An Introduction to Econophysics*. Cambridge University Press.

Meyer, J. A.; Wilson, S. W. (eds.) (1991): *From Animals to Animats. Proc. 1st Intern. Conf. on Simulation of Adaptive Behavior*. MIT Press, Cambridge, MA.

Müller, J. P.; Wooldridge, M. J.; Jennings, N. R. (eds.) (1997): *Intelligent agents III : agent theories, architectures, and languages*. Springer, Berlin.

Niedersen, U.; Schweitzer, F. (Hrsg.) (1993): *Ästhetik und Selbstorganisation* (*Selbstorganisation. Jahrbuch für Komplexität in den Natur- Sozial- und Geisteswissenschaften*, Band 4), Duncker & Humblot, Berlin.

Parisi, J.; Müller, S. C.; Zimmermann, W. (eds.) (1998): *A Perspective Look at Nonlinear Media - From Physics to Biology and Social Sciences*, Springer, Berlin.

Sakoda, J. M. (1971): The checkerboard model of social interaction. *Journal of Mathematical Sociology* 1, 119-132.

Schelling, T. (1969): Models of segregation. *American Economic Review* 59, 488-493.

Schimansky-Geier, L.; Mieth, M.; Rosé, H.; Malchow, H. (1995): Structure formation by active brownian particles. *Physics Letters A* 207, 140–146.

Schweitzer F. (1994): Natur zwischen Ästhetik und Selbstorganisationstheorie. In: *Zum Naturbegriff der Gegenwart*, volume 2, Frommann-Holzboog, Stuttgart, pp. 93–119.

Schweitzer, F. (ed.) (1997a): *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, part 1: Evolution of Complexity and Evolutionary Optimization, part 2: Biological and Ecological Dynamcis, Socio-Economic Processes, Urban Structure Formation and Traffic Dynamics, Gordon and Breach, London.

Schweitzer, F. (1997b): Wege und Agenten: Reduktion und Konstruktion in der Selbstorganisationstheorie. In: Krug, H. J.; Pohlmann, L. (Hrsg.), *Evolution und Irreversibilität* (*Selbstorganisation. Jahrbuch für Komplexität in den Natur- Sozial- und Geisteswissenschaften*, Band 8) Duncker & Humblot, Berlin, S. 113–135.

Schweitzer, F. (1997c): Structural and functional information - an evolutionary approach to pragmatic information. *World Futures: The Journal of General Evolution* 50, 533–550.

Schweitzer, F. (1997d): Active Brownian Particles: Artificial Agents in Physics. In: Schimansky-Geier, L.; Pöschel, T. (eds.): *Stochastic Dynamics* (Lecture Notes in Physics, vol. 484), Springer, Berlin, pp. 358-371

Schweitzer, F. (1998): Modelling migration and economic agglomeration with active brownian particles. *Advances in Complex Systems* 1, 11–37.

Schweitzer, F.; Helbing, D. (eds.) (2000): *Economic Dynamics from the Physics Point of View. Physica A* 287, no. 1-2

Schweitzer, F.; Hołyst, J. (2000): Modelling collective opinion formation by means of active brownian particles. *European Physical Journal B* 15, 723–732

Schweitzer, F.; Silverberg, G. (eds.) (1998): *Evolution und Selbstorganisation in der Ökonomie / Evolution and Self-Organization in Economics* (*Selbstorganisation. Jahrbuch für Komplexität in den Natur- Sozial- und Geisteswissenschaften*, Band 9). Duncker & Humblot, Berlin.

Schweitzer, F.; Steinbrink, J. (1997): Urban cluster growth: Analysis and computer simulation of urban aggregations. In: Schweitzer, F. (ed.): *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, Gordon and Breach, London, pp. 501–518.

Schweitzer, F.; Zimmermann, J. (2000): Coordination of decisions in a spatial agent model. *Physica A*, in press.

SFB 230 (1994): *Evolution of Natural Structures*, Proceedings of the 3rd International Symposium (Mitteilungen des SFB 230, Heft 9) Stuttgart.

Silverberg, G. (1997): Is there evolution after economics? In: Schweitzer, F. (ed.): *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, Gordon and Breach, London, pp. 415–425.

Silverberg, G.; Verspagen, B. (1994): Collective learning, innovation and growth in a boundedly rational, evolutionary world. *Journal of Evolutionary Economics* 4, 207–226.

Solomon, S.; Weisbuch, G.; de Arcangelis, L.; Jan, N.; Stauffer, D. (2000): Social percolation models. *Physica A* 277, 239–247.

Troitzsch, K. G.; Mueller, U.; Gilbert, G. N.; Doran, J. E. (eds.) (1996): *Social Science Microsimulation*, Springer, Berlin.

Vallacher, R.; Nowak, A. (eds.) (1994): *Dynamical Systems in Social Psychology.* Academic Press, New York.

Veit, H.; Richter, G. (2000): The FTA design paradigm for distributed systems. *Future Generation Computer Systems* 16, 727–740.

Weidlich, W. (1972): The use of statistical models in sociology. *Collective Phenomena* 1, 51–59.

Weidlich, W. (1991): Physics and social science – the approach of synergetics. *Physics Reports* 204, 1–163.

Weidlich, W. (2000): *Sociodynamics. A Systematic Approach to Mathematical Modelling in the Social Sciences.* Harwood Academic Publishers, London.

Weidlich, W.; Haag, G. (1983): *Concepts and Methods of a Quantitative Sociology: The Dynamics of Interacting Populations*, Springer, Berlin.