Multi-Agent Model of Biological Swarming

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Abstract

An agent-based approach is used to explain the formation of vortex swarms in biological systems. The dynamics of the multiagent system is described by $3N$ coupled equations, modeling for each agent its position, its velocity and its internal energy depot. The energy depot considers the conditions for active biological motion, such as energy take-up, metabolism, and energy conversion. The equation of motion results from a superposition of *deterministic* and *stochastic* terms (random noise). The deterministic part considers indirect interactions with other agents to describe local avoidance behavior, and external influences resulting from an attractive environmental potential. Stochastic computer simulations of the multi-agent system are shown in very good agreement with the behavior observed in Daphnia swarms.

1 Introduction

One of the major conceptual challenges of system biology is the understanding of the behavior at the system level from the interactions of the entities comprising the system. To reach this goal, discrete, individual-based or *agent-based* modeling has become a very promising and powerful methodology. Recently, different computer architectures in distributed artificial intelligence have been developed to simulate the collective behavior of interacting agents. However, due to their rather complex simulation facilities many of these simulation tools lack the possibility to investigate systematically and in depth the influence of specific interactions and parameters. Instead of incorporating only as much detail as is *necessary* to produce a certain emergent behavior, they put in as much detail *as possible*, and thus reduce the chance to understand *how* emergent behavior occurs and *what* it depends on.

In our paper, we investigate a prominent example of complex behavior in biological systems, namely *swarming*. Pioneering work has been done by Reynolds[1] that is based on behavioral rules for artificial creatures (boids), while our aim is to obtain the same behavior based on interactive forces between reactive agents. Therefore, in our approach, we use *Brownian agents*

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[1] http://www.red3d.com/cwr/boids/

[14] (see Sect. 2), that – in addition to their computational suitability – can be also investigated by means of analytical methods from statistical physics and mathematics. Swarming is a form of *collective motion* that may emerge from *local interactions* of a large number of individuals (agents). Swarms (also called herds, flocks, schools) can often be observed in certain mammals, fish, insects, and birds for various benefits such as enhaced feeding and mating as well as more successful predator avoidance. Detailed experimental investigations on swarming, however, are rare, either because of the size of the animals or because well defined conditions for experiments are difficult to realize. Fortunately, as Ordemann *et. al* have shown [9, 10], zooplankton is a suitable candidate for experiments on swarming, as briefly described in the following.

2   Modeling swarming in biological systems

Daphnia (water flea) is an ideal object to study swarming behavior under well defined lab conditions, because of its intermediate size and biological complexity. The sketch of the experimental setup (Fig. 1) shows a water tank penetrated vertically from an artificial light source. While it may seem that this condition is highly artificial, we note similarities to light conditions in field observations [10]. In particular, in such a tank, Ordemann *et. al* [10] have investigated the motion of both *single* Daphnia and *many* Daphnia.
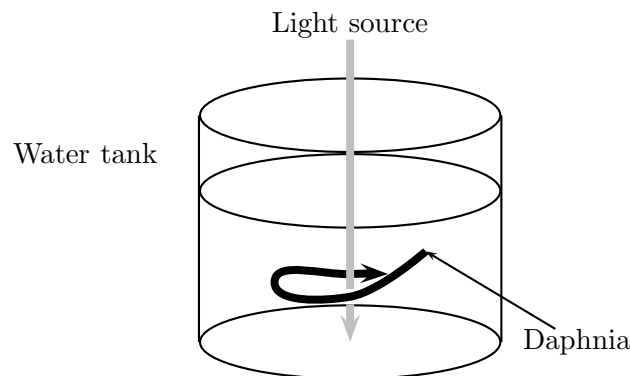

Figure 1: Sketch of the Daphnia experiments carried out by Ordemann *et. al* [10]. The trajectory indicates the cycling motion of a single animal.[6]

In such a setup it has been found that a *single* Daphnia starts to *cycle* (i.e. rotate) around the artificial light source, keeping its cycling direction for quite a while. In repeated experiments, however, the cycling direction may change to the opposite, which lead to the conclusion that

single Daphnia, while rotating around the light beam, do *not* have a preferred direction of motion.

Interestingly, the situation changes if instead of single Daphnia a larger number of animals is put in the water tank. In this case, the Daphnia start again with their cycling motion, but then *all* tend to move into the *same* direction of motion. From a physical perspective, a symmetry break is observed, i.e., the symmetry between the two possible cycling directions (left, right rotation) is clearly broken toward *one* of the possibilities (left *or* right rotation). Both of these possibilities have the same chance to occur, but only one of them is eventually realized.

It has been further observed that the Daphnia swarm, while rotating round the light beam in the same direction, mostly keeps a certain distance to the light source. This kind of swarming behavior is also called *vortex swarming.*

The vortex formation as well as the symmetry break in the cycling direction are clearly self-organized phenomena that result from the *collective* interaction of *many* animals. In order to understand this in more detail, we derive a multiagent model in the following.

Our approach is based on *Brownian agents* [14], each of them described by three state variables: spatial position $\boldsymbol{r}_i$, velocity $\boldsymbol{v}_i$ and internal energy depot e_i. The first two state variables describe the *movement* of the agent and can be observed from the outside. The agent's energy depot, however, is an *internal* variable describing its *capability* of *active movement.* Different from physical Brownian particles that move *passively* and *randomly* because of the impacts from surrounding molecules, the Brownian agent may move actively and in a preferred direction. But the term "Brownian" refers to the fact that the agent may still be subject to fluctuations that are described by a stochastic force, as explained further below.

The internal energy depot $e_i(t)$ of agent i has to consider that active motion is based on the take-up of energy from the environment, the storage of energy and conversion of stored energy into energy of motion. These three processes are summerized in the following balance equation [11]:

$$\mu \, \frac{d}{dt} e_i(t) = q(\boldsymbol{r}) - c \, e(t) - d_2 v^2 \, e(t) \; ; \quad d_2 > 0 \tag{1}$$

$q(\boldsymbol{r})$ is the flux of energy into the internal depot. If the availability of energy is heterogeneously distributed, the energy flux may depend on the space coordinate, $\boldsymbol{r}$. In this model we assume a *homogeneous* distribution of energy, i.e. a constant influx $q(\boldsymbol{r}) = q_0$. c describes the loss of energy due to internal dissipation, which is assumed to be proportional to the internal energy. The last term in eq. (1) considers the conversion of internal energy into kinetic energy with a rate, which should be a function of the actual velocity, $\boldsymbol{v}$, of the agent. Here, $d_2 \cdot v^2$ is assumed in agreement with physical equations for the energy balance. The formal parameter μ in eq. (1) can be used to describe the time scale of relaxation of the internal energy depot, as described below.

In order to discuss the dynamics of the two external state variables, position $\boldsymbol{r}_i$ and velocity $\boldsymbol{v}_i$, we have to consider (i) the influence of the internal energy depot on the (active) motion of the agent, and (ii) the influence of the environment. The experiments described above have used a vertical beam of light that causes an attractive force on the Daphnia, which tend to cycle around it. In order to cope with this, we may choose the very simple assumption of an *external potential* of the form

$$U(\boldsymbol{r}) = \frac{a}{2}\boldsymbol{r}^2 \tag{2}$$

which generates an attractive force $\boldsymbol{F} = -\boldsymbol{\nabla}U(\boldsymbol{r}) = -a\boldsymbol{r}$ towards the center, $\boldsymbol{r} = 0$. The two effects can then be put into dynamic equations for the change of the agent's state variables, which have the following form [3, 11]:

$$\frac{d}{dt}\boldsymbol{r}_i = \boldsymbol{v}_i \ ; \quad \frac{d}{dt}\boldsymbol{v}_i = -\gamma_0\,\boldsymbol{v}_i - \boldsymbol{\nabla}U(\boldsymbol{r})|_{r_i} + d_2 e(t)\boldsymbol{v}_i + \sqrt{2D}\,\boldsymbol{\xi}_i(t) \tag{3}$$

Here, for the mass $m = 1$ is used. Causes for the change of the variables are summarized on the right-hand side of the equations. The change of the agent's position, $\boldsymbol{r}_i$ is caused by the movement of the agent, described by the velocity $\boldsymbol{v}_i$, that in turn can be changed by four different forces: (i) friction, with γ_0 being the friction coefficient, (ii) attraction toward the center of the light beam, expressed by the gradient of the environmental potential, (iii) active motion in forward direction driven by the energy from the internal depot, (iv) a stochastic force ξ of strength D, which describes the influence of random events on the agent's motion.

Both terms (i) and (iv) are already known from Langevin's equation to describe the motion of (passive) Brownian particles. *Without* the stochastic force, the particle would eventually come to rest because its kinetic energy would be dissipated due to friction. The stochastic force, however, keeps it (passively) moving into a random direction, where $\xi(t)$ is assumed to be Gaussian white noise with $\langle \xi_i(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = \delta(t - t')$.

We conclude that in our approach of Brownian agents, the dynamics is described on the individual agent level by three coupled equations for the three state variables characterizing each agent, $\boldsymbol{r}_i$, $\boldsymbol{v}_i$, e_i, eqs. (3), (1). For theoretical investigations, these equations can be reduced to two by assuming that the internal energy depot relaxes very fast into a quasi-stationary equilibrium. This adiabatic approximation, i.e. the limit $\mu \to 0$ in eq. (1), results in

$$e_i(t) = \frac{q_0}{c + d_2 v_i^2} \tag{4}$$

and we can rewrite the equations of motion as:

$$\frac{d}{dt}\boldsymbol{r}_i = \boldsymbol{v}_i \ ; \quad \frac{d}{dt}\boldsymbol{v}_i = -\gamma(v_i^2)\,\boldsymbol{v}_i - \boldsymbol{\nabla}U(\boldsymbol{r})|_{r_i} + \sqrt{2D}\boldsymbol{\xi}_i(t) \tag{5}$$

$\gamma(v_i^2)$ is a *non-linear friction function*:

$$\gamma(v_i^2) = \gamma_0 - \frac{d_2\,q_0}{c + d_2 v_i^2} \tag{6}$$

which describes the active motion of the agent. It has a zero for

$$v_0^2 = \frac{q_0}{\gamma_0} - \frac{c}{d_2} \tag{7}$$

Active motion, i.e. $|v_0| > 0$ becomes possible only for a certain supercritical take-up of energy from the environment, $q_0 > c\gamma_0/d_2$. The actual motion of the agent is then a *compromise* between its active motion – which eventually would lead it everywhere, as long as internal energy is provided – and the environmental conditions which set some restrictions on this motion.

3   Simulating swarming without interaction

For the case of an attractive potential, eq. (2), Fig. 2 shows computer simulations for the active movement of a single agent, as described by the three eqs. (3), (1) for the state variables, $\boldsymbol{r}_i$, $\boldsymbol{v}_i$, e_i. The results clearly indicate the *cyclic motion* round the center, which has been also observed in single Daphnia motion, as explained above (*vortex motion*). Running the computer simulations for single agents with different initial conditions eventually results in the same kind of cyclic motion, but with different rotational directions, i.e. left-handed or right-handed rotations. Due to stochastic influences, also a change of the direction of motion becomes possible. Thus, we may conclude that our model of Brownian agents sufficiently describes the observed behavior of *single Daphnia*. We now turn to the case of many, i.e. $i = 1, ..., N$ Brownian agents, which is of importance for swarming. The dynamics of the multiagent system is then described by $3N$ coupled (stochastic) equations of the form (3), (1). In this case, we observe from computer simulations again the characteristic rotational motion, where, however, about half of the agents rotate clockwise, while the other half rotates counterclockwise. The two different cyclic directions can be clearly observed when looking at the *angular momentum* distribution, $\rho(L)$, where $\boldsymbol{L}$ (for $m = 1$) is defined as $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{v}$. As Fig. 3 shows, this is a *bimodal* distribution of about equal height, indicating the both left- and righthanded rotational directions with the same probability.

This simulation result does not quite agree with the observation of swarming in Daphnia, which apparently cycle into *one*, i.e. the same direction. The reasons for this mismatch are quite obvious: in our model, we have so far only considered "point-like" agents without any kind of mutual interaction, whereas in real biological systems the coherent motion of the swarm is certainly based on interactions between the entities. Thus, the question arises, which kind of interaction may lead to the break in the rotational symmetry, as observed in the Daphnia experiments.

So far, different forms of *global* or *local* interactions have been introduced into swarming models. We mention (i) local interactions via a self-consistent field that has been created by the agents and in turn influences their further movement and/or "behavior" [5, 13] – chemotactic response is
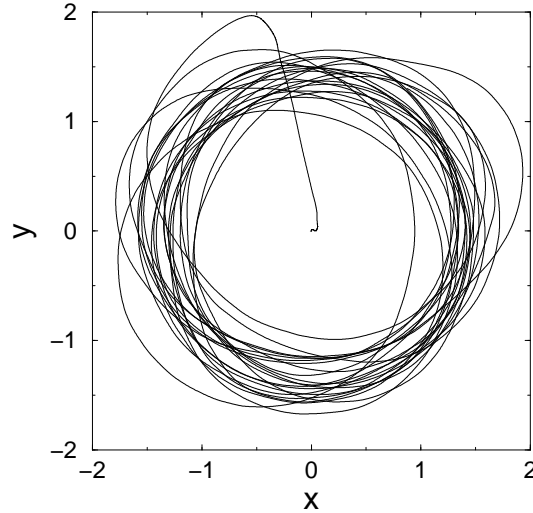
Figure 2: Trajectory ($t = 200$) of a single Brownian agent moving in an environmental potential, eq. (2) with supercritical take-up of energy, $q_0 > c\gamma_0/d_2$. Initial conditions: $\{x(0), y(0)\} = \{0, 0\}$, $\{v_x(0), v_y(0)\} = \{0, 0\}$, $e(0) = 0$, parameters: $\gamma = 5.0$, $d_2 = 1.0$, $q_0 = 10.0$, $c = 1.0$, $D = 0.005$, $a = 0.5$.[6]
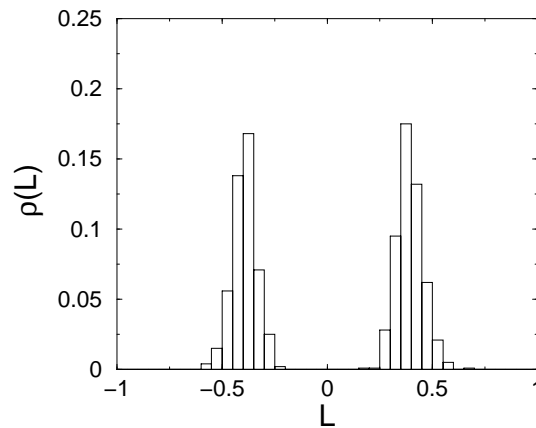

Figure 3: Angular momentum distribution $\rho(L)$ of $N = 1000$ Brownian agents after $t = 150$. The positive or negative sign of L indicates the right- or lefthanded rotation. Parameters: $q_0 = 10.0$, $c = 1.0$ $\gamma = 20.0$, $d_2 = 10.0$, $D = 0.001$, $a = 1.0$.[6]

a prominent example here, (ii) local interactions based on the coupling of the agents's individual velocity to a local average velocity [2, 15, 16] (iii) global interactions, such as the coupling of the agent's individual orientation (i.e. direction of motion) to the mean orientation of the swarm

[1, 2], or the coupling of the agent's individual position to the mean position (center of mass) of the swarm [7], further couplings via the mean momentum or mean angular momentum or a combined set of invariants of motion [1, 12], (iv) interactions based on hydrodynamics coupling between agents [4].

Despite the fact that some of these models simulate coherent swarm behavior or even rotation of the swarm into the same direction, there is evidence that many of the underlying assumptions for interactions can hardly be satisfied by *biological* observations, thus their biological relevance is highly questionable. Therefore, in the following section, we introduce local interactions between the agents that indeed match with biological reality.

4   Modeling swarming with avoidance behavior

Experiments on Daphnia swarming has shown that these animals tend to cycle into the same direction. The simple and obvious reason for this is that animals try to avoid as much as possible collisions with other animals – which would occur much more frequently if different animals cycled into opposite directions at the same time. Thus, a biologically satisfied assumtion is to include *avoidance behavior* in our model of swarming, in order to test, whether this would lead to the observed break in the rotational symmetry described above.

Daphnia are able to visually sense their environment to a certain degree, i.e. they can detect animals approaching them from the front, and then try to avoid collisions. In our model, we account for this by assuming that there is a short-range repulsive force between agents, to prevent their collisions, which will result from a repulsive *interaction potential* $V(r_i)$ around each agent i that depends on its actual position, r_i:

$$V(r_i) = p \cdot \exp\left(-\frac{R_i}{\sigma}\right) \tag{8}$$

p denotes the strength and σ the range of the potential, the latter being a measure of the sight, i.e. the range of *visibility*. R_i is a specific function of the distance between agents, as explained in the following. Since all agents are moving, agent i needs to account for the space that will be occupied by all other agents j in the vicinity during the next time step. This space needed, depends both on the agent's positions r_j *and* their velocity of motion, v_j, so R_i is a function of these. For further specification, we introduce the unit vector in the direction of motion of agent i, $\boldsymbol{n}_i^0 = \boldsymbol{v}_i/||\boldsymbol{v}_i||$; $\boldsymbol{n}_j^0$ is defined similarly. This allows to define a new *velocity-dependent* coordinate system for agent i, namely $\boldsymbol{y}_i$ and $\boldsymbol{x}_i$ defined by:

$$\boldsymbol{y}_i = \frac{v_i \boldsymbol{n}_i^0 - \delta v_j \boldsymbol{n}_j^0}{||v_i \boldsymbol{n}_i^0 - \delta v_j \boldsymbol{n}_j^0||} \quad ; \quad \boldsymbol{x}_i \perp \boldsymbol{y}_i \quad \text{and} \quad \langle \boldsymbol{x}_i, \boldsymbol{x}_i \rangle = 1 \ . \tag{9}$$

If $\delta > 0$, the direction of motion of agent j is also taken into account for agent i. The $\boldsymbol{x}_i$ can be constructed by the orthonomalization algorithm by GRAM-SCHMIDT. Using this coordinate system, the dependence of R_i on the position and velocity of agent j is now given as

$$R_i = \sqrt{\langle \boldsymbol{r}_i - \boldsymbol{r}_j, \boldsymbol{x}_i \rangle^2 + \beta^2 \langle \boldsymbol{r}_i - \boldsymbol{r}_j, \boldsymbol{y}_i \rangle^2} \tag{10}$$

with a velocity-dependent function:

$$\beta = \begin{cases} \beta' & : \langle \boldsymbol{r}_i - \boldsymbol{r}_j, \boldsymbol{y}_i \rangle \geq 0 \\ \dfrac{\beta'}{1 + \lambda \cdot v_i} & : \langle \boldsymbol{r}_i - \boldsymbol{r}_j, \boldsymbol{y}_i \rangle < 0 \end{cases} \tag{11}$$
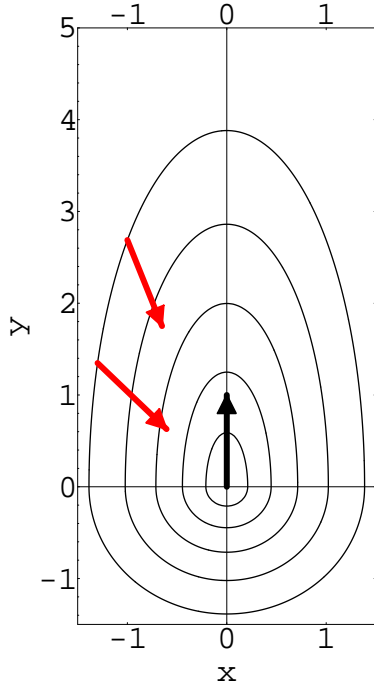

Figure 4: Equipotential lines of the repulsive potential $V(r_i)$, eq. (8). The black arrow indicates the agent in the origin, having a velocity of $v = \{0, 1\}$. The two gray arrows representing other agents have the same absolute value of 1 and point towards the origin.[6]

Eventually, with the known repulsive interaction potential $V(r_i)$, the force between any two agents i and j is given as:

$$\begin{aligned} \boldsymbol{f}_{ij} &= -\boldsymbol{\nabla} V(r_i) = \frac{V(r_i)}{\sigma \cdot R_i} \left(\langle \boldsymbol{r}_i - \boldsymbol{r}_j, \boldsymbol{x}_i \rangle \boldsymbol{x}_i + \langle \boldsymbol{r}_i - \boldsymbol{r}_j, \beta \boldsymbol{y}_i \rangle \beta \boldsymbol{y}_i \right) \tag{12} \\ &= \frac{p}{\sigma \cdot R_i} \exp\left(-\frac{R_i}{\sigma} \right) (\boldsymbol{r}_i - \boldsymbol{r}_j) \tag{13} \end{aligned}$$

The total force on agent i resulting from the assumed local interaction is then given as the sum over all 2-agent forces, $\vec{F}_i = \sum_{j \neq i} \boldsymbol{f}_{ij}$. This repulsive force of course changes eq. (5) for the
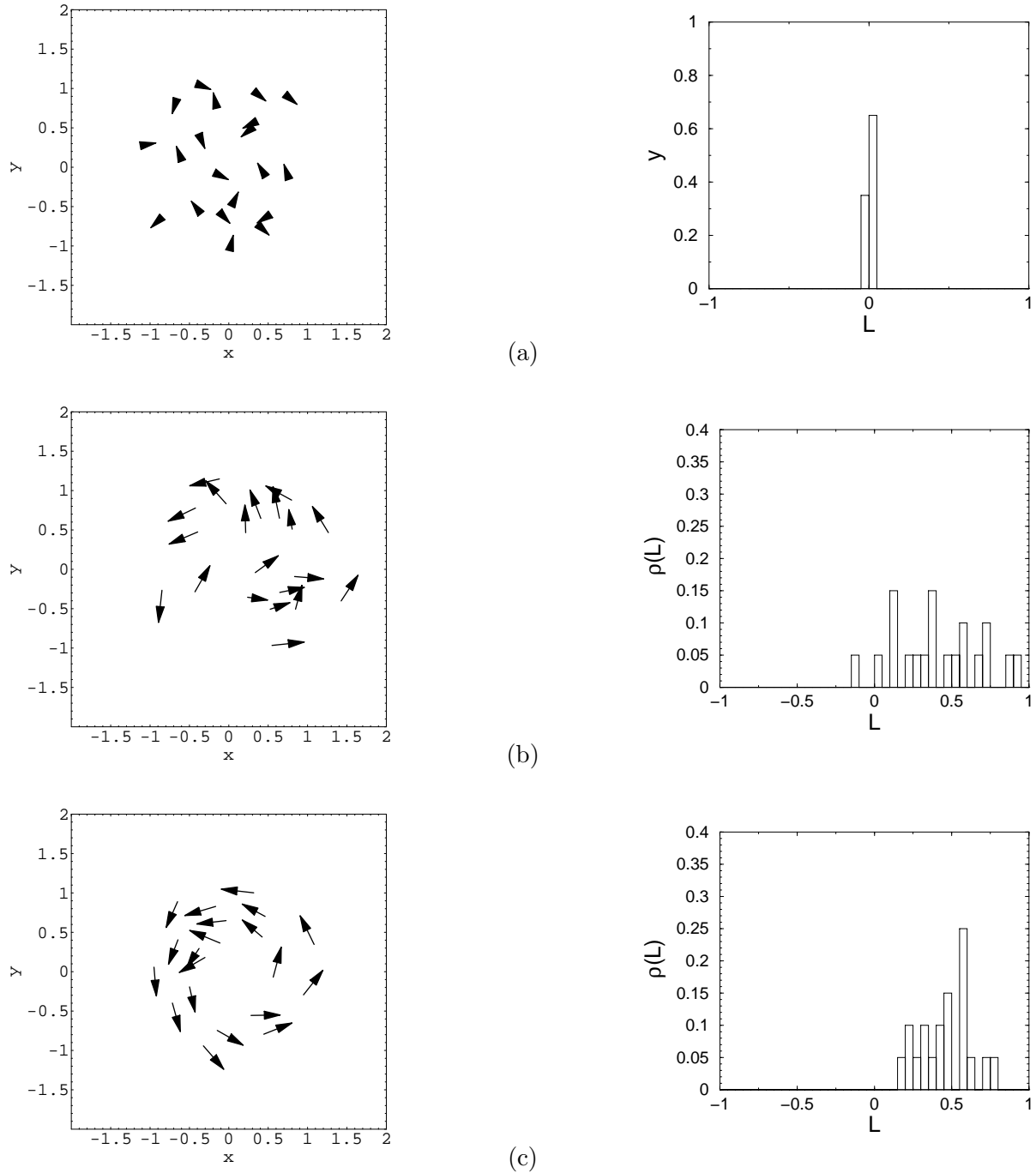
Figure 5: Spatial snapshots (left) and distribution of angular momentum $\rho(L)$ (right) for a multiagent system ($N = 20$) at three different times: (a) $t = 0$ (b) $t = 8$ and (c) $t = 55$. The length of the arrows indicates the velocities. [6]

agents by an additional term, i.e. in the extended avoidance model, the dynamics now read:

$$\frac{d}{dt}\boldsymbol{r}_i = \boldsymbol{v}_i \; ; \quad \frac{d}{dt}\boldsymbol{v}_i = -\gamma(v^2)\,\boldsymbol{v}_i - a\boldsymbol{r}_i + \sum_{i \neq j} \boldsymbol{f}_{ij} + \sqrt{2D}\xi(t) \tag{14}$$

We note that these assumptions lead to an asymmetric repulsive potential $V(r_i)$ around each agent. Two different forms of such an asymmetrical potential have been introduced in [6]. The one used in this paper – which has been originally used to simulate the movement of pedestrians [8] – is preferred because it leads to smoother movements of the agents. The potential defined by eq. (8) with eq. (10) can be seen in Fig. 4.

Fig. 5 shows spatial snapshots of a computer simulation of the multiagent system with respect to avoidance behavior, together with the respective distribution of the angular momenta $\rho(L)$. The results can be concluded as follows: (i) On the spatial level, we observe the emergence of a *coherent motion* of the multi-agent swarm out of a random initial distribution. This collective motion is characterized by a *unique cycling direction* (either left- *or* righthanded rotation), as can be also seen from the *unimodal* distribution, $\rho(L)$. (ii) We further observe the formation of a vortex, which is rather similar to the Daphnia swarm cycling round the light beam. (iii) While in one simulation all agents cycle in the same direction, we note that in different simulations the cycling direction can be also opposite, i.e. there is *no preferred cycling direction* for the swarm, which also agrees with the observations of the Daphnia swarm. (iv) We note that for certain parameters a spontaneous change in the rotating direction can be observed. This occurs in particular if agent i takes strongly the movement of agent j into account ($\delta \approx 0.5$).

5 Conclusions

Our goal was to model vortex swarming behavior by means of rather *minimal* assumptions that, however, should have a clear *biological relevance*. As we have shown the model of Brownian agents introduced in this paper can indeed produce a reasonable swarming behavior that resembles real Daphnia swarms. Although reasonable assumptions about local interactions (such as local repulsion) are taken into account, a definite justification of the model is still pending. This is due to the fact that data is difficult to gather, because defined lab conditions are hard to establish. Currently, Ordemann *et al.* are testing this model on real *Daphnia* swarms.

Acknowledgment

References

[1] A. Czirok, E. Ben-Jacob, I. Cohen, and T. Vicsek. Formation of complex bacterial colonies via self-generated vortices. *Physical Review E*, 54(2):1791–1801, 1996.

[2] A. Czirok and T. Vicsek. Collective behavior of interacting self-propelled particles. *Physica A*, 281:17–29, 2000.

[3] W. Ebeling, F. Schweitzer, and B. Tilch. Active Brownian particles with energy depots modelling animal mobility. *BioSystems*, 49:17–29, 1999.

[4] U. Erdmann and W. Ebeling. Collective motion of Brownian particles with hydrodynamic interactions. *Fluctuation and Noise Letters*, 2002 (submitted).

[5] D. Helbing, F. Schweitzer, J. Keltsch, and P. Molnár. Active walker model for the formation of human and animal trail systems. *Physical Review E*, 56(3):2527–2539, 1997.

[6] R. Mach, A. Ordemann, and F. Schweitzer. Modeling vortex swarming in Daphnia. *Journal of Theoretical Biology*, 2003 (submitted)

[7] A.S. Mikhailov and D. H. Zanette. Noise-induced breakdown of coherent collective motion in swarms. *Physical Review E*, 60:4571–4575, 1999.

[8] P. Molnár. *Modellierung und Simulation der Dynamik von Fussgängerströmen.* Shaker, Aachen 1995.

[9] A. Ordemann, G. Balazsi, and F. Moss. Motions of daphnia in a light field: Random walks with a zooplankton. *Nova Acta Leopoldina*, 2003 (in press)

[10] A. Ordemann. Vortex-swarming of the zooplankton daphnia. *The Biological Physicist*, 2 (3):5–10, August 2002.

[11] F. Schweitzer, W. Ebeling, and B. Tilch. Complex motion of Brownian particles with energy depots. *Physical Review Letters*, 80(23):5044–5047, 1998.

[12] F. Schweitzer, W. Ebeling, and B. Tilch. Statistical mechanics of canonical-dissipative systems and applications to swarm dynamics. *Physical Review E*, 64(2):021110–1 – 021110–12, August 2001.

[13] F. Schweitzer, K. Lao, and F. Family. Active random walkers simulate trunk trail formation by ants. *BioSystems*, 41:153–166, 1997.

[14] F. Schweitzer. *Brownian Agents and Active Particles. Collective Dynamics in the Natural and Social Sciences, Springer Series in Synergetics,* Springer Berlin 2003

[15] J. Toner and Yuhai Tu. Long-range order in a two-dimensional dynamical xy model: How birds fly together. *Physical Review Letters*, 75(23):4326–4329, December 1995.

[16] T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, and O. Shochet. Novel type of phase transition in a system of self-driven particles. *Physical Review Letters*, 75:1226–1229, 1995.