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Brownian Agent Models for Swarm and Chemotactic Interaction

Frank Schweitzer

Fraunhofer Institute for Autonomous Intelligent Systems
Schloss Birlinghoven, D-53754 Sankt Augustin, Germany

e-mail: schweitzer@ais.fhg.de, http://www.ais.fhg.de/~frank

Abstract

Brownian agents denote a particular class of agents that combines features of reactive and reflexive agent concepts. A Brownian agent is characterized by a set of state variables that include also internal degrees of freedom. The dynamics of these state variables is in general described by a stochastic equation that further considers direct and indirect interactions with other agents and external influences. An important state variable, the internal energy depot, captures the costs of each action of the agent. In the paper, we discuss two biological applications of the Brownian agent concept: (i) the formation of swarms which depends on the active motion of the agents and some appropriate coupling, (ii) the formation of aggregates based on chemotactic interaction of the agents. We also demonstrate one of the major advances of the Brownian agent concept, namely the derivation of macroscopic equation from the agent dynamics, which can be used to analyze and predict the behavior of the MAS.

1 Introduction

Discrete, individual-based or *agent-based* modeling has become a very promising and powerful methodology to describe the occurence of complex behavior in biological systems. This holds for instance for population dynamics [2, 5], but also for the collective behavior in social insects [1, 13]. Trail following in ants is one of the examples, where the interplay between individual properties and collective behavior has been successfully simulated by means of individual-based models [8, 19]. Also different forms of biological structures, namely biological aggregates in different species such as slime molds, bacteria, larvae, or in cells have been modeled within a individual-based approach [3, 4, 9, 12, 21].

While the patterns emerging are observable only on the "macroscopic" system level, the modelling effort aims to understand their emergence from the "microscopic" level of interacting individuals [16]. The advantage of such an individual-based approach is given by the fact that it is applicable also in cases where only a small number of agents govern the further evolution. Here deterministic approaches or mean-field equations are not sufficient to describe the behavior of the complex system. Instead, the influence of history, i.e. irreversibility, path dependence, the occurence of random events play a considerable role.

Recently, different computer architectures in distributed artificial intelligence have been developed to simulate the collective behavior of interacting indivudals or agents (cf. for instance the SWARM project at `http://www.swarm.org/`). However, due to their rather complex simulation facilities many of the currently available simulation tools lack the possibility to investigate systematically and in depth the influence of specific interactions and parameters. Instead of incorporating only as much detail as is *necessary* to produce a certain emergent behavior, they put in as much detail *as possible*, and thus reduce the chance to understand *how* emergent behavior occurs and *what* it depends on.

Therefore, it would be feasible to have multi-agent systems (MAS) that can be also investigated by means of *analytical methods* (from statistical physics or mathematics) – in addition to their computational suitability. The concept of *Brownian agents* [17] is one of the possible approaches to serve for this purpose. In addition to rather complex (direct and indirect, global and local) interactions among the agents, the concept also provides an appropriate represention of (i) physical space and time, (ii) external, environmental (boundary) conditions, and (iii) stochastic influences. This will be outlined in the following sections. In order to demonstrate the applicability of the Brownian agent concept, a variety of applications in physical, physico-chemical, biological and socio-economic systems have been investigated [17]. In this paper, we may pick some examples with close relation to biology, namely the formation of swarms in Sect. 3 and the aggregation of individuals based on chemotactic interaction in Sect. 5.

2 Brownian Agents

In distributed artificial intelligence, often a distinction is made between the *reflexive* and the *reactive* agent. The first one has an (internal) model or at least some knowledge about its environment that allows its to draw conclusions about some certain actions in a given situation. The reactive agent, on the other hand, simply "reacts" to signals from the environment without referring to internal knowledge. The *Brownian Agent* [17] combines features of both reactive and reflexive agent concepts. It is described by a set of state variables $u_i^{(k)}$ where the index $i = 1, ..., N$ refers to the individual agent i, while k indicates the different variables. These could be either *external* variables that can be observed from the outside, or *internal degrees of freedom* that can only be

indirectly concluded from observable actions. Important external variables are $u_i^{(1)} = \boldsymbol{r}_i$, which denotes the *space coordinate* (mostly a vector in the two-dimensional physical space), or $u_i^{(2)} = \boldsymbol{v}_i$, which is the individual *velocity* in the case of a moving agent. Both are assumed as *continuous* variables.

The internal degrees of freedom on the other hand that cannot be directly observed, could be continuous or discrete variables. For instance, the state variable $u_i^{(3)} = \theta_i \in \{-1, 0, +1\}$ may describe three different responses to certain environmental conditions or to incoming information. For example, agents with $\theta = -1$ may not be affected by a particular signal, while agents with $\theta = +1$ may respond to it. An important continuous state variable in the context of Brownian agents is the *internal energy depot* $u_i^{(4)} = e_i$, which determines whether agent i may perform a certain action or not. This includes the assumption that all actions – be it active motion or communication or environmental changes – need to use "energy". In general, this term describes not just the physical free energy that is dissipated e.g. during active motion, it intends to cover also other resources needed to perform a certain action.

Noteworthy, the different (external or internal) state variables can change in the course of time, either due to impacts from the surrounding, or due to an internal dynamics. Thus, in a most general way, we may express the dynamics of the different state variables as follows:

$$\frac{d\,u_i^{(k)}}{dt} = f_i^{(k)} + \mathcal{F}_i^{\rm stoch} \tag{1}$$

For the Brownian agents, it is assumed that the causes for the temporal change of u_i may be described as a *superposition* of *deterministic* and *stochastic* influences, imposed on agent i. This picks up the ingenious idea first used by Langevin in early 1900 to describe the motion of *Brownian particles* – and is basically the reason why this agent concept is denoted as *Brownian* agent. A Brownian particle moves due to the impacts of the surrounding molecules whose motion however can be observed only on a much smaller time and length scale compared to the motion of the Brownian particle. Thus, Langevin invented the idea to sum up all these impacts in a stochastic force with certain statistical properties.

For the Brownian agent, we will exploit Langevins idea in a similar manner, i.e. we will sum up influences which may exist on a microscopic level, but are not observable on the time and length scale of the Brownian agent, in a stochastic term $\mathcal{F}_i^{\rm stoch}$, while all those influences that can be directly specified on these time and length scales are summed up in a *deterministic* term $f_i^{(k)}$. Such a distinction basically defines the level of coarse-grained description for the multi-agent system. The "cut" may prevent us from considering too much "microscopic" details of the MAS, while focussing on particular levels of description. The summed up stochastic influences might result from a more fine-grained deterministic description – but instead of taking this into detailed account, just some specific statistical (gross) properties are considered on the coarse-grained level. Notheworthy, the

strength of the stochastic influences may also vary for different agents and may thus depend on local parameters or internal degrees of freedom.

The *deterministic* part $f_i^{(k)}$ contains all specified influences that cause changes of the state variable $u_i^{(k)}$. This could be

- non-linear interactions with other agents $j \in N$ - thus $f_i^{(k)}$ can be in principle a function of all state variables
$$\underline{u} = \left\{ u_1^{(1)}, u_1^{(2)}, ..., u_2^{(1)}, ..., u_j^{(k)}, ..., u_N^{(k)} \right\} \tag{2}$$
describing any agent (including agent i),
- external conditions - such as forces resulting from external potentials, or the in/outflux of resources etc. These circumstances shall be expressed as a set of (time-dependent) *control parameters*
$$\underline{\sigma} = \{\sigma_1, \sigma_2, ...\} \tag{3}$$
- an *eigendynamics* of the system that does not depend on the action of the agents. In the example of an ecosystem this eigendynamics may describe day/night or seasonal cycles, or the agent-independent diffusion of resources within the system. This is expressed in an *explicite time-dependence* of $f_i^{(k)}$.

Hence, in general we have $f_i^{(k)} = f_i^{(k)}(\underline{u}, \underline{\sigma}, t)$. In order to set up a Brownian multi-agent system we need to specify (i) the relevant state variables $u_i^{(k)}$, (ii) the dynamics for changing them, i.e. $\dot{u}_i^{(k)}$, (iii) the external conditions, i.e. $\sigma_1, ..., \sigma_n$, or a possible eigendynamics of the system. Thus, basically the dynamics of the MAS is specified on the level of the individual agent, not on a macroscopic level. In the following, we will discuss two different examples from a biological context in order to demonstrate the applicability of the Brownian agent concept.

3 Swarming of Brownian Agents

A swarm can seen as a multi-agent system with two important features: (i) *direct or indirect interactions* such as additional coupling, that would account for the typical *correlated motion* of the agents, (ii) *active motion* of the agents, i.e. some *energetic conditions* must be satisfied in order to keep the swarm moving.

In the following, we consider an ensemble of $i = 1, ..., N$ Brownian agents, each of them described by three state variables: spatial position $\boldsymbol{r}_i$, velocity $\boldsymbol{v}_i$ and internal energy depot e_i. The dynamics

of these variables is given by the following set of coupled equations:

$$\begin{aligned}
\frac{d\,u_i^{(1)}}{dt} &= \frac{d\,\boldsymbol{r}_i}{dt} = f_i^{(1)} = \boldsymbol{v}_i \\
\frac{d\,u_i^{(2)}}{dt} &= \frac{d\,\boldsymbol{v}_i}{dt} = f_i^{(2)} + \mathcal{F}_i^{\rm stoch} = \mathcal{F}_i^{\rm diss} + \mathcal{F}_i^{\rm ext} + \mathcal{F}_i^{\rm int} + \mathcal{F}_i^{\rm stoch} \\
\frac{d\,u_i^{(4)}}{dt} &= \frac{d\,e_i}{dt} = f_i^{(4)} = q_i(\boldsymbol{r}_i,t) - p_i(\boldsymbol{r}_i,t) - c_i\; e_i(t)
\end{aligned} \tag{4}$$

Here the change of the space coordinate results from the velocity of the agent without any additional stochastic influences, which are summed up in the dynamic equation for the velocity itself. For the stochastic force we may use a standard expression from statistical physics,

$$\mathcal{F}_i^{\rm stoch} = \sqrt{2\,S}\;\boldsymbol{\xi}_i(t) \tag{5}$$

where S denotes the strength of the stochastic force and the random function $\boldsymbol{\xi}_i(t)$ is assumed to be Gaussian white noise. $\mathcal{F}_i^{\rm diss}$ denotes the non-conservative (dissipative) forces that may affect the active motion of the agent. Here we have to consider (i) the friction γ_0 that decelerates the agent's motion, and (ii) power from the internal energy depot that is used for the acceleration of motion [7]:

$$\mathcal{F}_i^{\rm diss} = -\gamma_0\,\boldsymbol{v} + d_i(\boldsymbol{v}_i)\,e_i(t)\,\boldsymbol{v} \tag{6}$$

$d_i(\boldsymbol{v}_i)$ is the agent specific rate of converting depot energy into kinetic energy, that may depend on the agent's actual velocity, $\boldsymbol{v}_i$. In the following we will assume $d_i(\boldsymbol{v}_i) = d_2 v^2$, where d_2 is a constant conversion rate.

$\mathcal{F}_i^{\rm ext}$ describes the influence of external forces and will be omitted here. $\mathcal{F}_i^{\rm int}$ eventually describes influences that may result from interactions with other agents. For the example of a *harmonic swarm* [6, 11, 18], we may assume a mutual interaction of the agents due to a harmonic potential that depends on the spatial positions of the agents. This results in linear attractive forces between all pairs of agents. The force acting on agent i reads:

$$\mathcal{F}_i^{\rm int} = -a\left(\boldsymbol{r}_i - \boldsymbol{R}(t)\right)\;; \quad \boldsymbol{R}(t) = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{r}_i(t) \tag{7}$$

where $\boldsymbol{R}$ denotes the center of mass of the swarm.

In the dynamics of the internal energy depot $e_i(t)$ of the Brownian agent the term $q_i(\boldsymbol{r}_i,t)$ describes the influx of ressources into the agent's depot, for example the take-up of energy that can later be used for active motion. These ressources may not be homogeneously distributed in the system or may not be available at all time [7], however, in this paper we may assume $q_i(\boldsymbol{r}_i,t) = q_0 = \text{const}$. The term $p_i(\boldsymbol{r}_i,t)$ on the other hand describes different kind of "outfluxes" from the agent's depot,

i.e. what the depot energy is used for. Specifically, we will consider two different processes in the following two sections:

$$p_i(\boldsymbol{r}_i,t) = s_i(\boldsymbol{r}_i,t) + d_i(\boldsymbol{v}_i)\; e_i(t) \tag{8}$$

$s_i(\boldsymbol{r}_i,t)$ describes for example *environmental changes* performed by the agent at its current position. In Sect. 4 we will assume that the agent is able to change an "adaptive landscape" or to establish a "communication field" to interact with other agents. Again, $s_i(\boldsymbol{r}_i,t) = s_0 = \text{const.}$ is chosen for simplicity. The second term $d_i(\boldsymbol{v}_i)e_i(t)$ is used to describe the active (accelerated) motion of the agent as described above. Eventually, the term $c_i e_i(t)$ in eq. (4) describes the internal dissipation of the energy depot at a specific loss rate c_i. In a biological context, internal dissipation is analogous to metabolism of the organism. According to eq. (1), it is possible to consider also explicite stochastic influences on the internal energy depot. This is neglected here since the equation for the depot is implicitely coupled to the equations of motion, where stochastic forces are taken into account. With the assumed specifications, the two-dimensional motion of a swarm of N Brownian agents can be simulated on the "individual level" by means of the $5N$ coupled (stochastic) equations (4). Fig. 1 presents snapshots of a computer simulation of 2.000 agents. (A movie of these computer simulations – with the same parameters, but a different random seed – can be found at `http://ais.gmd.de/~frank/swarm-tb.html`). In the simulations, we have assumed that the agents are initially at rest and at the same spatial position. Due to a supercritcal take-up of energy, expressed by the condition $(q_0 - s_0) > \gamma_0 c/d_2$ [7], the agents are able to move actively, the interaction however prevents the swarm from simply dispersing in space. Thus, the collective motion of the swarm becomes rather complex, as a compromise between *spatial dispersion* (driven by the energy pumping) and *spatial concentration* (driven by the mutual interaction) [6, 11, 18].

4 Brownian Agents in an Adaptive Landscape

In this section, we assume that the Brownian agents use their internal energy depot e_i mainly for environmental changes, $s_i(\boldsymbol{r}_i,t) = s_0$, and not for an accelerated motion, i.e. $d_2 \equiv 0$. Further, we assume that the different state variables $\boldsymbol{r}_i$, $\boldsymbol{v}_i$, e_i may change on different time scales which allows a so-called adiabatic approximation for the fast variables, as known e.g. from synergetics.Treating the internal energy depot and the velocity as quasistationary variables, we find for the dynamics of eq. (4):

$$\begin{aligned}
\frac{d\,u_i^{(1)}}{dt} &= \frac{d\,\boldsymbol{r}_i}{dt} = \boldsymbol{v}_i^0 \\
\frac{d\,u_i^{(2)}}{dt} &= \frac{d\,\boldsymbol{v}_i}{dt} = 0 \;\;\Leftrightarrow\;\; \boldsymbol{v}_i^0 = \frac{1}{\gamma_0}\mathcal{F}_i^{\rm int} + \sqrt{2\,D}\;\boldsymbol{\xi}_i(t) \\
\frac{d\,u_i^{(4)}}{dt} &= \frac{d\,e_i}{dt} = 0 \;\;\Leftrightarrow\;\; e_i^0 = \frac{q_0 - s_0}{c}
\end{aligned} \tag{9}$$

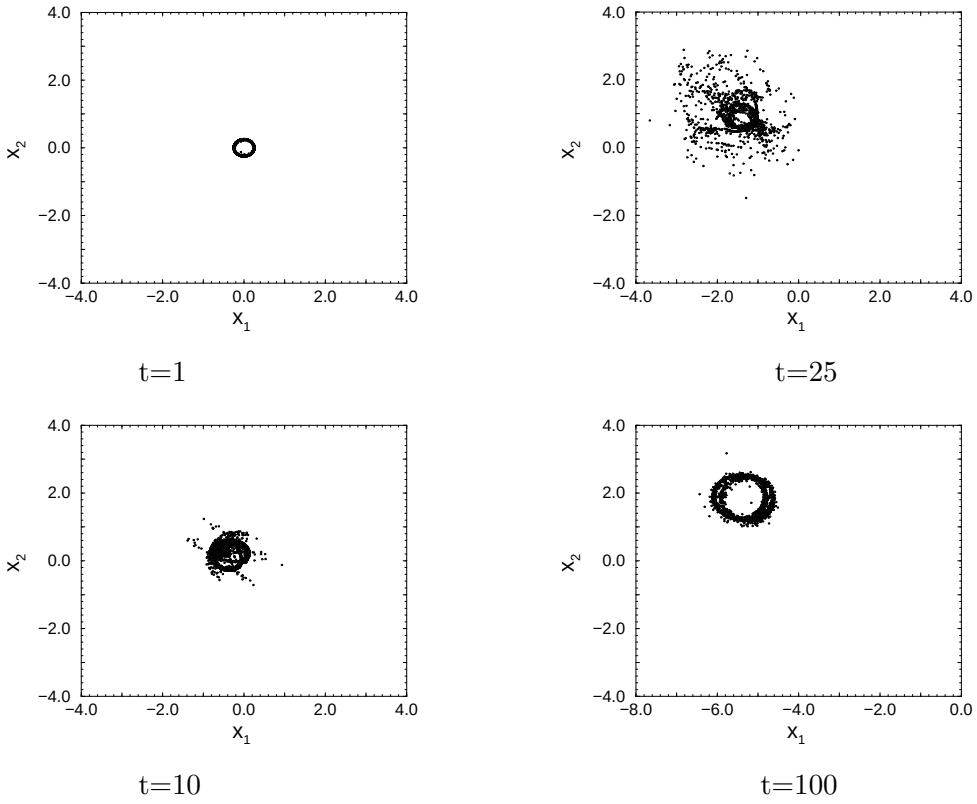

Figure 1: Snapshots (spatial coordinates) of a swarm of 2.000 Brownian agents moving according to eq. (4). t gives the different times. Note that the picture for $t = 100$ has a shifted x_1-axis. Initial conditions at $t = 0$: $\{x_{1i}, x_{2i}\} = \{0.0, 0.0\}$, $\{v_{1i}, v_{2i}\} = \{0.0, 0.0\}$ for all agents. Parameters: $a = 1$, $S = 10^{-8}$, $s_0 = 0$, $q_0 = 10$; $c = 1.0$; $\gamma_0 = 20$, $d_2 = 10$. [6]

$D = S/\gamma_0^2$ refers to the spatial diffusion coefficient of the agents. For the interaction of the agents, we may assume that they *indirectly* interact via an *adaptive landscape*. Every action of each agent is assumed to change the state of the adaptive landscape - either *locally* or *globally*, dependent on the model under consideration. On the other hand, the changes of the landscape may affect the actions of other agents in near or far distance. This way, a *non-linear feedback* occurs that is of importance for all processes of structure formation and self-organization.

In our approach, the adaptive landscape is modeled as a self-consistent spatial field, $h^e(\boldsymbol{r}, t)$ that may consist of different *components*, $h_\theta(\boldsymbol{r}, t)$ [10, 14, 15]. In a biological context, the field can for example represent a chemical field of different pheromones that are produced e.g. by ants or other insects in order to communicate to their mates. In this respect, the field can be envisioned as a communication field that contains spatial information produced by the insects. The response to the chemical field is then known as *chemotaxis*, i.e. the insects are either attracted or rejected by the

chemical. Such a reactive behavior is a basic feature of phenomena such as trail formation in ants [8, 19] or aggregation of slime molds and myxobacteria [4, 9, 12, 21].

In order to capture these features of chemical communication, we assume that the field components may follow a general dynamics [14, 15, 20]:

$$\frac{\partial}{\partial t}h_\theta(\boldsymbol{r},t) = \sum_{i=1}^{N} s_i\ \delta_{\theta,\theta_i}\ \delta(\boldsymbol{r}-\boldsymbol{r}_i)\ -\ k_\theta h_\theta(\boldsymbol{r},t)\ +\ D_\theta \Delta h_\theta(\boldsymbol{r},t). \tag{10}$$

In eq. (10) every agent contributes to the field with an amount of s_i. δ_{θ,θ_i} is the Kronecker Delta indicating that the agents contribute only to the field component which matches their internal parameter θ_i. $\delta(\boldsymbol{r}-\boldsymbol{r}_i)$ means Dirac's Delta function used for continuous variables to indicate that the agents contribute to the field only at their current position, $\boldsymbol{r}_i$. The *information* generated this way has a certain life time $1/k_\theta$, further it can spread throughout the system where D_θ represents the diffusion constant for the dissemination of information. Note, that the parameters describing the communication field do not necessarily have to be the same for the different agents i or internal parameters θ. In the following we may assume $s_i \equiv s_0$. Further, we wish to emphasize the fact that the *effort* of the agent for establishing the communication field, denoted by s_0, is also related to the internal energy depot e_i, eq. (9) of the agent. This shall reflect that every change of the agent's environment, such as generation of information or changes of the adaptive landscape in general, has some *costs* and therefore needs to be considered in the "energetic" balance equation.

In accordance with the chemotactic dynamics, the nonlinear feedback between the field $h^e(\boldsymbol{r},t)$ and the agent's behavior shall be determined by spatial gradients (for other applications, different assumptions can of course be made):

$$\mathcal{F}_i^{\rm int} = \alpha_i\ \frac{\partial h^e(\boldsymbol{r},t)}{\partial \boldsymbol{r}}\bigg|_{\boldsymbol{r}_i} \tag{11}$$

We note that the gradient of the effective field $h^e(\boldsymbol{r},t)$ is in general a specific function of the different field components $h_\theta(\boldsymbol{r},t)$: $\boldsymbol{\nabla} h^e(\boldsymbol{r},t) = \boldsymbol{\nabla} h^e(\ldots, h_\theta(\boldsymbol{r},t), h_{\theta'}(\boldsymbol{r},t), \ldots)$ In eq. (11) the agent's individual response parameter α_i can be further used to describe different responses, for instance (i) attraction to the field, $\alpha_i > 0$, or repulsion, $\alpha_i < 0$, (ii) response only if the local value of the field is above a certain threshold, (iii) response only if the agent is in a specific internal state.

5 "Heatbugs" revisited

In the specification of Sect. 4 the Brownian agent model captures essential features of the so-called "heatbug" simulation, one of the standard examples to demonstrate the capabilities of the SWARM simulation platform:

> "Each agent in this model is a heatbug. The world has a spatial property, heat, which diffuses and evaporates over time. Each heatbug puts out a small amount of heat, and also has a certain ideal temperature it wants to be. The system itself is a simple time stepped model: each time step, the heatbug looks moves to a nearby spot that will make it happier and then puts out a bit of heat. One heatbug by itself can't be warm enough, so over time they tend to cluster together for warmth." (`http://www.swarm.org/examples-heatbugs.html`)

The "spatial property" heat is formally described by eq. (10) for the spatio-temporal field $h_\theta(\boldsymbol{r},t)$. Since there is only one field component, we may simply choose $\theta = 0$, which in turn also means that all agents have the same internal degree of freedom, $\theta_i \equiv 0$. The amount of heat each agent puts out, is a constant s_0 in our model, while in the original heatbug simulation it can vary between a lower and an upper value. Additionally, in the heatbug simulation the agent may have a minimum and a maximum ideal temperature which is not considered here.

Apart from these minor differences, the Brownian agent model of Sect. 4 (that has been already published in 1994 [20]) provides a formal description of the heatbug simulation. Moreover, it is also applicable to many cases of *biological aggregation* based on *chemotaxis* where biological species, such as insect larvae [3] or myxobacteria [4, 21] gather guided by chemical signals originated by the individuals. For an analytical investigation of the aggregation dynamics of the "heatbugs" *et al.*, we may start from a stochastic description, where $P(\underline{r},t) = P(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N,t)$ gives the probability to find the N Brownian agents at time t at the positions $\boldsymbol{r}_1,...,\boldsymbol{r}_N$, with $N = \text{const}$. Considering further eq. (9) for the dynamics of the individuals, we can derive from the probability distribution $P(\underline{r},t)$ a macroscopic equation for the agent density, $n(\boldsymbol{r},t)$ [20]:

$$\frac{\partial}{\partial t}\, n(\boldsymbol{r},t) = \frac{\partial}{\partial \boldsymbol{r}} \left\{ -\frac{\alpha}{\gamma_0} \frac{\partial h_0(\boldsymbol{r},t)}{\partial \boldsymbol{r}} n(\boldsymbol{r},t) \;+\; D \frac{\partial n(\boldsymbol{r},t)}{\partial \boldsymbol{r}} \right\} \tag{12}$$

Using $n(\boldsymbol{r},t)$ and $\theta = 0$, eq. (10) becomes:

$$\frac{\partial}{\partial t}\, h_0(\boldsymbol{r},t) = s_0\, n(\boldsymbol{r},t) - k_0\, h_0(\boldsymbol{r},t) + D_0\, \frac{\partial^2 h_0(\boldsymbol{r},t)}{\partial r^2} \tag{13}$$

Both macroscopic equations (12) and (13) can be used to further analyze the aggregation dynamics [20]. Let us assume in an adiabatic approximation that the field $h_0(\boldsymbol{r},t)$ relaxes faster, compared to the distribution of the agents $n(\boldsymbol{r},t)$ into its stationary state, and the diffusion coefficient of the field D_0 is very small:

$$\frac{\partial h_0(\boldsymbol{r},t)}{\partial t} \approx 0 \;\; ; \quad h_0(\boldsymbol{r},t) = \frac{s_0}{k_0}\, n(\boldsymbol{r},t); \quad \text{if} \quad D_0 \to 0 \tag{14}$$

Eq. (14) means that the spatio-temporal distribution of the field follows quickly the spatio-temporal distribution of the agents. Then eq. (12) for the agent density $n(\boldsymbol{r},t)$ can be rewritten in terms of

a usual diffusion equation:

$$\frac{\partial n(\boldsymbol{r},t)}{\partial t} = \frac{\partial}{\partial \boldsymbol{r}} \left\{ D_{\rm eff} \frac{\partial n(\boldsymbol{r},t)}{\partial \boldsymbol{r}} \right\} \tag{15}$$

where the *effective diffusion coefficient*, $D_{\rm eff}$ in the quasistationary limit of eq. (14) can be expressed as follows:

$$D_{\rm eff} = D - \frac{\alpha}{\gamma_0} \frac{\partial h_0(\boldsymbol{r},t)}{\partial n(\boldsymbol{r},t)} n(\boldsymbol{r},t) = D - \frac{\alpha}{\gamma_0} \frac{s_0}{k_0} n(\boldsymbol{r},t) = D - \frac{\alpha}{\gamma_0} h_0(\boldsymbol{r},t) \tag{16}$$

We note that $D_{\rm eff} = 0$ for a certain *equilibrium density* of the field, $h_0^{\rm eq}$. We may thus introduce a

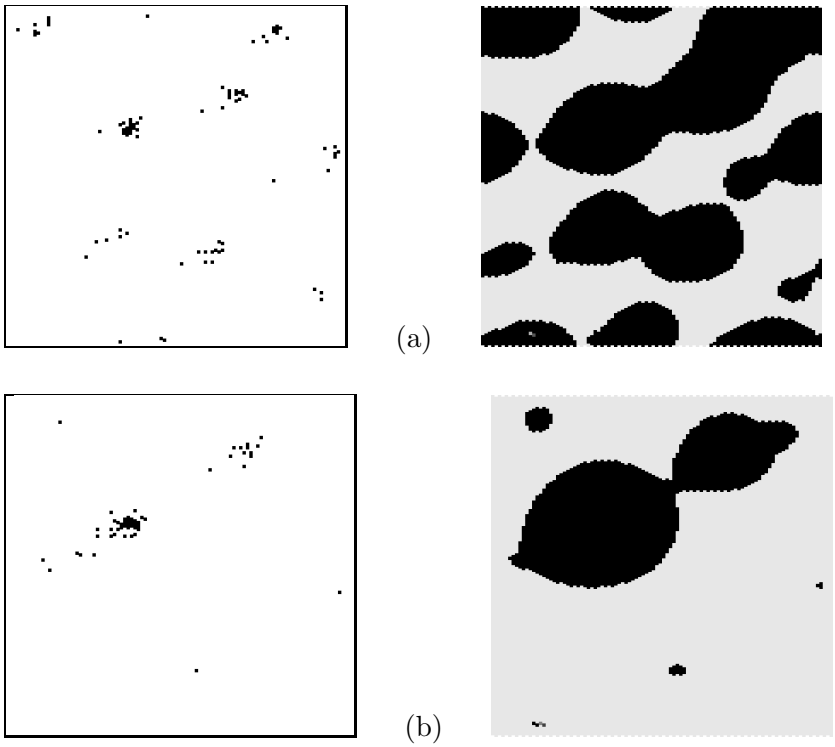

Figure 2: (left) Spatial positions of 100 agents on a surface of $A = 100 \times 100$ (periodic boundary conditions), (right) corresponding spatial distribution of the reduced effective diffusion coefficient $\sigma(\boldsymbol{r},t)$, eq. (17). The black area indicates $\sigma < 0$, which means an attraction area for the agents, the grey area indicates $\sigma > 0$. Time in simulation steps: (a) $t = 5.000$, (b) $t = 50.000$. Parameters: $s_0 = 80$, $k_0 = 0.001$, $D_0 = 0.01$, $h^{\rm eq} = 200$, $\alpha/\gamma_0 = 1$. [20]

reduced variable:

$$\sigma(\boldsymbol{r},t) = \frac{D_{\rm eff}}{D} = 1 - \frac{h_0(\boldsymbol{r},t)}{h_0^{\rm eq}} \tag{17}$$

$\sigma > 0$ means a spreading of agents over the whole surface as in a usual diffusion process. $\sigma < 0$, on the other hand results in a lump of agents concentrating only in certain regions. The transition from

$\sigma > 0$ to $\sigma < 0$ is driven by the *local field production* of the agents, i.e. σ becomes negative if $h_0(\boldsymbol{r}, t)$ locally exceeds the equilibrium value, $h_0^{\rm eq}$. Depending on the spatio-temporal density $h_0(\boldsymbol{r}, t)$, the local value of $\sigma(\boldsymbol{r}, t)$ can at the same time have quite different effects at different places. This is shown in Fig. 2 that presents computer simulations of the spatial agent positions (left side) together with the corresponding distribution of σ (right side). The inhomogeneously distributed black areas indicate $\sigma < 0$, resulting in a bound motion of the agents and eventually in an aggregation in that area. As shown in Fig. 2, the attraction area is decreasing with time in diversity as well as in size, indicating a selection process among the attraction areas, which has been discussed in detail in [17, 20].

6 Conclusions

The two examples discussed above have shown how the concept of Brownian agents can be used to (i) *simulate* and (ii) *analytically investigate* the dynamic behavior of certain spatial *multi-agent systems*. It is one of the major advantages of the Brownian agent concept that, by means of formal methods known from statistical physics, we are able to derive from the agent-based (*microscopic*) dynamics *macroscopic equations* for the system dynamics. This allows a more detailed theoretical investigation of the dynamics of the multi-agent system and some predictions of the collective behavior, for example bifurcations in the behavior at certain critical parameters.

While on one hand considerable advantanges result from the use of *both* a microscopic simulation and their corresponding macroscopic investigation, we note that on the other hand not all kind of MAS may be mapped to a Brownian agent model and thus not all kind of questions can be answered by this approach. This is no surprise. As any other agent approach, the Brownian agent concept is based on certain modeling reductions regarding the system elements and their interactions that need to be carefully understood.

The Brownian agent concept has proven its use in a number of applications where (positive and negative) local feedback processes play a considerable role, but an internal *evolution* of the agents can be neglected: for example physico-chemical pattern formation, certain behavior in insect societies (such as aggregation, foraging behavior and trail formation), traffic dynamics, human behavior (pedestrian dynamics, panics), urban and economic aggregation, opinion formation and coordination of decisions in social systems [17]. The Brownian agent approach would be less appropriate in cases where agents display "intelligent" features, such as logical deduction, complex behavioral adaptation to the environment, development of individual strategies to pursue a certain goal, or developent of an individual world view. The question however is, to what degree these specifications need to be taken into account in order to explain or to reproduce an observed emergent behavior. Moreover, there is certainly a *trade-off* between a most realistic computer simulation that includes as much microscopic details as possible, and the possibility to match such a system with a tractable

analytical model. While the latter one is most desirable, it would need some compromises in the design of the microscopic simulations.

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