

Swarms of Particle Agents with Harmonic Interactions

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Abstract

Agent-based modeling is a powerful methodology to describe the occurrence of complex behavior in biological systems. The interaction of a large number of individuals (agents) may for example lead to the emergence of new forms of collective motion. In this paper, we investigate a particle-based approach to the coherent motion of a swarm with parabolic (i.e. harmonic) interactions between the agents. It is based on generalized Langevin equations for the particle agents, which take into account (i) energetic conditions for active motion, (ii) linear attractive forces between each two agents. The complex collective motion observed can be explained as the result of these different influences: the active motion of the agents, which is driven by the energy-take up, would eventually lead to a spatial dispersion of the swarm, while the mutual interaction of the agents results in a tendency of spatial concentration. In addition to particle-based computer simulations, we also provide a mathematical framework for investigating the collective dynamics.

Key words: active motion, collective behavior, Brownian particles, energy supply, swarming

1 Particle-Based Models and Biology

Discrete, individual-based modelling has become a very promising and powerful methodology to describe the occurrence of complex behavior in biological systems. This holds for instance for population dynamics (DeAngelis and Gross, 1992; Durrett and Levin, 1994a,b), but also for the collective behavior in social insects (Pasteels and Deneubourg, 1987; Bonabeau *et al.*, 1999). Trail following in ants is one of the examples, where the interplay between individual properties and collective behavior has been successfully simulated by means of particle-based models (Calenbuhr and Deneubourg,

1990; Edelstein-Keshet, 1994; Edelstein-Keshet *et al.*, 1995; Schweitzer *et al.*, 1997). Also different forms of biological structures, namely biological aggregates in different species such as slime molds, bacteria, larvae, or in cells have been modeled within a particle-based approach (Ben-Jacob *et al.*, 1994; Grünbaum and Okubo, 1994; Othmer and Stevens, 1997; Stevens, 1993, 1995; Stevens and Schweitzer, 1997; Flierl *et al.*, 1999; Deutsch, 1999; Deutsch and Lawniczak, 1999).

While the patterns emerging are observable only on the “macroscopic” system level, the modelling effort aims to understand their emergence from the “microscopic” level of interacting individuals (Schweitzer, 1997b). The advantage of such an individual-based approach is given by the fact that it is applicable also in cases where only a small number of actors (individuals, particles, or *agents* in general) govern the further evolution. Here deterministic approaches or mean-field equations are not sufficient to describe the behavior of the complex system. Instead, the influence of history, i.e. irreversibility, path dependence, the occurrence of random events/stochastic fluctuations play a considerable role.

The fact that most of the complex features of collective behavior may emerge from rather simple interactions between a large number of individuals – without any external guidance or control and with no central coordination – has recently been dubbed “swarm intelligence” (Bonabeau *et al.*, 1999). This computational approach for solving distributed problems has already found applications in network optimization (“ant colony optimization”) (Bonabeau *et al.*, 2000).

But the metaphor of a “swarm” serves also as a blueprint for different computer architectures in distributed artificial intelligence (cf. for instance the SWARM project at <http://www.swarm.org/>). However, due to their rather complex simulation facilities many of the currently available simulation tools lack the possibility to investigate systematically and in depth the influence of specific interactions and parameters. Instead of incorporating only as much detail as is *necessary* to produce a certain emergent behavior, they put in as much detail *as possible*, and thus reduce the chance to understand *how* emergent behavior occurs and *what* it depends on.

Therefore, it would be also feasible to have multi-agent or “swarm” models that can be also investigated by means of analytical methods (from statistical physics or mathematics) – in addition to their computational suitability. In this respect, a number of different approaches have been developed to serve as a framework for individual-based modeling. They all have to deal with the problem how to represent (i) physical space and time, (ii) external, environmental (boundary) conditions, (iii) discrete, local individual interactions, (iv) stochastic influences appropriately. Cellular automata (or lattice gas models in physics) for instance use a discretized space and time concept (Phipps, 1992; Deutsch and Lawniczak, 1999), regarding their interaction rules, they can be deterministic or stochastic.

But physics with its long-lasting history in describing interacting many-particle systems, has also contributed “microscopic” models that are *discrete* on the particle level, but *continuous* in space and time, namely molecular dynamics as a simulation technique. Another such particle-based approach we deal with in this paper, is based on *active Brownian particles*. (Schimansky-Geier *et al.*, 1995; Schweitzer *et al.*, 1998; Ebeling *et al.*, 1999; Schweitzer *et al.*, 2000; Erdmann *et al.*, 2000). These are *particle agents* with the ability to take up energy from the environment, that can be used for different “activities”, such as active motion, chemical communication, environmental changes.

While such features are evidently inspired by biology, we want to note explicitly, that we do *not* intend to model a particular biological system. Our aim is merely to extend a known (physical) dynamics by some plausible arguments. By adding some more complexity to a particle system, we investigate the possibilities to obtain dynamic phenomena, which may find an analogy also in biological systems - this way (maybe?) bridging the gap between physical and biological dynamics.

The basic dynamics of the particle agents is a modified Langevin equation, i.e. a *stochastic* equation that reflects both random and deterministic influences on the particles. The take-up of energy and the dissipative forces which result e.g. from the friction of the moving particles are considered in a *dissipation function*, that depends on the velocity of the particle.

While “simple” Brownian particles do not interact, active Brownian particles or particle agents have the ability to interact with each other due to local or global couplings. Different forms of interactions have been investigated so far: (i) interactions via a self-consistent field that has been created by the particles and in turn influences their further movement and/or “behavior” (Schimansky-Geier *et al.*, 1995, 1997; Helbing *et al.*, 1997), (ii) interactions via global couplings, such as coupling to the center of mass, coupling via the mean momentum or mean angular momentum or a combined set of invariants of motion for a canonical-dissipative dynamics (Czirok *et al.*, 1996; Mikhailov and Zanette, 1999; Schweitzer *et al.*, 2001).

In this paper, we will concentrate on just one particular coupling, namely to the center of mass. We show that such a global coupling results from linear attractive forces between each two particles. An *ensemble* of N particle agents is then called a *harmonic swarm*, since the resulting interaction potential is of quadratic type. By means of computer simulations, we find that such a *swarm* displays a rather *complex collective motion*. This is the result of two counteractive influences: (i) the *active* motion of the particle agents, which is driven by the energy-take up and allows the particles to move in a “high-velocity mode” (Tilch *et al.*, 1999; Schweitzer *et al.*, 2000) – this would eventually lead to a *spatial dispersion* of the swarm, (ii) the mutual attraction of the particle agents, resulting in a tendency of *spatial concentration*.

The paper is organized as follows: In Sect. 2, we first investigate the motion of a *single* active Brownian particle in a harmonic external potential. This will lead to some general insights of the conditions for “active motion” of particle agents. In Sect. 3, we deal with the motion of a swarm of *free* particle agents, which do not interact, in order to discuss the velocity distribution function in comparison to the Maxwellian distribution of “simple” Brownian particles. In Sect. 4, we introduce harmonic attractive interactions between the particles, which lead to a so-called “harmonic” swarm and investigate the complex motion of the swarm. We further show that for certain approximations many of the results discussed for the special cases of Sect. 2 und Sect. 3 can be rediscovered. In Sect. 5, we conclude the discussion by pointing to some future directions of a particle-based modeling within the framework of active Brownian particles and particle agents.

2 Langevin dynamics of one particle agent in an external harmonic field

In order to discuss first some basic features of the active motion of particle agents, we restrict the discussion in the following to a single particle moving in an external potential $U(\mathbf{r})$. In general $U(\mathbf{r})$ can be regarded as an environmental potential, which may also consist of contributions that can be interactively changed by the particles (Lam, 1995; Schweitzer and Schimansky-Geier, 1996; Schweitzer, 1997a, 2001). Here, we simply consider an external harmonic potential in two dimensions (Ebeling *et al.*, 1999):

$$U(x_1, x_2) = \frac{a}{2}(x_1^2 + x_2^2) \quad (1)$$

The parabolic potential, eq. (1), originates a force directed to the minimum of the potential. In a biological context, it simply models a “home”, and the moving object always feels a driving force pointing back to its “nest” (Ebeling *et al.*, 1999).

The corresponding one-particle Hamiltonian which considers both the kinetic and the potential energy of the particle reads:

$$H = \frac{m}{2}(v_1^2 + v_2^2) + \frac{a}{2}(x_1^2 + x_2^2) \quad (2)$$

For simplicity, $m = 1$ is used in the following.

The dynamics of the particle agent is described by a generalized LANGEVIN equation that reads (Ebeling *et al.*, 1999; Schweitzer *et al.*, 2000):

$$\dot{\mathbf{r}} = \mathbf{v}; \quad \dot{\mathbf{v}} = -g(v^2)\mathbf{v} - \nabla U(\mathbf{r}) + \sqrt{2D}\boldsymbol{\xi}(t) \quad (3)$$

The variable \mathbf{r} is used for the space coordinate of the particle, and \mathbf{v} stands for its velocity. The last term of eq. (3) denotes a stochastic force with a strength D that can be expressed by the fluctuation-dissipation theorem (EINSTEIN relation):

$$D = k_B T \gamma_0 \quad (4)$$

T is the temperature, k_B is the BOLTZMANN constant and γ_0 is the friction coefficient of the moving particle. The random function $\boldsymbol{\xi}(t)$ is assumed to be Gaussian white noise.

The function $g(v^2)$ in eq. (3) denotes a *non-linear dissipation function* which may depend on the velocity of the particle as follows (Erdmann *et al.*, 2000; Schweitzer *et al.*, 2001):

$$g(v^2) = \gamma_0 - \frac{q_0 d_2}{c + d_2 v^2} \quad (5)$$

The parameters q_0 , d_2 and c will be explained below. The meaning of the velocity dependence of $g(v^2)$ becomes obvious when looking at Fig. 1.

Provided that the relation

$$q_0 d_2 > \gamma_0 c \quad (6)$$

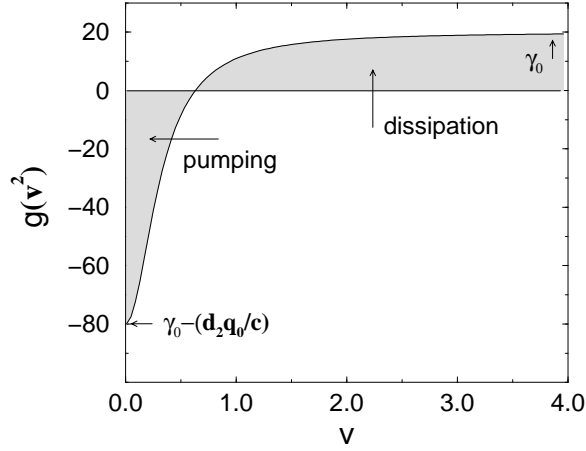


Figure 1: Velocity-dependent dissipation function, $g(v^2)$, eq. (5) vs. v . The velocity ranges for “pumping” ($g(v^2) < 0$) and “dissipation” ($g(v^2) > 0$) are indicated. Parameters: $q_0 = 10$; $c = 1.0$; $\gamma_0 = 20$, $d_2 = 10$.

is fulfilled, we find a zero of the dissipation function for the real velocity \mathbf{v}_0 :

$$\mathbf{v}_0^2 = \frac{q_0}{\gamma_0} - \frac{c}{d_2} \quad (7)$$

which allows us to express the dissipation function, eq. (5) as:

$$g(v^2) = \gamma_0 \frac{(v^2 - \mathbf{v}_0^2)}{(q_0/\gamma_0) + (v^2 - \mathbf{v}_0^2)} \quad (8)$$

We see that for $v^2 \gg \mathbf{v}_0^2$, i.e. in the range of large velocities, $g(v^2)$ approaches the “normal” friction coefficient, γ_0 . This range is denoted as *passive friction*, which also characterizes the motion of “simple” Brownian particles. However, for $v^2 \leq \mathbf{v}_0^2$, i.e. in the range of small velocities, $g(v^2)$ can also become *negative*, which means that the motion of the particle is accelerated. This is of course only possible for $\mathbf{v}_0 > 0$, i.e. for a supercritical influx of energy. In this case, the passive mode of motion of “usual” Brownian particles could be transformed into *active mode of motion* which is characterized by different new features (Tilch *et al.*, 1999; Schweitzer *et al.*, 2000; Erdmann *et al.*, 2000). The parameter q_0 denotes here the take-up of energy from the environment. c represents the internal dissipation (“metabolism”) of the particles, while d_2 describes the conversion of energy taken up into kinetic energy. These parameters are related to “activities” which clearly exceed the concept of a pure physical particle, therefore the term *particle agent* is chosen. This concept combines both physical features (such as the stochastic dynamics) and biological features (energy take-up, active motion). Due to the pumping mechanism considered in our model, the conservation of energy clearly does not hold for particle agents, i.e. we now have a non-equilibrium, canonical-dissipative system (Feistel and Ebeling, 1989; Schweitzer *et al.*, 2001).

Negative friction plays an important role e.g. in technical constructions, but also in biological motion. For example, Schienbein and Gruler (1993) have introduced a velocity dependent dissipation

function with a zero v_0 that describes the active motion of different cell types, such as granulocytes, monocytes or neural crest cells

$$g(\mathbf{v}) = \gamma_0 \left(1 - \frac{v_0}{v} \right) \quad (9)$$

Here, the speed v_0 expresses the fact that the motion of cells is not only driven by stochastic forces, instead cells are also capable of self-driven motion. We note that the velocity-dependent dissipation function, eq. (5) avoids the singularity for $\mathbf{v} \rightarrow 0$ which appears in eq. (9).

For the given external potential, eq. (1) and the dissipation function, eq. (5), the dynamics of an particle agent in a two-dimensional space is explicitly described by four coupled first-order differential equations:

$$\begin{aligned} \dot{x}_1 = v_1 \quad ; \quad \dot{v}_1 &= - \left(\gamma_0 - \frac{d_2 q_0}{c + d_2 (v_1^2 + v_2^2)} \right) v_1 - a x_1 + (2D)^{1/2} \xi(t) \\ \dot{x}_2 = v_2 \quad ; \quad \dot{v}_2 &= - \left(\gamma_0 - \frac{d_2 q_0}{c + d_2 (v_1^2 + v_2^2)} \right) v_2 - a x_2 + (2D)^{1/2} \xi(t) \end{aligned} \quad (10)$$

As shown by Ebeling *et al.* (1999), in the deterministic case ($D = 0$) the individual particle can move on limit cycles with the amplitude

$$r_0 = \frac{v_0}{a^{1/2}} \quad (11)$$

In the two-dimensional space $\{x_1, x_2\}$ the stationary velocity v_0 , eq. (7), where the friction is just compensated by the energy supply, defines a cylinder in the four-dimensional state space $\{x_1, x_2, v_1, v_2\}$, which attracts all deterministic trajectories of the dynamic system. Hence, we find a periodic motion of the particle on a four-dimensional limit cycle. The projection of this limit cycle to the $\{v_1, v_2\}$ plane is given by the circle:

$$v_1^2 + v_2^2 = v_0^2 = \text{const.} \quad (12)$$

while the projection to the $\{x_1, x_2\}$ plane also corresponds to a circle

$$x_1^2 + x_2^2 = r_0^2 = \text{const} \quad (13)$$

where r_0 is given by eq. (11). The energy for the motion on the limit cycle is then given by:

$$E_0 = \frac{m}{2} (v_1^2 + v_2^2) + \frac{a}{2} (x_1^2 + x_2^2) = \frac{m}{2} v_0^2 + \frac{a}{2} r_0^2 \quad (14)$$

Ebeling *et al.* (1999) have shown that any initial value of the energy converges (at least in the limit of strong pumping) to

$$H \longrightarrow E_0 = m v_0^2 \quad (15)$$

This corresponds to an equal distribution between kinetic and potential energy. As for the harmonic oscillator in one dimension, both parts contribute with the same amount to the total energy. This result was obtained by Ebeling *et al.* (1999) based on the assumption that the energy is a slow

(adiabatic) variable which allows a phase average with respect to the phases of the rotation. In explicit form we may represent the motion on the limit cycle in the four-dimensional space by the four equations:

$$\begin{aligned} x_1 &= r_0 \sin(\omega_0 t + \phi) & ; & & v_1 &= r_0 \omega_0 \cos(\omega_0 t + \phi) \\ x_2 &= r_0 \cos(\omega_0 t + \phi) & ; & & v_2 &= -r_0 \omega_0 \sin(\omega_0 t + \phi) \end{aligned} \quad (16)$$

The frequency ω_0 is given by the time the particle needs for one period moving on the limit circle with radius r_0 with constant speed v_0 . This leads to the relation:

$$\omega_0 = \frac{r_0}{v_0} = \left(\frac{m}{a} \right)^{1/2} = \omega \quad (17)$$

This means, the particle oscillates even in the limit of strong pumping (at least in our approximation) with a frequency given by the linear oscillator frequency ω .

The trajectory defined by eq. (16) is like a hoop in the four-dimensional space. Thus, most projections to the two-dimensional subspaces are circles or ellipses; however there are two subspaces, namely $\{x_1, v_2\}$ and $\{x_2, v_1\}$, where the projection is like a rod (Erdmann *et al.*, 2000). A second limit cycle is obtained by time reversal, $t \rightarrow -t$, $v_1 \rightarrow -v_1$, $v_2 \rightarrow -v_2$. This limit cycle also forms a hula hoop which is different from the first one in that the projection to the $\{x_1, x_2\}$ plane has the opposite rotation direction compared to the first one. However both limit cycles have the same projections to the $\{x_1, x_2\}$ and to the $\{v_1, v_2\}$ plane. The separatrix between the two attractor regions is given by the following plane in the four-dimensional space Erdmann *et al.* (2000):

$$(\omega_0 x_1 - v_1) + (\omega_0 x_2 - v_2) = 0 \quad (18)$$

3 Langevin dynamics and velocity distribution function for free swarms

After summarizing some main results for the motion of a single active particle, we now turn to the case of a many-particle system, i.e. we investigate an ensemble of N particle agents which are regarded as a “swarm”. In this section, we assume that there are no interactions between the particles, and no external potential shall be present. The Hamiltonian for the swarm reads then:

$$H = \sum_{i=1}^N \frac{m}{2} v_i^2 \quad (19)$$

Without interactions the dynamics of the individuals in the swarm is independent and we may reduce the description level from the N -particle distribution function to the one-particle distribution function $P(\mathbf{r}_i, \mathbf{v}_i)$:

$$P(\mathbf{r}_1 \dots \mathbf{r}_N, \mathbf{v}_1 \dots \mathbf{v}_N) = \prod_{i=1}^N P(\mathbf{r}_i, \mathbf{v}_i) \quad (20)$$

The Langevin equation for each particle agent reads with $m = 1$ and in the absence of an external potential:

$$\dot{\mathbf{r}}_i = \mathbf{v}_i; \quad \dot{\mathbf{v}}_i = -g(v_i^2) \mathbf{v}_i + (2D)^{1/2} \boldsymbol{\xi}_i(t) \quad (21)$$

For simplicity we consider for the moment the swarm in a system of coordinates where the center of mass is at rest. For the dissipation function $g(v_i^2)$, the ansatz of eq. (5) is used again. This means that the motion of the active particles is driven by two forces: a stochastic force with the strength D , and the acceleration/deceleration resulting from the pumping/dissipation of energy due to a velocity-dependent dissipation function $g(v^2)$ (Schweitzer *et al.*, 1998; Ebeling *et al.*, 1999; Schweitzer *et al.*, 2001).

This additional pumping will result in deviations from the known MAXWELLIAN velocity distribution of an equilibrium canonical system. The probability density for the velocity $P(\mathbf{v}, t)$ obeys a Fokker-Planck equation, which reads for the special case of the dissipation function, eq. (5):

$$\frac{\partial P(\mathbf{v}, t)}{\partial t} = \frac{\partial}{\partial \mathbf{v}} \left[\left(\gamma_0 - \frac{d_2 q_0}{c + d_2 v^2} \right) \mathbf{v} P(\mathbf{v}, t) + D \frac{\partial P(\mathbf{v}, t)}{\partial \mathbf{v}} \right] \quad (22)$$

We mention that Fokker-Planck equations with nonlinear friction functions are discussed in detail by Klimontovich (1994). For the stationary solution of eq. (22), we find (Erdmann *et al.*, 2000):

$$\begin{aligned} P^0(\mathbf{v}) &= C \left(1 + \frac{d_2 v^2}{c} \right)^{\frac{q_0}{2D}} \exp \left(-\frac{\gamma_0}{2D} v^2 \right) \\ &= C \exp \left(-\frac{G_0(v^2)}{D} \right) \end{aligned} \quad (23)$$

where C results from the normalization condition. The stationary solution can be expressed by means of a velocity-dependent *dissipation potential* $G_0(v^2)$ that reads for the case of eq. (5) explicitly:

$$G_0(v^2) = \gamma_0 \frac{v^2}{2} - \frac{q_0}{2} \ln \left(1 + \frac{d_2 v^2}{c} \right) \quad (24)$$

Compared to the Maxwellian velocity distribution of “simple” Brownian particles, a new prefactor appears now in eq. (23) which results from the additional pumping of energy. For subcritical pumping, $q_0 d_2 < c \gamma_0$, only an *unimodal velocity distribution* results, centered around the maximum $\mathbf{v}_0 = 0$ (cf. Fig. 2 left). However, for supercritical pumping, $q_0 d_2 > c \gamma_0$, we find in the two-dimensional case a *crater-like velocity distribution*, the maxima of which are given by the roots of v_0^2 , eq. (7) (cf. Fig. 2 right). The distribution clearly indicates strong deviations from the Maxwell distribution (Erdmann *et al.*, 2000; Schweitzer *et al.*, 2001). We note that these non-Maxwellian velocity distributions have been also observed experimentally in cells, such as granulocytes (Franke and Gruler, 1990; Schienbein and Gruler, 1993).

As Fig. 2 shows, the stationary velocity distribution is centered around $\mathbf{v} = 0$ both for a subcritical and a supercritical pumping. In the limiting case of strong activation, i.e. relatively weak noise $D \sim T \rightarrow 0$ and/or strong pumping, we find a δ -distribution:

$$P^0(\mathbf{v}) = \text{const.} \delta \left(v_0^2 - \mathbf{v}^2 \right) \quad (25)$$

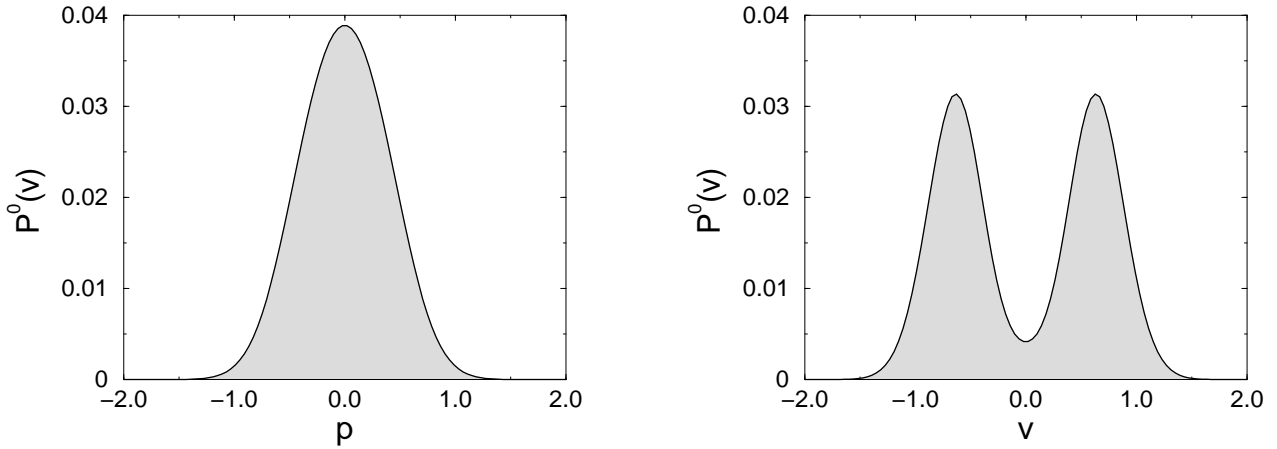


Figure 2: Stationary velocity distribution $P^0(v)$, eq. (23) (shown in a one-dimensional projection) for two different values of the conversion parameter d_2 : (left) $d_2 = 1.0$, (right) $d_2 = 10.0$; $D = 2$, other parameters see Fig. 1.

which is characteristic for a microcanonical ensemble. An interesting feature of the probability distribution, eq. (25) can be observed when looking at the integrated probability density according to one definite velocity component, say e.g. v_1 , $P_1^0(v_1)$. This is a projection obtained by integration of eq. (25) with respect to the other velocity components. In the most general d -dimensional case, the integrated distribution is given by (Rateitschak *et al.*, 2000):

$$P_1^0(v_1) = \frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{d-1}{2}\right) v_0^{d-2}} \left(v_0^2 - v_1^2\right)^{\frac{d-3}{2}} \quad (26)$$

where $\Gamma(x)$ denotes the Gamma function and d is the dimensionality of the system. The d dependence leads to the notable result that for one-dimensional and two-dimensional systems, i.e. $d = 1$ and $d = 2$, the shape of the probability distribution is dip-like, as shown in Fig. 2 right). However, for $d = 3$ the distribution is constant for $v_1 < v_0$, i.e. the characteristic dip disappears at $v_1 = 0$.

That means, only for one-dimensional and two-dimensional systems and supercritical pumping the velocity distribution of active particles shows some clear maxima at $v^2 \approx v_0^2$ and a clear minimum at $v^2 = 0$, indicating the tendency of the particles to move with a non-Maxwellian velocity. In the following we restrict our consideration to $d = 2$ again.

4 Langevin dynamics for harmonic swarms

Let us now turn to a swarm of particle agents which interact mutually due to a harmonic potential that depends on the spatial positions of the particles. This type of swarms is called *harmonic swarm* in the following. In addition to the kinetic energy, eq. (19), we have to consider a quadratic

interaction term, and the Hamiltonian for the ensemble of N particle agents reads now:

$$H = \sum_{i=1}^N \frac{m}{2} v_i^2 + \frac{a}{4} \sum_{i=1}^N \sum_{j=1}^N (\mathbf{r}_i - \mathbf{r}_j)^2 \quad (27)$$

The interaction term results in linear attractive forces between all pairs of agents. The force acting on particle i reads:

$$\mathbf{F}_i = -a (\mathbf{r}_i - \mathbf{R}(t)) \quad (28)$$

where \mathbf{R} denotes the center of mass of the swarm:

$$\mathbf{R}(t) = \frac{1}{N} \sum_{i=1}^N \mathbf{r}_i(t) \quad (29)$$

The LANGEVIN equation for the motion of the particle agents reads in this case:

$$\dot{\mathbf{r}}_i = \mathbf{v}_i; \quad \dot{\mathbf{v}}_i = -g(v_i^2) \mathbf{v}_i - a (\mathbf{r}_i - \mathbf{R}(t)) + (2D)^{1/2} \boldsymbol{\xi}_i(t) \quad (30)$$

For the dissipation function $g(v^2)$, eq. (5) is used again. In order to get some insight into the dynamics of the swarm, Fig. 3 presents snapshots of a computer simulation of eq. (30) for 2.000 particle agents. (A movie of these computer simulations – with the same parameters, but a different random seed – can be found at <http://ais.gmd.de/~frank/swarm-tb.html>). Here, we have assumed that the particles are initially at rest and at the same spatial position. Due to a supercritical take-up of energy, the agents are able to move actively, the interaction however prevents the swarm from simply dispersing in space. Thus, the collective motion of the swarm becomes rather complex, as a compromise between *spatial dispersion* (driven by the energy pumping) and *spatial concentration* (driven by the mutual interaction).

With the assumed harmonic interaction, the motion of the swarm can be considered as a superposition of two motions: (i) the motion of the center of mass itself, and (ii) the motion of the particle agents relative to the center of mass. Taking into account that the noise acting on the different particles is not correlated, the center of mass for the assumed coupling obeys a force-free motion,

$$\dot{\mathbf{R}} = \mathbf{V}; \quad \dot{\mathbf{V}} = -\frac{1}{N} \sum_{i=1}^N g(v_i^2) \mathbf{v}_i \quad (31)$$

Because of the nonlinearities in the dissipation function $g(v^2)$ both motions (i) and (ii) cannot be simply separated (cf. also Mikhailov and Zanette 1999). However, Mikhailov and Zanette (1999) and Schweitzer *et al.* (2001) have also shown that for the initial condition $v_i^2(t=0) < v_0^2$, eq. (7) the motion of the center of mass comes to rest after some time. This can be also seen in Fig. 4 which shows the result of the computer simulations presented in Fig. 3. We find for the squared velocity of the center of mass $V^2(t) \rightarrow 0$, while the averaged squared velocity of the swarm reaches the known stationary velocity, $v^2(t) = 1/N \sum v_i^2(t) \rightarrow v_0^2$, eq. (7).

Consequently, after a rather short transient time, we may use an adiabatic approximation that assumes $R(t)$ and $V(t)$ for the center of mass as slowly varying quantities. It means $\dot{R}(t) \approx 0$

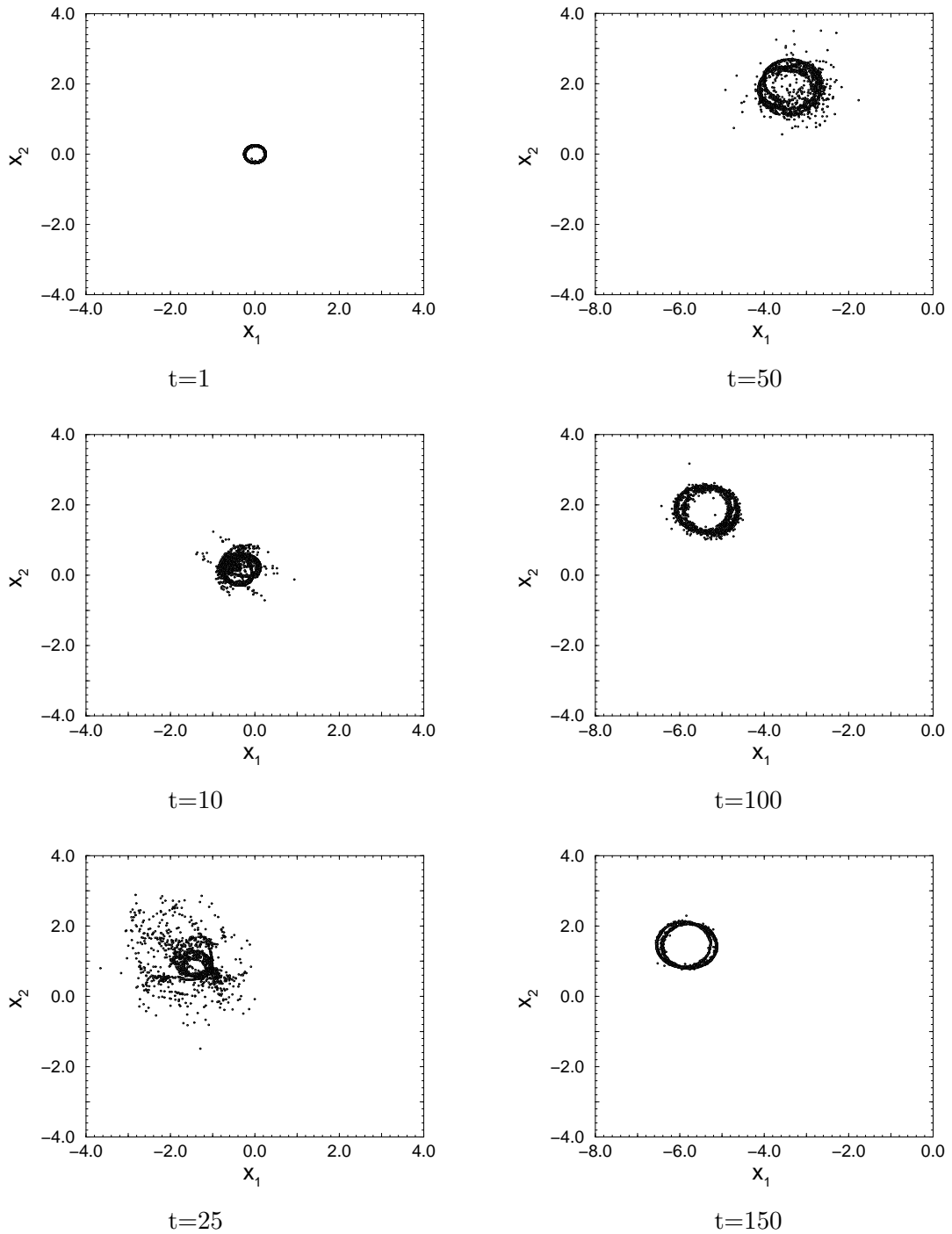


Figure 3: Snapshots (spatial coordinates) of a swarm of 2000 particle agents moving according to eq. (30). t gives the different times. Note that the pictures in the right column have a shifted x_1 -axis compared to the left column. Initial conditions at $t = 0$: $\{x_{1i}, x_{2i}\} = \{0.0, 0.0\}$, $\{v_{1i}, v_{2i}\} = \{0.0, 0.0\}$ for all particles. Parameters: $a = 1$, $D = 10^{-8}$, $q_0 = 10.0$; $c = 1.0$; $\gamma_0 = 20$, $d_2 = 10$.

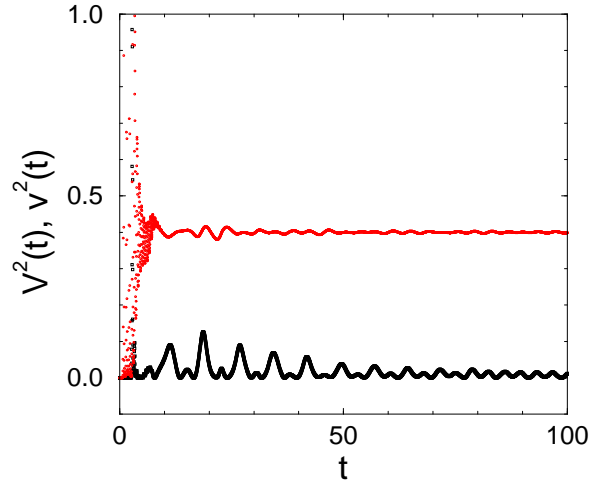


Figure 4: Squared velocity of the center of mass, $P^2(t) = \left(N^{-1} \sum_i \mathbf{p}_i(t)\right)^2$ (black dots) and averaged squared velocity $p^2(t) = N^{-1} \sum p_i^2(t)$ (grey dots) for the simulation shown in Fig. 3.

and $\dot{V}(t) \approx 0$ compared to the motion of the individual particles. Then the coupling between the movement of the center of mass and the movement of the agents relative to it can be neglected.

In this case however, the dynamics for the *relative* motion of the interacting particles is equivalent to the motion of *free* (or uncoupled) particles in an external parabolic potential $U(x_1, x_2) = a(x_1^2 + x_2^2)/2$ with the origin $\{0, 0\}$. Therefore, many of the results obtained in Sect. 2 for a single particle moving in a parabolic potential can be rediscovered. On the other hand, for the *relative* motion of the particles also some of the results obtained in Sect. 3 for the motion of a free swarm are still valid. For example, in the adiabatic approximation the form of the stationary distribution function $P^0(\mathbf{v})$, eq. (23) can be obtained again for the *relative velocity* of the harmonic swarm.

In particular, we also note the motion of the swarm on two limit cycles, the radius of which is given by eq. (11). These limit cycles can be clearly observed in Fig. 3 at $t = 150$. They result from a *spontaneous symmetry break* in the motion of the swarm during the early stage. One of the limit cycles refers to the left-handed, the other one to the right-handed direction of motion in the two-dimensional space. This finding also agrees with the theoretical investigations of the deterministic case (Ebeling *et al.*, 1999).

5 Conclusions and Future Directions

In this paper, we have investigated a particle-based model for swarming, i.e. the coherent collective motion of an ensemble of “individuals” (particles, agents). This genuine biological phenomenon, widely observed in schools of fish, flocks of birds, herds of hoof animals, or swarms of insects, recently also attracted the interest of *physicists*. Here, the question how a long-range order between

the moving entities can be established is of particular interest. Consequently, some of the more biologically centered questions of swarming behavior, namely about the reasons of swarming or the group size dependence, have been dropped so far in physical swarm models. The main focus was rather on the emergence of coherent motion in a “swarm” of locally or globally coupled particles (Czirok *et al.*, 1996, 1999; Mikhailov and Zanette, 1999; Czirok and Vicsek, 2000; Schweitzer *et al.*, 2001).

But long-range or short-range coupling of the particles is only one of the prerequisites that account for swarming. Another one is the *active motion* of the particles. Of course, particles can also move passively, driven by thermal noise, by convection, currents or by external fields. This kind of driving force however does not allow the particle to change its direction of motion, or velocity *etc. itself*. Recent models of *self-driven* particles which are used to simulate swarming behavior (Vicsek *et al.*, 1995; Mikhailov and Meinköhn, 1997; Helbing and Vicsek, 1999) usually just postulate that the entities move with a certain non-zero velocity, without considering the *energetic* implications of active motion. In order to do so, we need to consider that the many-particle system is basically an *open* system which is driven into non-equilibrium.

The approach of particle agents (active Brownian particles) discussed in this paper provides a suitable framework to consider both the *energetic conditions* for active motion and the *interactions* between the particles. It is based on a stochastic equation of motion for each individual, that considers the take-up/dissipation of energy in a *dissipation function* and the forces resulting from non-linear interactions. In this paper we have shown that both influences may result in a rather complex motion of the swarm. The collective motion observed on the “macroscopic” level shows interesting analogies to swarming phenomena in biological systems, but we note again that we do *not* intend to model a particular biological system. We are basically interested in *which extensions* to a known (physical) dynamics might bridge the gap towards a more complex (biological) dynamics.

A number of applications in very different fields have proved that such a stepping stone strategy is quite promising. Models based on Brownian dynamics are already used for describing new types of complex dynamics in micro-biological systems, such as Brownian motors or directed transport (Derenyi and Vicsek, 1995; Hänggi and Bartussek, 1996; Astumian, 1997; Tilch *et al.*, 1999). Adding features like take-up and storage of energy to the Brownian particles, or the capability to “communicate” via the exchange of information, we may observe even more complex phenomena. In particular, a particle-based approach to *interactive structure formation* can be developed (Schimansky-Geier *et al.*, 1995, 1997; Schweitzer, 1997a). It is based on the idea that the active particles (or *active walkers* within a discrete approximation) are able to generate a self-consistent field, which in turn influences their further movement and physical or chemical behavior. This non-linear feedback between the particles and the field generated by themselves results in the emergence of structure on the macroscopic level. Hence, these models have been used to simulate a broad variety of patterns, ranging from physical to biological and social systems (Lam, 1995; Lam *et al.*, 1995; Schweitzer and Schimansky-Geier, 1996; Schweitzer *et al.*, 1997; Helbing *et al.*, 1997; Schweitzer, 1998; Schweitzer and Holyst, 2000).

Generalizing the approach of particle agents by including more complex possibilities of direct and indirect interactions, we may be able to define a rather general tool – denoted as *Brownian agents*

– for describing and simulating complex interactive systems (Schweitzer, 2001). In general, agents are regarded as relatively autonomous entities which, dependent on the context of the model, may represent local processes, individuals, species, agglomerates, chemical components, etc. These entities have a set of different rules to interact with each other. Which of the rules applies for a specific case, may also depend on local variables, which in turn can be influenced by the (inter)action of the agents. A *multi-agent system* (MAS) then may consist of a *large number* of agents, which can be also of *different types*. The complex behavior of the multi-agent system as a whole basically depends (i) on the complexity of the agent (i.e. the range of possible actions), (ii) on the complexity of the interaction.

Different from complex agents that are used e.g. in software technologies or in economic applications, a *Brownian agent* is meant to be a subunit with an “intermediate” complexity. This means on one hand that the agent is not assumed as a “physical” particle only reacting to external forces, but on the other hand should not already have the same complex capabilities as the whole system. Instead the agent should be characterized by some “activity”, which of course may depend on the system the model is applied to. Some possible ideas are given in this paper.

The restriction to a rather “minimalistic” agent design raises the problem that some types of “biological agents” may have a much larger variety of properties and interactions than “Brownian agents”. But, as the results in this paper and the different examples mentioned elucidate, particular classes of collective behavior can to a certain extent be described within the framework provided by statistical physics.

References

- Astumian, R. D. (1997). Thermodynamics and kinetics of a brownian motor. *Science* **276**: 917–922.
- Ben-Jacob, E., Schochet, O., Tenenbaum, A., Cohen, I., Czirók, A. and Vicsek, T. (1994). Generic modelling of cooperative growth patterns in bacterial colonies. *Nature* **368**: 46–49.
- Bonabeau, E., Dorigo, M. and Théraulaz, G. (1999). *Swarm Intelligence: From Natural to Artificial Systems*. Santa Fe Institute Studies on the Sciences of Complexity. New York: Oxford University Press.
- Bonabeau, E., Dorigo, M. and Théraulaz, G. (2000). Inspiration for optimization from social insect behavior. *Nature* **406**: 39–42.
- Calenbuhr, V. and Deneubourg, J. L. (1990). A model for trail following in ants: individual and collective behaviour. In: *Biological Motion* (Alt, W. and Hoffmann, G., (eds.)). Berlin: Springer, pp. 453–469.
- Czirók, A., Barabasi, A. L. and Vicsek, T. (1999). Collective motion of self-propelled particles: Kinetic phase transition in one dimension. *Physical Review Letters* **82** (1): 209–212.
- Czirók, A., Ben-Jacob, E., Cohen, I. and Vicsek, T. (1996). Formation of complex bacterial colonies via self-generated vortices. *Physical Review E* **54** (2): 1791–1801.

- Czirok, A. and Vicsek, T. (2000). Collective behavior of interacting self-propelled particles. *Physica A* **281**: 17–29.
- DeAngelis, D. L. and Gross, L. J. (eds.) (1992). *Individual-based Models and Approaches in Ecology: Populations, Communities, and Ecosystems*. New York: Chapman and Hall.
- Derenyi, I. and Vicsek, T. (1995). Cooperative transport of brownian particles. *Physical Review Letters* **75/3**: 374–377.
- Deutsch, A. (1999). Principles of morphogenetic motion: swarming and aggregation viewed as self-organization phenomena. *J. Biosci.* **24** (1): 115–120.
- Deutsch, A. and Lawniczak, A. (1999). Probabilistic lattice models of collective motion and aggregation; from individual to collective dynamics. *Mathematical Biosciences* **156**: 255–269.
- Durrett, R. and Levin, S. A. (1994a). The importance of being discrete (and spatial). *Theoretical Population Biology* **46**: 363–394.
- Durrett, R. and Levin, S. A. (1994b). Stochastic spatial models: a user’s guide to ecological applications. *Philosophical Transactions of the Royal Society of London B* **343**: 329–350.
- Ebeling, W., Schweitzer, F. and Tilch, B. (1999). Active brownian particles with energy depots modelling animal mobility. *BioSystems* **49**: 17–29.
- Edelstein-Keshet, L. (1994). Simple models for trail following behaviour: Trunk trails versus individual foragers. *J. Math. Biol.* **32**: 303–328.
- Edelstein-Keshet, L., Watmough, J. and Ermentrout, G. B. (1995). Trail following in ants: individual properties determine population behaviour. *Behav. Ecol Sociobiol* **36**: 119–133.
- Erdmann, U., Ebeling, W., Schimansky-Geier, L. and Schweitzer, F. (2000). Brownian particles far from equilibrium. *European Physical Journal B* **15** (1): 105–113.
- Feistel, R. and Ebeling, W. (1989). *Evolution of Complex Systems. Self-Organization, Entropy and Development*. Dordrecht: Kluwer.
- Flierl, G., Grünbaum, D., Levin, S. and Olson, D. (1999). From individuals to aggregations: the interplay between behavior and physics. *Journal of Theoretical Biology* **196**: 397–454.
- Franke, K. and Gruler, H. (1990). Galvanotaxis of human granulocytes: electric field jump studies. *Europ. Biophys. J.* **18**: 335–346.
- Grünbaum, D. and Okubo, A. (1994). Modelling social animal aggregation. In: *Frontiers in Theoretical Biology* (Levin, S. A., (ed.)), vol. 100 of *Lecture Notes in Biomathematics*. New York: Springer.
- Hänggi, P. and Bartussek, R. (1996). Brownian rectifiers: How to convert brownian motion into directed transport. In: *Nonlinear Physics of Complex Systems - Current Status and Future Trends* (Parisi, J., Müller, S. C. and Zimmermann, W., (eds.)). Berlin: Springer, pp. 294–308.

- Helbing, D., Schweitzer, F., Keltsch, J. and Molnár, P. (1997). Active walker model for the formation of human and animal trail systems. *Physical Review E* **56/3**: 2527–2539.
- Helbing, D. and Vicsek, T. (1999). Optimal self-organization. *New Journal of Physics* **1**: 13.1–13.17.
- Klimontovich, Y. L. (1994). Nonlinear Brownian motion. *Physics-Uspekhi* **37** (8): 737–766.
- Lam, L. (1995). Active walker models for complex systems. *Chaos, Solitons & Fractals* **6**: 267–285.
- Lam, L., Veinott, M. C. and Pochy, R. (1995). Abnormal spatiotemporal growth. In: *Spatio-Temporal Patterns in Nonequilibrium Complex Systems* (Cladis, P. E. and Palfy-Muhoray, P., (eds.)). Reading, MA: Addison-Wesley, pp. 659–670.
- Mikhailov, A. and Zanette, D. H. (1999). Noise-induced breakdown of coherent collective motion in swarms. *Physical Review E* **60**: 4571–4575.
- Mikhailov, A. S. and Meinköhn, D. (1997). Self-motion in physico-chemical systems far from thermal equilibrium. In: *Stochastic Dynamics* (Schimansky-Geier, L. and Pöschel, T., (eds.)), vol. 484 of *Lecture Notes in Physics*. Berlin: Springer, pp. 334–345.
- Othmer, H. G. and Stevens, A. (1997). Aggregation, blowup and collapse: the abc’s of taxis in reinforced random walks. *SIAM J. of Applied Mathematics* **57/4**: 1044–1081.
- Pasteels, J. M. and Deneubourg, J. L. (eds.) (1987). *From Individual To Collective Behavior in Social Insects*, vol. 54 of *Experientia Supplementum*. Basel: Birkhäuser.
- Phipps, M. J. (1992). From local to global: The lesson of cellular automata. In: DeAngelis and Gross (1992), pp. 165–186.
- Rateitschak, K., Klages, R. and Hoover, W. G. (2000). The nosé-hoover thermostated lorentz gas. *Journal of Statistical Physics* **101**: 61–77.
- Schienbein, M. and Gruler, H. (1993). Langevin equation, fokker-planck equation and cell migration. *Bull. Mathem. Biology* **55**: 585–608.
- Schimansky-Geier, L., Mieth, M., Rosé, H. and Malchow, H. (1995). Structure formation by active brownian particles. *Physics Letters A* **207**: 140–146.
- Schimansky-Geier, L., Schweitzer, F. and Mieth, M. (1997). Interactive structure formation with brownian particles. In: *Self-Organization of Complex Structures: From Individual to Collective Dynamics* (Schweitzer, F., (ed.)). London: Gordon and Breach, pp. 101–118.
- Schweitzer, F. (1997a). Active brownian particles: Artificial agents in physics. In: *Stochastic Dynamics* (Schimansky-Geier, L. and Pöschel, T., (eds.)), vol. 484 of *Lecture Notes in Physics*. Berlin: Springer, pp. 358–371.

- Schweitzer, F. (ed.) (1997b). *Self-Organization of Complex Structures: From Individual to Collective Dynamics. Part 1: Evolution of Complexity and Evolutionary Optimization, Part 2: Biological and Ecological Dynamics Socio-Economic Processes, Urban Structure Formation and Traffic Dynamics*. London: Gordon and Breach.
- Schweitzer, F. (1998). Modelling migration and economic agglomeration with active brownian particles. *Advances in Complex Systems* **1/1**: 11–37.
- Schweitzer, F. (2002). *Brownian Agents and Active Particles*. Springer Series in Synergetics. Berlin: Springer.
- Schweitzer, F., Ebeling, W. and Tilch, B. (1998). Complex motion of brownian particles with energy depots. *Physical Review Letters* **80/23**: 5044–5047.
- Schweitzer, F., Ebeling, W. and Tilch, B. (2001). Statistical mechanics of canonical-dissipative systems and applications to swarm dynamics. *Physical Review E* **64/2** (2001): 021110 (1-12)
- Schweitzer, F. and Holyst, J. (2000). Modelling collective opinion formation by means of active brownian particles. *European Physical Journal B* **15** (4): 723–732.
- Schweitzer, F., Lao, K. and Family, F. (1997). Active random walkers simulate trunk trail formation by ants. *BioSystems* **41**: 153–166.
- Schweitzer, F. and Schimansky-Geier, L. (1996). Clustering of active walkers: Phase transitions from local interactions. In: *Fluctuations and Order: The New Synthesis* (Millonas, M., (ed.)). New York: Springer, pp. 293–305.
- Schweitzer, F., Tilch, B. and Ebeling, W. (2000). Uphill motion of active brownian particles in piecewise linear potentials. *European Physical Journal B* **14** (1): 157–168.
- Stevens, A. (1993). Aggregation of myxobacteria - a many particle system. In: *Proc. First European Conference of Mathematics Applied to Biology and Medicine*. Winnipeg: Wuerz Publishing, pp. 519–524.
- Stevens, A. (1995). Trail following and aggregation of myxobacteria. *J. of Biol. Systems* **3**: 1059–1068.
- Stevens, A. and Schweitzer, F. (1997). Aggregation induced by diffusing and nondiffusing media. In: *Dynamics of Cell and Tissue Motion* (Alt, W., Deutsch, A. and Dunn, G., (eds.)). Basel: Birkhäuser, pp. 183–192.
- Tilch, B., Schweitzer, F. and Ebeling, W. (1999). Directed motion of brownian particles with internal energy depot. *Physica A* **273** (3-4): 294–314.
- Vicsek, T., Czirok, A., Ben-Jacob, E., Cohen, I. and Shochet, O. (1995). Novel type of phase transition in a system of self-driven particles. *Physical Review Letters* **75**: 1226–1229.