Coordination of Decisions in a Spatial Model of Brownian Agents

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Abstract

Brownian agents denote a particular class of heterogeneous agents that combines features of reactive and reflexive agent concepts. As one major advance, the Brownian agent concept allows the derivation of macroscopic equations from the agent dynamics, which can be used to analyze and predict the behavior of the MAS. As an application of the concept, we discuss a binary choice problem where individual decisions are based on different local information generated by the agents. The spatial coordination of decisions in a multi-agent community is investigated both analytically and by means of stochastic computer simulations. We find that dependent on two essential parameters describing the local impact and the spatial dissemination of information either a definite stable minority/majority relation (single-attractor regime) or a broad range of possible values (multi-attractor regime) occurs. In the latter case, the outcome of the decision process becomes rather diverse and hard to predict, both with respect to the fraction of the majority and their spatial distribution. We also show that a more "efficient" information dissemination of a subpopulation provides a suitable way to stabilize their majority status and to reduce "diversity" and uncertainty in the decision process.
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1 Introduction

Discrete, individual-based or *agent-based* modeling has become a very promising and powerful methodology to describe the occurence of complex behavior in economic and social systems [1, 11, 24]. While the patterns emerging are observable only on the "macroscopic" system level, the modelling effort aims to understand their emergence from the "microscopic" level of interacting individuals [22]. The advantage of such an individual-based approach is given by the fact that it is applicable also in cases where only a small number of agents govern the further evolution. Here deterministic approaches or mean-field approximations are not sufficient to describe the behavior of the complex system. Instead, the influence of history, i.e. irreversibility, path dependence, the occurence of random events play a considerable role.

As one example, in this paper we investigate the spatial coordiation of decisions in a multi-agent system. Decision making is one of the fundamental processes in economy, but also in social systems. Among the various factors that may influence the decision of agents we mention the *information* available on a particular subject – such as the price or the quality of a particular product, in an economic context, or the benefits and harms that might result from the decision, in a social context, but also information about the decisions of others. Considering the *bounded rationality* of agents, decisions are not taken upon complete a priori information, but on incomplete, limited information that involves uncertainties and is disseminated with finite velocity. This however would require to model the information flow between the agents explicitely. A possibile approach to this problem is given by the *spatio-temporal communication field* [25, 26], that is also used in this paper (cf. Sect. 2).

Based on incomplete information, how does an agent make her decision on a particular subject? The "rational" agent usually calculates her private utility and tries to maximize it. But in a world of uncertainty it turns out that the maximization of private utilities can be only achieved by some supplemented strategies. In order to reduce the risk of making the wrong decision, it often seems to be appropriate just to copy the decisions of others. Such an *imitation* strategy is widely found in biology, but also in cultural evolution and in economics, where late entrants quite often size markets from pioneers [21]. In the case where agents can observe the *payoffs* generated by other agents *information contagion* [2] has been presented as an explanation for particular patterns of macrobehavior in economic systems, for example path-dependence and lock-in-effects [13, 28]. Information contagion however involves the transmission of two different information, the *decision* made by an agent, and the *payoff* received. The situation becomes different, when agents only observe the choices of other agents and tend to imitate them, without complete information about the possible consequences of their choices. This is commonly denoted as *herding behavior* which plays a considerable role in economic systems [3, 10], in particular in financial markets [16], but also in human and biological systems where *panic* can be observed [5].

In social systems, herding behavior may result from the many (internal or external) interdepencencies of an agent community that push or pull the individual decision into a certain direction, such as peer pressure or external influences. The *social impact theory* [7, 18] that intends to describes the transition from "private attitude to public opinion" has covered some these collective effects in a way that can be also formalized within a physical approach [12, 15]. One of the modelling impacts of the social impact theory was the "rediscovering of physical space" in sociology, i.e. distance matters for social influence [6, 14, 17]. Hence, instead of mean-field approaches where all actions of agents are coupled via a mean field, spatial models become of increasing interest. In addition to the question of how individual decisions of agents may affect the macrobehavior of the system, now the question becomes important how these desisions may organize themselves *in space*, i.e. what kind of *spatial patterns* may be observed on the global level.

2 Agent Model of Decision Making

2.1 Concept of Brownian Agents

Recently, different computer architectures in distributed artificial intelligence have been developed to simulate the collective behavior of interacting individuals or agents (cf. for instance the SWARM project at `http://www.swarm.org/`). However, due to their rather complex simulation facilities many of the currently available simulation tools lack the possibility to investigate systematically and in depth the influence of specific interactions and parameters. Instead of incorporating only as much detail as is *necessary* to produce a certain emergent behavior, they put in as much detail *as possible*, and thus reduce the chance to understand *how* emergent behavior occurs and *what* it depends on.

Therefore, it would be feasible to have multi-agent systems (MAS) that can be also investigated by means of *analytical methods* (from statistical physics or mathematics) – in addition to their computational suitability. The concept of *Brownian agents* [23] is one of the possible approaches to serve for this purpose. It denotes a particular class of agents that combines features of reactive and reflexive agent concepts.

A Brownian agent is characterized by a set of state variables that include also internal degrees of freedom. The change of these state variables is in general described by a stochastic dynamics that further considers direct and indirect interactions with other agents and external influences. To be specific, let us consider a 2-dimensional spatial system with the total area A, where a community of N agents exists. In general, N can be changed by birth and death processes but A is assumed fixed. Each agent i shall be treated as a rather autonomous entity which is assigned two individual variables: its position in space, r_i, which should be a continuous variable, and its current "opinion", θ_i (with respect to a definite aspect or problem). The latter one is a discrete valued variable representing an *internal degree of freedom* (which is a rather general view of "opinion").

For N = const., the community of agents may be described by the time-dependent canonical N-particle distribution function

$$P(\underline{\theta}, \underline{r}, t) = P(\theta_1, \boldsymbol{r}_1, ..., \theta_N, \boldsymbol{r}_N, t), \tag{1}$$

which gives the probability to find the N agents with the opinions $\theta_1, ..., \theta_N$ in the vicinity of $\boldsymbol{r}_1, ..., \boldsymbol{r}_N$ on the surface A at time t. Different from [25] we assume in this paper that the agents do *not* migrate, i.e. their positions $\boldsymbol{r}_i$ do not change stochastically. The time depentent change of $P(\underline{\theta}, \underline{r}, t)$ is then given by the following master equation [29]:

$$\frac{\partial}{\partial t} P(\underline{\theta}, \underline{r}, t) = \sum_{\underline{\theta}' \neq \underline{\theta}} \Big[w(\underline{\theta}|\underline{\theta}') P(\underline{\theta}', \underline{r}, t) - w(\underline{\theta}'|\underline{\theta}) P(\underline{\theta}, \underline{r}, t) \Big] \tag{2}$$

Eq. (2) describes the "gain" and "loss" of agents with the coordinates $\boldsymbol{r}_1, ..., \boldsymbol{r}_N$ due to opinion changes, where $w(\underline{\theta}|\underline{\theta}')$ means any possible transition within the opinion distribution $\underline{\theta}'$ which leads to the assumed distribution $\underline{\theta}$. After specifying the possible opinion changes in the next section, eq. (2) can be solved by means of stochastic computer simulations [23]. We note however the possibility to derive from eq. (2) *macroscopic* equations for e.g. the spatial distribution or the global fraction of agents sharing a particular opinion [25, 29].

2.2 Modeling Communication

As one example, let us imagine the separate disposal of recycling material. Each agent in the system needs to decide whether she will cooperate in the recycling campaign or defect. Then, there are only two (opposite) opinions, i.e. $\theta_i \in \{+1, -1\}$, or $\{+, -\}$ to be short. $\{+\}$ shall indicate the cooperating agent, and $\{-\}$ the defecting agent.

From the classical economic perspective, the agents' decision about her opinion may depend on an estimate of her *utility*, i.e. what she may gain compared to her own effort, if she decides to cooperate or not. Here, we neglect any question of utility and may simply assume that the agent will more likely do what others do with respect to the specific problem, i.e. she will decide to cooperate in the recycling campaign if most of her neighbors will do so, and defect if most of their neighbors have the same opinion in this case. This type of herding behavior in decision processes – a special kind of the imitation strategy – is well known from different fields, as discussed in Sect. 1.

This example raises the question about the interaction between agents at different locations, i.e. how is agent i at position r_i affected by the decisions of other agents at closer or far distant locations? In a checkerboard world, commonly denoted as cellular automaton, a common assumption is to consider only the influence of agents, which are at the (four or eight) nearest neigbour sites or also at the second-nearest neighbor sites, etc. Contrary, in a *mean-field approximation*, all agents are considered as influencial via a mean field, which affects each agent at the same time in the same manner.

Our approach will be different from these ones in that we will consider a continuous space and a gradual, time delayed interaction between all agents. We assume that agent i at position r_i is not directly affected by the decisions of other agents, but only receives information about their decisions via a *communication field* generated by the agents with the different opinions. This field is assumed a scalar *multi-component spatio-temporal field* $h_\theta(\boldsymbol{r},t)$, which obeys the following equation:

$$\frac{\partial}{\partial t} h_\theta(\boldsymbol{r},t) = \sum_{i=1}^{N} s_i \; \delta_{\theta,\theta_i} \; \delta(\boldsymbol{r}-\boldsymbol{r}_i) \; - \; k_\theta h_\theta(\boldsymbol{r},t) \; + \; D_\theta \Delta h_\theta(\boldsymbol{r},t). \tag{3}$$

Every agent contributes permanently to this field with her personal "strength" or influence, s_i. δ_{θ,θ_i} is the Kronecker Delta indicating that the agents contribute only to the field component which matches their opinion θ_i. $\delta(\boldsymbol{r}-\boldsymbol{r}_i)$ means Dirac's Delta function used for continuous variables, which indicates that the agents contribute to the field only at their current position, $\boldsymbol{r}_i$. The *information* generated this way has a certain life time $1/k_\theta$, further it can spread throughout the system by a diffusion-like process, where D_θ represents the diffusion constant for information dissemination. We have to take into account that there are two different opinions in the system, hence the communication field should also consist of two components, $\theta \in \{+1,-1\}$, each representing one opinion. Note, that the parameters describing the communication field, s_i, k_θ, D_θ do not necessarily have to be the same for the two opinions.

The *spatio-temporal communication field* $h_\theta(\boldsymbol{r},t)$ is used to reflect some important features of communication in social systems:

(i) the existence of a *memory*, which reflects the past experience. In our model, this memory exist as an external memory, the lifetime of which is determined by the decay rate of the field, k_θ.

(ii) the *dissemination of information* in the community with a *finite* velocity. It means that the information will eventually reach each agent in the whole system, but of course at different times.

(iii) the influence of *spatial distances* between agents. Thus, the information generated by a specific agent at position r_i will affect agents at a closer spatial distance earlier and thus with larger weight, compared to far distant agents.

The communication field $h_\theta(\boldsymbol{r},t)$ influences the agent's decisions as follows: At a certain location $\boldsymbol{r}_i$ agent i with e.g. opinion $\theta_i = +1$ is affected by two kinds of information: the information $h_{\theta=+1}(\boldsymbol{r}_i,t)$ resulting from agents who share her opinion, and the information $h_{\theta=-1}(\boldsymbol{r}_i,t)$ resulting from the opponents. The diffusion constants D_θ determine how fast she will receive any information, and the decay rate k_θ determines, how long a generated information will exist. Dependent on the information received locally, the agent has two opportunities to act: she can *change her opinion* or she can keep it. A possible ansatz for the transition rate to change the opinion reads [25]:

$$w(-\theta_i|\theta_i) = \eta \, \exp\left\{-\frac{h_\theta(\boldsymbol{r}_i,t) - h_{-\theta}(\boldsymbol{r}_i,t)}{T}\right\} \tag{4}$$

The probability to change opinion θ_i is rather small, if the local field $h_\theta(\boldsymbol{r}_i, t)$, which is related to the support of opinion θ_i, overcomes the local influence of the opposite opinion. Here, η defines the time scale of the transitions. T is a parameter which represents the *erratic circumstances* of the opinion change, based on an incomplete or incorrect transmission of information. Note, that T is measured in units of the communication field. In the limit $T \to 0$ the opinion change rests only on the difference $\Delta h(\boldsymbol{r}_i, t) = h_\theta(\boldsymbol{r}_i, t) - h_{\theta'}(\boldsymbol{r}_i, t)$, leading to "rational" decisions (cf also [4]), i.e. decisions that are totally determined by the external information. In the limit $T \to \infty$, on the other hand, the influence of the information received is attenuated, leading to "random" decisions. We note that T can be also interpreted in terms of a "social temperature" [8, 25], i.e. it is a measure for the randomness in social interaction.

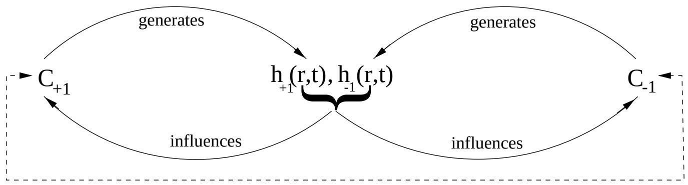

Figure 1: Circular causation between the agents, C_{-1}, C_{+1}, and the two-component communication field, $h_\theta(\boldsymbol{r}, t)$.

In order to summarize our model, we note the non-linear feedback between the agents and the communication field as shown in Fig. 1. The agents generate the field, which in turn influences their further decisions. In terms of synergetics, the field plays the role of an order parameter, which couples the individual actions, and this way initiates coherent behavior within the agent community.

3 Fast Information Dissemination

In this section, we will neglect any spatial effects of the agents distribution and the communication field [25]. This case may have some practical relevance for communities existing in small systems with short distances between different agents. In particular, in such small communities a very fast dissemination of information may hold, i.e. spatial inhomogenities in the communication field are equalized immediately. Thus, in this section, the discussion can be restricted to subpopulations with a certain opinion rather than to agents at particular locations. Let us define the fraction x_θ of a subpopulation θ and the respective mean density $\bar{n}_\theta$ in a system of size A consisting of N agents:

$$x_\theta(t) = \frac{N_\theta(t)}{N} \;; \quad \bar{n}_\theta(t) = \frac{N_\theta(t)}{A} \tag{5}$$

where the total number of agents sharing opinion θ at time t fulfils the condition $N_+(t) + N_-(t) = N = \text{const.}$, $x_+(t) = 1 - x_-(t)$. In the mean-field approach, the communication field $h_\theta(\boldsymbol{r}, t)$ can be approximated by a mean value $\bar{h}_\theta(t)$ which obeys the following dynamic equation:

$$\frac{\partial \bar{h}_\theta(t)}{\partial t} = -k_\theta \bar{h}_\theta(t) + s_\theta \bar{n}_\theta \tag{6}$$

Here, we have assumed that agents with the same opinion θ will have the same influence $s_i \to s_\theta$. The dynamic equation for the fraction of subpopulation θ can be derived from eq. (2) in the mean–field approximation as follows [25]:

$$\dot{x}_\theta \;=\; (1 - x_\theta)\, \eta\, \exp(a) - x_\theta\, \eta\, \exp(-a)\; ; \quad a = \left[\bar{h}_\theta(t) - \bar{h}_{-\theta}(t)\right] / T \tag{7}$$

Via $\Delta\bar{h}(t) = \bar{h}_\theta(t) - \bar{h}_{-\theta}(t)$, this equation is coupled to eq. (6). Let us for the moment assume that the parameters decribing the communication field are the same for both components, i.e.

$$s_+ = s_- \equiv s\; ; \quad k_+ = k_- \equiv k\; ; \quad D_+ = D_- \equiv D \tag{8}$$

The stationary solutions for the fraction of each subpopulation can be obtained from $\dot{x}_\theta = 0$, $\dot{h}_\theta = 0$. It is shown in Fig. 2 for $x_+^{\rm stat}$ dependent on a parameter κ that results from the value a, eq. (7). In the stationary limit a can be expressed as:

$$a = \kappa \left(x_+ - \frac{1}{2} \right) \quad \text{with} \quad \kappa = \frac{2s\,\bar{n}}{k\,T} \tag{9}$$

The parameter κ plays the role of a bifurcation parameter that includes the specific *internal conditions* within the community, such as the population density, the individual strength of the opinions, the life time of the information generated or the randomness T.

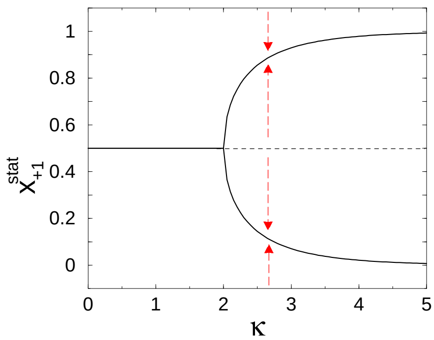

Figure 2: Stationary solutions for x_+ (eq. 7) for different values of κ. The bifurcation at the critical value $\kappa^c = 2$ is clearly visible. For $\kappa = 2.66$ used for some of the computer simulations we find in the mean field limit the stationary values $x_+ = 0.885$ and $x_+ = 0.115$ for the majority and the minority status, respectively. [27]

In [25] we found that depending on κ different stationary values for the fraction of the subpopulations exist. For $\kappa < 2$, $x_+ = 0.5$ is the only stationary solution, which means a stable community where both opposite opinions have the same influence. However, for $\kappa > 2$, the equal distribution of opinions becomes unstable, and a separation process towards a preferred opinion is obtained, where $x_{\pm} = 0.5$ plays the role of a separatrix. Then, two stable solutions are found where both opinions coexist with different fractions in the community, as shown in Fig. 2. Hence, each subpopulation can exist either as a *majority* or as a *minority* within the community. Which of these two possible situations is realized, depends in a deterministic approach on the initial fraction of the subpopulation.

From the critical condition $\kappa^c = 2$ we can derive by means of eq. (9) a *critical population size*,

$$N^c = k\,A\,T/s, \tag{10}$$

where for larger populations an equal fraction of opposite opinions is certainly unstable.[1] I.e., after a certain population growth, the community tends towards one of these opinions, thus necessarily separating into a majority and a minority.

4 Spatial Influences on Decisions

4.1 Results of Computer Simulations

The previons section has shown within a mean-field approach the emergence of a minority/majority relation in the agents community. With respect to the example of the recycling campaign adressed in the beginning, it means that *either* most of the agents decide to cooperate *or* most of them defect. The question remains how the cooperators and the defectors organize themselve in space. In order to consider the spatial dimension of the system explicitely, let us consider N agents randomly distributed in a system of size A with random initial opinions. They get information about the opinions of other agents by means of the two-component communication field $h_\theta(\boldsymbol{r}, t)$, eq. (3), which now explicitely considers space and therefore "diffusion" of information. The two-dimensional system is here treated as a torus, i.e. we assume periodic boundary conditions. Further, for the parameters we use again eq. (8).

Fig. 3 shows a snapshot of the spatial distribution of the cooperators and the defectors after a sufficient simulation time. We note that besides some stochastic fluctuations the observed coordination pattern remains stable also in the long run. Evidently, we find again the emergence of a minority and a majority, but interestingly their fractions could be not very different, as in the case of Fig. 3. The two different groups organize themselve in space in such a way that they are separated. Thus, besides the existence of a global majority, we find regions in the system which are dominated by the minority. From this we can conclude a *spatial coordination of decisions*, i.e. agents which share the

[1]We note that this critical value has been derived based on a mean field analysis and therefore does not consider finite size or discrete effects.

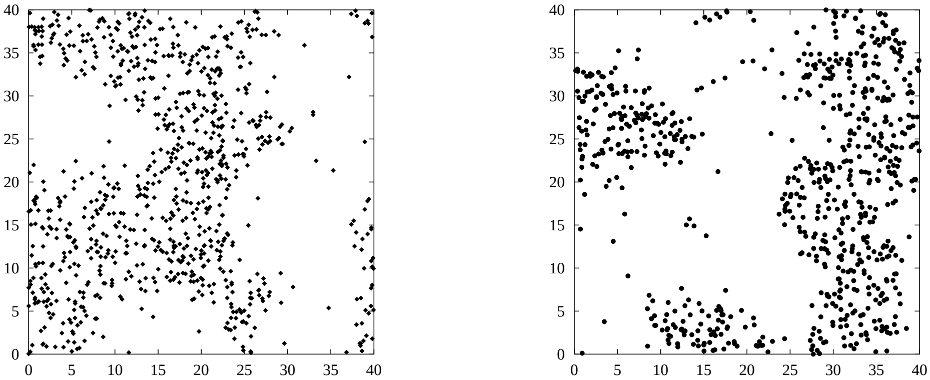

Figure 3: Snapshot of the spatial distribution of cooperators ($\diamond$ left) and defectors ($\circ$ right) at $t = 5 \cdot 10^4$. System size $A = 1600$, total number of agents $N = 1600$, $s = 0.1$, $k = 0.1$, $T = 0.75$, i.e. $\kappa = 2.66$, $D = 0.06$. In this particular realization, the frequency of collaborators is $x_+ = 0.543$ and the frequency of defectors is $x_- = 0.456$, respectively, which is very different from the mean field limit, Fig. 2. [27]

same opinion are spatially concentrated in particular regions. With respect to the example of the recycling campaign this means that those agents who cooperate (or defect in the opposite case), are mostly found in a spatial domain of a like-minded neighborhood.[2]

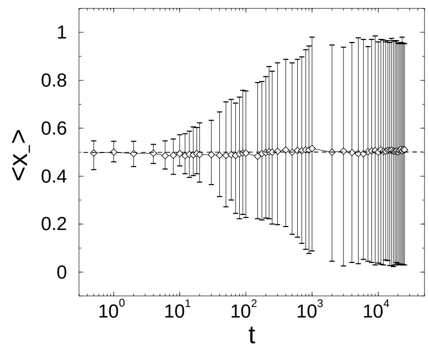

Figure 4: Relative subpopulation size $\langle x_- \rangle$ averaged over 20 runs together with minimum/maximum values. $N = 400$, $A = 400$, other parameters see Fig. 3. [27]

Running the simulations of the agent system different times with the same set of parameters but just different initial random seeds reveals an interesting effect that can be observed in Fig. 4. It shows the global fraction of agents of subpopulation $\{-1\}$ over time averaged over 20 runs. The

[2]This result might remind on the famous simulations of segregation in social systems [19, 20] - however, we would like to note that in our case the agents do *not migrate* toward supportive places; they rather *adapt* to the opinion of their neighborhood based on the information received.

mean value gives an estimate of the chance that subpopulation $\{-1\}$ becomes the minority or majority in the system, while the error bars give an estimate about the possible values. We find that the chance to become majority is about 50 percent, i.e. only random events decide about its status. Regarding the possible values, we see that instead of a single fixed majority/minority relation for the spatially extended system a *large range* of such relations exist, which even in the presence of fluctuations are stable over a very long time. We call this the *multi-attractor regime* and conclude from these simulations that the spatially extended system – under certain conditions – possesses multiple attractors for the collective dynamics that makes the outcome of the decision process hard to predict. This holds not only for the global minority/majority ratio, but also for the possible spatial patterns that correspond to the different attractors. This shall be discussed in more detail in the following section.

4.2 Results of Analytical Investigations

In [27] we have investigated analytically the attractor structure of the spatially extended system. While the mean-field case is characterized by just one bifurcation parameter, $\kappa_1 = 2$, we found that in the spatial case a new bifurcation parameter $\kappa_2(D/k)$ appears. It depends on the scaled diffusion constant D/k, a measure of the spatial coupling, as shown in Fig. 5.

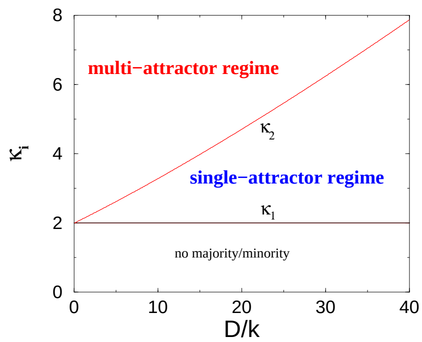

Figure 5: Change of the critical bifurcation values κ_i' (i=1,2) dependent on D/k.

κ, eq. (9) includes the specific internal conditions within the agent community, namely the population density $\bar{n}$, the production rate of information per agent s, the lifetime of information k and the randomness T that can be envisioned as a measure of the incompleteness or incorrect transformation of information. Defining $\nu = s\bar{n}/k$ as the net information density, $\kappa = \nu/T$ describes the relation between the mean information available at any location and its impact $\sim 1/T$ – in other words, the *efficiency* of the information produced. We recall that the limit $T \to 0$ means a large impact of the available information leading to "rational" decisions, whereas in the limit $T \to \infty$ the influence of the information is attenuated, leading to "random" decisions.

In order to gain at least some impact of the available information, a supercritical value of $\kappa > \kappa_1 = 2$ is needed. For $\kappa < \kappa_1$ the equal distribution of both opinions is the stable state also for the spatially heterogeneous system and only random decisions occur. For $\kappa > \kappa_1$ a majority and a minority within the agent community emerges, which organizes itself in space in the way shown e.g. in Fig. 3. Whether this minority/majority relation is characterized by a fixed value (*single-attractor regime*) or by a multitude of possible values (*multi-attractor regime*) further depends on the spatial coupling, D/k.

For a given value of κ (i.e. fixed internal conditions), we find the multi-attractor regime only for $\kappa > \kappa_2(D/k)$, i.e. in the case of *small* values of D. This means that the spatial couplings, expressed in terms of D, are not large enough to *globally* organize the system. Since κ characterizes the average *local* situation in a spatially extended system in terms of a net information production, this can be also interpreted in a way that the impact resulting from the information *dissemination* does not overcome the impact resulting from the *local* information production. Then a variety of possible spatial decision patterns can be found, and the outcome of the decision process becomes certainly unpredictable, *both* with respect to the fraction of the majority *and* to the spatial distribution.

For values of κ below $\kappa_2^c(D)$, however, these local effects become smaller, and the spatial couplings are able to organize the whole system. Thus only one minority/majority relation occurs on the global level, which relates to randomly different, but very similar spatial patterns. If we put these results in the context of a social system, we could conclude that strong local influences, expressed in a high information efficiency, can prevent the global system from being equalized and "globalized" by some ruling information. While such a *diversity* might be among the wanted effects, we note again that this on the other hand makes the system difficult to predict.

5 Breaking the Symmetry of Decisions

5.1 Influence of External Support

From the investigations in the previous section, we found the emergence of different majority/minority relations in a spatially extended agent system. So far, however, fluctuations during the initial period may decide whether the cooperators or the defectors will appear as the majority. If we start from an unbiased initial distribution, i.e. an equal distribution between both opinions, there are different ways to break the symmetry towards e.g. cooperation.

In [25] we have considered two similar cases: (i) the existence of a *strong leader* in the community, who possesses a strength s_l which is much larger than the usual strength s of the other individuals [8, 9, 25], (ii) the existence of an external field, which may result from government policy, mass media, etc. which support a certain opinion with a strength s_m. In the case of fast information dissemination discussed in Sect. 3 the additional influence $s^\star := \{s_l/A, s_m/A\}$ mainly effects the mean communication field, eq. (6), due to an extra contribution, normalized by the system size A. We found within the mean-field approach that at a critical value of $s^\star$ the possibility of a minority status completely vanishes. This is shown in Fig. 6 that shall be compared to the bifurcation diagram in Fig. 2.

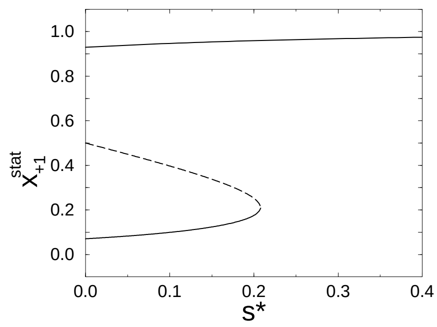

Figure 6: Stationary solutions for x_+ for different values of $s^\star$ and a fixed supercritical value $\kappa^c < \kappa = 3$. The dashed line represents the separation line for the initial conditions, which lead either to a minority or to a majority status of the supported subpopulation. [25]

Hence, for a certain supercritical external support, the supported subpopulation will grow towards a majority, regardless of its initial population size, with no chance for the opposite opinion to be established. This situation is quite often realized in communities with one strong political or religious leader ("fundamentalistic dictatorships"), or in communities driven by external forces, such as financial or military power ("banana republics").

5.2 Increase the Efficiency of Communication

Another possibility to break the symmetry of decisions exploits the different properties of the information dissemination in the system, as expressed in terms of the parameters s_θ, k_θ, D_θ of the communication field. For instance, we may assume that the information generated by of one of the subpopulations is distributed *faster* in the system than the information generated by the other one. Alternatively, we may also consider different life times of the different components of the communication field. To be consistent, we have to choose [27] that both the ratios

$$\frac{k_\theta}{s_\theta} = \beta \, ; \quad \frac{D_\theta}{s_\theta} = \gamma \tag{11}$$

need to be constant for both components $\theta = \{+1, -1\}$. In this case, eq. (3) for the dynamics of the multi-component communication field can be rewritten as:

$$\frac{\partial}{\partial \tau} h_\theta(\boldsymbol{r}, \tau) = \sum_{i=1}^{N} \delta_{\theta,\theta_i} \, \delta(\boldsymbol{r} - \boldsymbol{r}_i) \; - \; \beta \, h_\theta(\boldsymbol{r}, \tau) \; + \; \gamma \, \Delta h_\theta(\boldsymbol{r}, \tau). \tag{12}$$

where the time scale τ is now defined as $\tau = t\,(D_\theta/\gamma)$. If both parameters β and γ are kept constant, the dynamics of the respective component of the communication field occurs on a different time scale τ, dependent on the value of D_θ. An increase in the diffusion constant D_θ then models indeed the information dissemination on a faster time scale. This effect can be understood by means of computer simulations where the ratio

$$d = \frac{D_+}{D_-} \tag{13}$$

is varied.

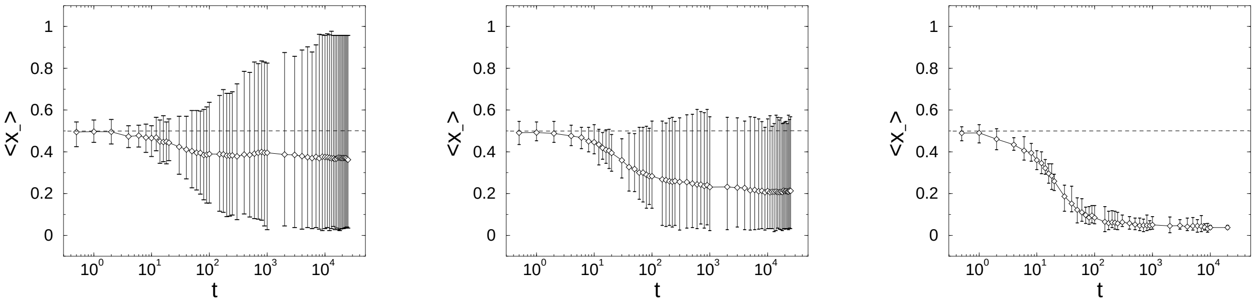

Figure 7: Relative subpopulation size $\langle x_- \rangle$ averaged over 20 runs together with minimum/maximum values: (left) $d = 1.1$, (middle) $d = 1.2$, (right) $d = 1.5$. Other parameters see Fig. 4 ($d = 1.0$). [27]

Fig. 7 shows the total fraction of agents of subpopulation $\{-1\}$ over time averaged over 20 runs, for different values of d and shall be compared to Fig. 4 for $d = 1$. Again, the mean value gives an estimate of the chance that subpopulation $\{-1\}$ becomes the minority or majority in the system, while the error bars give an estimate about the possible values. For $d = 1$ the chance for subpopulation $\{-1\}$ to become the majority in the system is about 50 percent (see Fig. 4), but with increasing d (i.e. with an increasing information diffusion of the *other* subpopulation) there is a clear trend towards the minority status for subpopulation $\{-1\}$. Inspite of this trend, we find that for $d \in \{1.1; 1.2\}$ there are still possibilities that the subpopulation $\{-1\}$ ends up as the majority in the system – even with a slower communication. Only for $d > 1.4$, theses possibitities vanish, i.e. the deviations become small enough to allow only one stable size of the minority subpopulation.

This on the other hand means that an increase/decrease of the ratio $d = D_+/D_-$ forces a crossover between the *multi-attractor* regime, where different values for a stable minority/majority ratio are possible, and the *single-attractor* regime, where only one stable minority/majority ratio exists. More efficient communication (in terms of d) enables the supported subpopulation to largely reduce the chance to become the minority and also largely reduce the uncertainty about their total fraction in the system. Another feature to be noticed by comparing Fig. 4 with Fig. 7 is the decrease of the initial time lag when the decision about which subpopulation becomes the majority is yet pending.

I.e. with increasing d there is a considerably reduced period of time for early fluctuations to break the symmetry toward one of the subpopulations.

6 Conclusions

In this paper, we have investigated the coordination of (binary) decisions in a spatially distributed agent community. We were mainly interested in how the majority and the minority of agents making a particular decision emerge in a spatially heterogeneous system and how they organize themselves in space. We observed that the formation of minority/majority subpopulations goes along with a *spatial separation process*, i.e. besides the existence of a global majority, there are regions that are dominated by the minority. Hence, a *spatial coordination* of decisions among the agents occurs. Further, we found that – different from the mean-field case – a large range of possible global minority/majority relations can be observed that refer to different spatial coordination patters.

We have investigated analytically and by means of computer simulations, under which conditions these multiple steady states occur and stable exist: (i) there should be a supercritical population density, cf. eq. (10), i.e. $\kappa > \kappa_1 = 2$, (ii) the spatial coupling in terms of information dissemination should be weak enough to prevent the system from being "globalized", i.e. $\kappa > \kappa_2(D/k)$, (iii) the dissemination of information generated by the different subpopulations should occur on comparable time scales, i.e. $d \approx 1$, in order to prevent the system from beeing "enslaved" by a dominating opinion. Tot put it differently, "efficient" information dissemination provides a suitable way to stabilize the majority status of a particular subpopulation – or to avoid "diversity" and uncertainty in the decision process.

Finally, we want to add that the toy model of communicating agents investigated in this paper may be easily modified or extended to describe other processes. Without giving up the whole framework, we may consider e.g. other types of information distribution in the system, i.e. eq. (3) for the communication field may be replaced for example by a more network-type communication among the agents. Another possible modification is regarding the decision process described in this paper by means of eq. (4). Here, we may envision various dependences on the information received from likeminded or opponent agents.

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