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Urban Cluster Growth: Analysis and Computer Simulations of Urban Aggregations

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1 Introduction

The formation of urban structures, like settlements, trail systems, transport and supply networks, shows, from a structural perspective, significant analogies to phase transitions, known from physics, like cluster formation, aggregation and percolation phenomena, but also to self-organizing processes, in which new system qualities emerge due to the dynamic interaction of sub-entities.

Usually, one is convinced, that the development of urban areas is determined by numerous factors, difficult to grasp, such as cultural, sociological, economic, political, ecological etc. ingredients. On the other hand, on a certain level of abstraction, one can also find many common features between different urban structures, even from far distant regions, as well as between urban and natural structures (Batty, Longley, 1994; Becker *et al.*, 1994, Humpert, Brenner, 1992, Schaur, 1991).

These investigations lead us to the hypothesis, that there is a common level to describe the formation of these different structures. From our perspective, the basic dynamics of structure formation creates these analogies, regardless of the different sub-entities involved (cf. Haken, 1978), and we are convinced that the theory of self-organization provides a suitable quantitative approach to it (Helbing *et al.*, 1994, Schweitzer, 1996).

In this chapter, the approach of self-organization should be applied to the field of *urban structure formation* on a very basic level, which only considers structural or morphological features of urban aggregates. However, if we can obtain very simple rules for the self-organization of urban aggregations, this might shed some new light on the established ideas about hierarchical planning of urban settlements. Hence, there could be a quest for the system-immanent forces which allow the generation of urban structures in a new way, by incorporating the ideas of self-organization.

This turn away from total planning, is not only a question of developing new theories, it is challenged by the pressure resulting from the population increase in the developing countries. This population

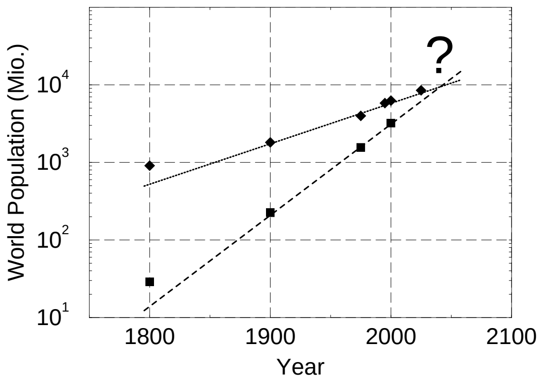

Figure 1: Growth of the world population (diamonds) and the urban population (squares) in a logarithmic plot. The dotted and dashed lines indicate the approximation by an exponential growth law, which may result in a fictive urban catastrophe by the year of 2050.

growth leads to an urban growth which cannot be controlled or regulated any further (cf. Fig. 1). Hence, established methods of urban planning become an illusion when confronted with these realities. What could be an alternative? To give new power to the structure generating forces existing also in urban systems, but to determine the conditions (e.g. the topological conditions) which set limits to the self-organization process. This, however, needs a better understanding of how urban self-organization proceeds, and a quantitative description to simulate the processes involved.

2 Analysis of Urban Clusters

2.1 Description

From the perspective of physics, urban aggregations, like cities with the surrounding satellite towns and commercial areas, or large agglomerations of mega-cities, can be described as special kinds of *clusters* on a two-dimensional surface. Using a black-and-white reduction, these clusters represent the build-up area (black pixel = occupied site, build up, white pixel = empty site, no building). Of course, this 2-bit characteristics of the urban area is a major reduction of the complex structure. No internal structures are considered here, in particular, there is **no** valuation of the types of buildings (garage or skyscraper) and **no** valuation of the importance of the area (downtown or suburb).

For our investigations, we used digitized maps, where the length scale: 1 : 50.000 or 1 : 100.000 defines the resolution. Based on these "black-and-white" maps, we (i) analyse the *structural features* of the build-up area (morphology, shape), and (ii) simulate the dynamics of the build-up area (occupation, growth) in a quantitative way.

Recently, for a description of the structural features and the kinetics of urban clusters, *fractal concepts* have been used (FRANKHAUSER, 1991, 1992, 1994). The existence of fractal properties over several spatial scales indicates a hierarchical organization of the urban aggregates (FRANKHAUSER, SADLER, 1992). Previous attempts to simulate the growth of urban clusters by means of fractal aggregation mechanisms, are mainly based on DLA- or DBM models (DLA - diffusion limited aggregation, DBM - dielectrical breakdown model) (BATTY, 1991). However, the existing results can only partially convince. Namely, DLA models only produce connected clusters, with the largest growth potential at the tips of the cluster, far away from the center. In DBM models, the compactness of the clusters can be changed with an additional parameter entering the aggregation probabilities (BATTY, 1991). However, it is not considered that the compactness of urban clusters change in the course of time. Also in DBM models only connected clusters occur, and phenomena specific for urban agglomerations, such as the burn-out of urban growth, the emergence of new growth zones, or the coexistence of urban clusters, cannot be sufficiently described within this approach.

Another class of kinetic models for urban growth, based on cellular automata, is discussed in the chapter of WHITE, ENGELEN in this volume (cf. also WHITE, ENGELEN, 1993, 1994). In models which mainly focus on the simulation of "urban land-use patterns", the urban growth is more or less determined by an economically based utility function, as discussed in the chapter of WEIDLICH in this volume (cf. also WEIDLICH, 1991).

In this chapter, the structural aspects of urban cluster growth and their relation to physical models are investigated. Some of the results are based on previous analyses of the spatial distribution of settlement areas (SCHWEITZER, STEINBRINK, 1996). Here, we focus on the cluster distribution of urban aggregates, and on a spatial model of urban aggregation, based on reaction-diffusion systems. This model should be capable to describe different effects of urban growth, like the burn-out of growth, or the shift of growth zones.

2.2 Rank-Size Distribution of Urban Clusters

Usually, urban aggregates are not just one merged cluster, but consist of a large number of separated clusters of different sizes. In a cluster analysis, the size and the number of the different separated clusters forming the urban aggregate can be calculated (SCHWEITZER, STEINBRINK, 1994). Then, the clusters are sorted with respect to their size, thus determining their rank number (the largest cluster gets rank 1, etc.). As a result, Fig. 2 presents the development of the rank-size cluster distribution for Berlin.

Fig. 2 shows, that the rank-size distribution approaches a power law in time, indicated by a straight slope, which can be described by the following equation:

$$\log N(q) = \log N(1) + \alpha \log q \quad \text{or} \quad N(q) = N(1) q^{\alpha}; \quad \alpha < 0 \tag{1}$$

$N(q)$ is the size of the cluster with rank q and $N(1)$ is the size of the largest cluster which serves as normalization. This power law of the rank-size distribution is also known as PARETO distribution (FRANKHAUSER, 1991, GÜNTHER *et al.*, 1992), with α being the PARETO exponent. The PARETO distribution is a characteristic feature for hierarchically organized systems; this means in

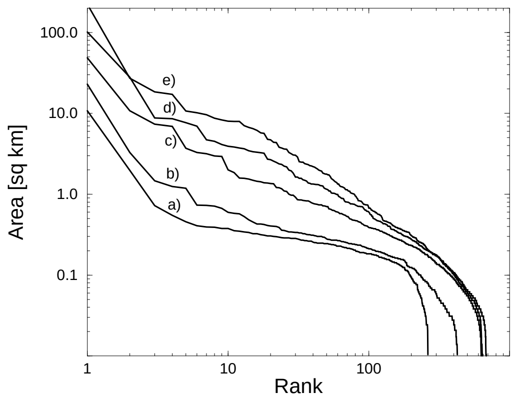

Figure 2: Evolution of the rank-size distribution of connected build-up areas (clusters) of Berlin in a double-logarithmic plot (a: 1800, b: 1875, c: 1910, d: 1920, e: 1945).

the considered case that the urban settlement area is formed by clusters of all sizes in the course of time.

If the evolution of urban aggregates occurs that way that the cluster distribution approaches a PARETO distribution in time, we may conclude that the establishment of this distribution could be a measure for a fully developed urban aggregate - and, on the other hand, the deviation from this distribution could be a measure for the *developmental potential* with respect to the morphology of the settlement.

Looking at the rank-size distributions of different urban aggregations (cf. Fig. 3), we find that this developed stage is sometimes already reached (cf. Fig. 3 d - Philadelphia)

Table 1 contains the PARETO exponents α which result from Fig. 3 due to eq. (1).

City	Pareto Exponent
Munich 1965	- 1.23 $\pm$ 0.02
Daegu 1988	- 1.31 $\pm$ 0.03
Moscow 1980	- 1.15 $\pm$ 0.02
Philadelphia 1980	- 1.32 $\pm$ 0.01

Table 1: PARETO exponent α for different urban agglomerations. The values are obtained from Fig. 3 by linear regression of the first 100 ranks.

Our investigations show that the PARETO exponents of different urban agglomerations have similar values. This indicates that the PARETO exponent could serve, in the scope of certain error limits, as an *structural measure of developed urban aggregates*, which may complement the fractal dimension.

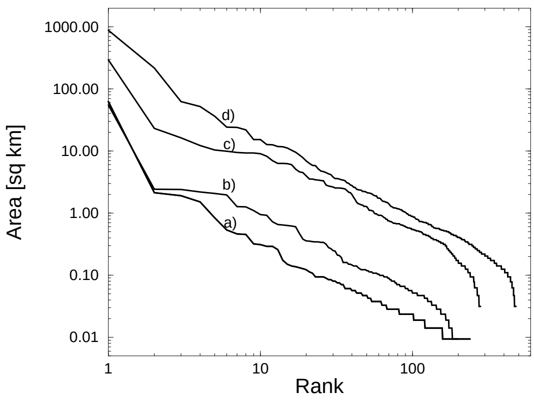

Figure 3: Rank-size distributions of connected build-up areas in a double-logarithmic plot: (a) Daegu (1988), (b) Munich (1965) (c) Moscow (1980), (d) Philadelphia (1980)

In the following, we want to derive a kinetic model to describe the evolution of the rank-size distribution towards a PARETO distribution.

3 Simulation of the Rank-Size Distribution of Urban Clusters

3.1 The Master Equation

For urban aggregates which consist of clusters of different sizes, we introduce the cluster-size distribution $\mathbf{n}$ as follows: $\mathbf{n} = \{n_1, n_2, ..., n_k, ..., n_A\}$. Here, n_k is the size (the number of pixels) of the cluster with number k, and a total number of A clusters exist ($k = 1, ..., A$). The total number of pixels is obtained by summation:

$$N_{tot}(t) = \sum_{k=1}^{A} n_k \tag{2}$$

and should, for an evolving settlement, increase in the course of time, as shown in Fig. 4 for the urban cluster of Berlin. For the years from 1870 to 1945, the total increase of the urban area of Berlin can be approximated by an exponential growth law.

The cluster distribution $\mathbf{n}$ can be changed due to two fundamental processes: (1) the formation of new clusters, (2) the growth of already existing clusters (SCHWEITZER, STEINBRINK, 1995, 1996). In principle, we should also consider the shrinkage and disappearence of already existing clusters. However, the probability of these processes is rather small for growing urban areas; therefore they are neglected in the following.

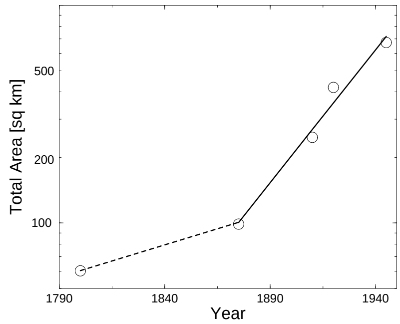

Figure 4: Growth of the settlement area of Berlin in a logarithmic plot

The formation of new clusters can be described by a symbolic reaction $A \xrightarrow{w_1} A+1$, which means that the total number of clusters A increases by one at that time. The probability for forming a new cluster during the next time intervall, w_1, should certainly depend on the total growth of the urban settlement, $N_{tot}(t)$, which can be approximated using empirical data (cf. Fig. 4). Thus, we find the ansatz:

$$w_1 = w(A+1, t+1|A, t) = c(N_{tot}) \tag{3}$$

However, for simplicity we assume a constant probability for the formation of new clusters, i.e. $c = const.$, in the following.

The growth of the already existing clusters can be described by the symbolic reaction $n_k \xrightarrow{w_k} n_k + 1$, which means that the size n of cluster k increases by one at that time. The probability that this process occurs during the next time intervall, w_k, should depend on the already existing size of the cluster and on the existing cluster distribution. Hence, we find the following ansatz:

$$w_k = w(n_k+1, t+1|n_k, t) = \gamma \frac{n_k}{N_{tot}}, \qquad \gamma = 1 - c(N_{tot}) \tag{4}$$

In physical terms, the dependence on $N_{tot}(t)$ represents a *global coupling* between the growth processes of all separated clusters. Therefore, the growth probability of a specific cluster within the urban aggregate is not independent on the other clusters existing.

By means of the factor γ, which is related to the probability of cluster formation, c, the ratio between the formation of new clusters and the growth of existing clusters can be weighted. Usually, we have chosen values of $\gamma = 0.90...0.99$, which means $c = 0.1...0.01$, accordingly.

In a given time intervall, the morphology of the urban aggregate can be changed due to numerous processes of formation and growth of clusters which occur simultanously. Since all possible processes have a certain probability, the result of the evolution is not clear determined.

Instead of a deterministic approach we have to use a *probabilistic approach* considering the uncertainity of the future and the limited predictability of evolution. The probability distribution $P(\mathbf{n},t) = P(n_1, n_2, ..., n_k, ..., n_A, t)$ gives the probability that a particular cluster distribution $\{n_1, n_2, ..., n_k, ..., n_A\}$ will be found after a certain time. The change of this probability distribution in the course of time can be described by the following *master equation*:

$$\begin{aligned} \frac{\partial P(\mathbf{n},t)}{\partial t} = & -\sum_k w_k(n_k,t)P(n_1,...,n_k,...,t) \\ & +\sum_{k\neq 1} w_k(n_k-1,t)P(n_1,...,n_k-1,...,t) \\ & -w_1(A,t)P(n_1,...,n_A,t) \\ & +w_1(A-1,t)P(n_1,...,n_{A-1},t) \end{aligned} \quad (5)$$

In the next section, the master equation will be solved by means of computer simulations. It is rather complicated to derive analytical results for this equation, since all possible processes depend on the others, too, because of the global coupling, given by eq. (2). However, this coupling can be neglected using the assumption, that the total number of clusters, A, is indeed large, but much less then the total number of pixels, N_{tot}. This is basically a restriction for the probability of formation of new clusters, c, which has to be small enough. With the assumption: $1 \ll A \ll N_{tot}$, the master equation can be resolved:

$$P(n_1,...,n_k,...,t) \approx \prod_{k=1}^{A} P(n_k,t) \quad (6)$$

Eq. (6) means, that instead of the probability of the whole cluster distribution, $P(\mathbf{n},t)$, only the probability to find clusters of a certain size, $P(n_k,t)$ has to be considered now. With this probability, the mean size of a cluster k at a given time can be calculated:

$$\langle n_k(t)\rangle = n_k P(n_k,t) \quad (7)$$

and we obtain the result (GÜNTHER *et al.*, 1992):

$$\langle n_k(t)\rangle \sim N_{tot}\, k^{-(1-c)} \quad (8)$$

Eq. (8) agrees with the PARETO-ZIPF distribution, eq. (1). Here, the PARETO exponent α is given by $\alpha = c - 1 < 0$. We note, that the descent of the PARETO-ZIPF distribution expressed by α depends remarkably on the probability for the formation of new clusters, c.

We conclude that the change of the cluster distribution, described by the master equation, eq. (5), occurs that way that eventually, in the average, clusters of all sizes exist, which lead to a hierarchical composition of the urban aggregate.

3.2 Results of Computer Simulations

In order to show a real example for the evolution of the rank-size cluster distribution, we have simulated the development of Berlin for the years between 1910 and 1945 (SCHWEITZER, STEINBRINK, 1995, 1996). The initial state for the simulation is given by the urban cluster of Berlin (1910) (cf. the black area in Fig. 5). The increase of the total build-up area, $N_{tot}(t)$, between 1910 and 1945 is known from the empirical data (cf. also Fig. 4).

The stochastic simulations of the urban growth proceed as follows:

(i) At time t, all possible probabilities for the formation of new clusters and the growth of existing clusters are calculated.

(ii) The probability w for every process is compared with a random number drawn from the intervall $(0,1)$. Only if w is larger than the random number, the related process is carried out, i.e. either cluster growth: $n_k \to n_k + \Delta n$, or cluster formation: $\Delta n \to n_{A+1}$ occur. Here, Δn is the the amount of pixels which is just optically recognized for the given resolution of the map (in the simulation of the growth of Berlin, these are 40 pixels). For the formation of new clusters, a random place is chosen; for the cluster growth, the number of pixels are randomly distributed on the rim of the cluster.

(iii) Afterwards, the simulation continues with the next time intervall unless the total increase of the build-up area is reached: $\sum \Delta n = N_{tot}(t_{fin})$

The first simulations which have been carried out that way, however proved that the empirical rank-size distribution for Berlin could not be reproduced. The reason for that is obvious: With respect to the growth probabilities of eq. (4), during the simulations, the largest cluster (rank 1) dominates the whole growth process. During its growth, which is proportional to its size, this cluster also merges with the smaller clusters surrounding it, which leads again to a jump in its size. Eventually, the whole cluster distrubution collapses. However, such an evolution was not found empirically. Instead, separate clusters are also found in large urban agglomerations, which leads us to the conclusion that the growth probabilities have to be modified at least for clusters with small ranks.

Our simulations (SCHWEITZER, STEINBRINK, 1995, 1996) have proved that the PARETO distribution for the urban area of Berlin could be reproduced with the additional assumption, that the cluster with rank 1 does not grow automatically proportional to its size, but only due to coagulation with neighboring clusters; whereas cluster with rank 2 or higher grow again proportional to their size, due to eq. (4), and of course due to coagulation.

The results of the computer simulations are shown in Fig. 5. They clearly indicate that the rank-size cluster distribution of Berlin could be well reproduced for the year of 1945, which confirms our kinetic assumptions for the simulation.

We want to point out again, that from the empirical data only the information about the total increase of the build-up area has been drawn, which could have been also calculated from an

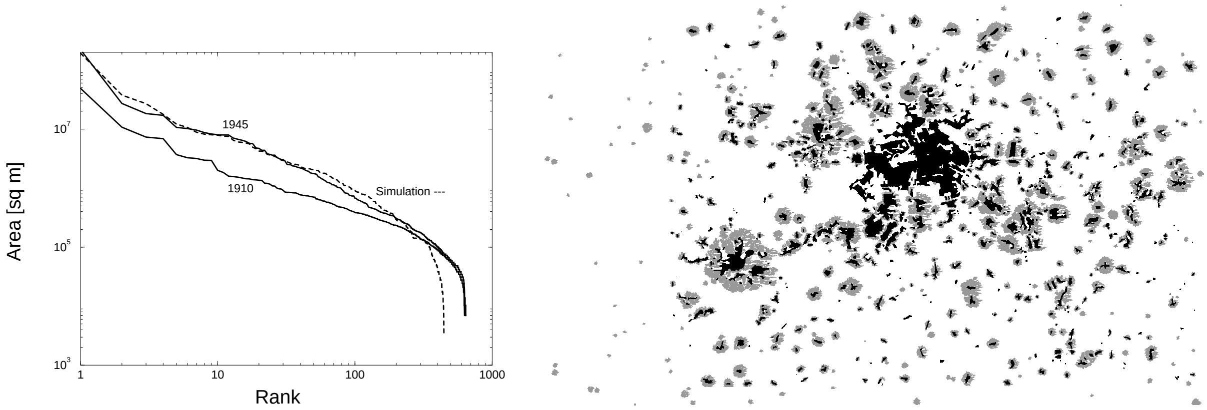

Figure 5: (left) Simulation of the rank-size cluster distribution of Berlin (1910 - 1945), (right) Simulated spatial distribution of the urban clusters for Berlin (1945), black: Initial state (Berlin 1910), grey: simulated growth area. Parameter: c=0.04

exponential growth law. We did not use any information which of the different clusters grew and by which amount - this process has been entirely simulated by the master equation. The simulations give an estimate how much a cluster of a particular size should grow during the time interval considered - independent on its definite location within the urban aggregate. The results sufficiently agree with the real evolution of the urban aggregate, however, they are not able to reproduce the *real spatial* distribution, since no informations about spatial coordinates are considered in the kinetic assumptions.

In a previous paper (Schweitzer, Steinbrink, 1995, 1996), the model presented has been also used to simulate the future evolution of the South-Korean city Daegu, an urban aggregation which, with respect to its structural morphology, today is similar to an early stage of the evolution of Berlin.

4 Kinetic Model of Urban Cluster Growth

4.1 Reaction-Diffusion Approach to Urban Cluster Growth

The computer simulations discussed in the previous section, leave us with two major questions: *Why* do clusters of rank 1 stop to grow? ("burn-out"), *When* do large clusters stop to grow? A possible insight into the specific dynamics of urban cluster growth could be provided by a 3 component reaction-diffusion system, the A-B-C model, which, in a symbolic manner, represents important features of the urban evolution.

(1) Local Aggregation of Particles

The basic assumption of our model is a non-linear feedback between the existing urban aggregate and its further growth, which is mediated by an attraction field created by the settlement. The aggregation process should be described by the particle species C, which should exist in two different sub-species: $C \Rightarrow C_0, C_1$. Here, C_1 represents the particles which are already aggregated (no motion), hence the concentration field $c_1(r,t)$ describes the build-up area at a given time. The sub-species C_0, on the other hand, represents the growth-units which are not aggregated yet, $c_0(r,t)$ being the corresponding concentration field. The transformation of a moving growth unit into a precipitated (non-moving) build-up unit can be expressed by the symbolic reaction: $C_0 \xrightarrow{\gamma} C_1$, γ being the reaction rate.

We further assume that the *existing* urban aggregate creates a specific spatio-temporal field $h(r,t)$ which should represent the attraction of the urban area. We are convinced that this attraction exists - however we are not specific about the reasons for this attraction, there could be economic, political, cultural reasons, or just a certain way of life, which makes the urban area attractive. Within our model we simply assume that the attraction field is steadily produced by the existing build-up area at a rate q. On the other hand, an attraction which is not maintained can also fade out in the course of time, expressed by the rate μ. Finally, we consider that the attraction of an urban area can spread out into the surrounding area, expressed by the diffusion constant D_h. These assumptions result in the following reaction-diffusion equation for the change of the attraction field $h(r,t)$:

$$\frac{\partial h(r,t)}{\partial t} = -\mu h(r,t) + q\, c_1(r,t) + D_h \Delta h(r,t) \tag{9}$$

The non-linear feedback between the existing urban aggregate and its further growth is given by the assumption, that the growth units C_0 which are not aggregated so far, are effected by the attraction field of the existing aggregations in a twofold way:

(i) The motion of the growth units C_0 at position r_i is influenced by the gradient of the attraction field, so that they intend to come close to the maxima of the attraction (cf. also SCHWEITZER, SCHIMANSKY-GEIER, 1994, SCHIMANSKY-GEIER *et al.*, 1996). The resulting LANGEVIN equation reads:

$$\frac{dr_i}{dt} = \lambda \left.\frac{\partial h(r,t)}{\partial r}\right|_{r_i} + \sqrt{2k_B T\beta}\,\xi_i(t) \tag{10}$$

where λ represents the strength of the response to the gradient and the second term of the right-hand side is a stochastic force which keeps the particles moving.

(ii) In dependence on the local value of the attraction field, the growth units can precipitate and transform into a build-up unit - either by attachment to an existing cluster or by formation of a new one. The probability of transformation of the growth units, γ, should depend on the normalized local attraction:

$$\frac{\partial c_1(r,t)}{\partial t} = \gamma\, c_0(r,t) \quad ; \quad \gamma(r,t) = \frac{h(r,t)}{h_{max}(t)} \tag{11}$$

To summarize the non-linear feedback, we conclude that the build-up units do not move, but create an attraction field, which effects the movement of the growth units, which do not create an attraction field. Growth units could be transformed into build-up units, thus further increasing the attraction of the urban aggregation.

(2) Local Depletion of Free Space

The demand for new build-up areas could not always be satisfied at desired places because of the local depletion of free space, which is the second important feature effecting urban growth. Let the species A represent the free space, the density being $a(r,t)$. Initially, for $t = 0$, we shall assume that the free space is equally distributed: $a(r,t_0) = a_0 = A_0/S$, where S is the surface area. However, for $t > 0$ the decrease of free space leads to the emergence of depletion zones in A.

Further, we assume that the species B represents the *demand* for build-up areas, with a spatial density $b(r,t)$. This demand could result from different reasons, which we do not discuss, and could be concentrated at different locations. Hence, the production of demand, expressed by the symbolic reaction, $\stackrel{\beta}{\longrightarrow} B$, could reflect different situations:

$$\begin{array}{lll} \beta = & b_0/S;\; b_0 = B_0\,\Delta t & \Rightarrow \text{demand equally distributed} \\ \beta = & b_0\,\delta(r-r_0) & \Rightarrow \text{demand in the main center } r_0 \\ \beta = & \sum_i b_{0i}\delta(r-r_i);\; \sum_i b_{0i} = B_0\,\Delta t & \Rightarrow \text{demand in different centers } r_i \end{array}$$

Here, B_0 is the total demand in the considered time-space intervall. In addition to a linear production of demand, also a nonlinear production could be considered, e.g.: $B \stackrel{\beta}{\longrightarrow} 2\,B$.

As pointed out, not every demand for a build-up area can be satisfied at the place where it is initiated; it has to match some free space available. Therefore, we assume that the species B diffuses with a constant D_b from the centers, as long as it meets particles of the A species, which represent the free space and which do not diffuse. Only B and A, both demand and free space, can create the growth units C_0 which account for the urban growth, expressed by the symbolic reaction: $A + B \stackrel{\alpha}{\longrightarrow} C_0$. Here, α describes the reaction rate at which the free space disappears. Due to the local depletion of A, the B species has to move further apart from the centers of demand to create some growth units C_0 in outer regions, which in turn try to move back to the urban centers with respect to the attraction field $h(r,t)$. Along their way, the moving growth units pass different aggregations, which create a local attraction, such as suburbs. This increases the probability that the growth units precipitate before they have reached the very centers of the urban aggregate with the highest attraction.

The dynamics for the concentrations of the A, B, C_0 species can be described by the following set of coupled reaction-diffusion equations for A, B and FOKKER-PLANCK equation for C_0:

$$\frac{\partial a(r,t)}{\partial t} = -\alpha\, a(r,t)\, b(r,t) \tag{12}$$

$$\frac{\partial b(r,t)}{\partial t} = \beta - \alpha\, a(r,t)\, b(r,t) + D_b\, \Delta b(r,t) \tag{13}$$

$$\frac{\partial c_0(r,t)}{\partial t} = \frac{\partial}{\partial r}\{\lambda \frac{\partial h(r,t)}{\partial r} + D_c \frac{\partial c_0}{\partial r}\} + \beta\, a(r,t)\, b(r,t) - \gamma\, c_0 \tag{14}$$

Together with eqs. (9) (11), the eqs. (12)-(14) have to be solved simultaneously to describe the reinforced urban growth with respect to local attraction and local burn-out.

4.2 Setup for the Computer Simulations

In order to demonstrate the applicability of the kinetic model introduced in Sect. 4.1., we have simulated the evolution of the urban area of Berlin.

For the computer simulation, three input parameters must be known: the free space available in the region, A_0, at the start time t_{start}, the demand for build-up areas, B_0, during the simulated time intervall $t_{end} - t_{start}$, and the existing urban aggregation, $c_1(t_{start}, r)$, at time t_{start}, in order to calculate the initial attraction field. Whereas the cluster distribution of the urban aggregate is given from empirical data (maps), the values for A_0, B_0 can be obtained from a plot which shows the evolution of the total build-up area (cf. Fig. 6).

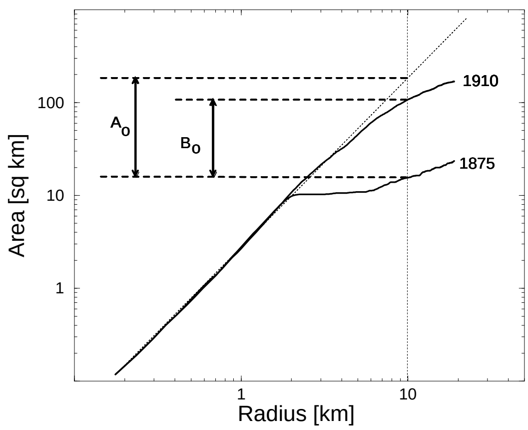

Figure 6: Total build-up area $N(R)$ of Berlin vs. distance R from the center in a double-logarithmic plot for different years

The straight slope of the curves in Fig. 6 indicates that the increase of the total build-up area approaches a characteristic power law, $N(R) \propto R^{D_f}$, in the course of time, which can be used to calculate the fractal dimension, D_f, of the urban aggregate. It has been found that during the urban evolution the fractal dimension remains constant. We count on that fact when calculating the free space available in the region. If we consider a certain spatial extension (e.g. $R^* = 10$ km), then the fractal dimension determines the *maximum value* of the build-up area within that area, $N_{max}(R^*)$ where N is the number of occupied pixels. The free space available in the region at a certain time, is just the difference between the actual value of the build-up area, $N(R^*, t)$, and $N_{max}(R^*)$: $A_0(R^*,t) = N_{max}(R^*) - N(R^*,t)$ (cf. Fig. 6). Assuming, that the free space in the region is initially equally distributed, the mean density of the free space is also given by: $a_0 = A_0/S = N_{max}(R^*)/4\pi R^{*2}$. For the area of Berlin, the mean density of free space is calculated as $a_0 = 0.43$ pixels/lattice unit.

The demand for build-up areas in a certain time intervall can be derived from the time series of Fig. 6 by calculating the difference between the build-up areas in the beginning and in the end: $B_0 = N(R^*, t_{end}) - N(R^*, t_{start})$. For future predictions, the value of $N(R^*, t_{end})$ is of course not known from empirical data, however, as proved by Fig. 4, the value can also be estimated by an exponential growth law. For our simulations of the urban area of Berlin, we have simply assumed a constant demand for build-up areas during the time intervall, which is initiated only in the center of Berlin (r_0). Hence, the production of B is described by: $\beta = b_0\,\delta(r - r_0)$, with $b_0 = B_0\,\Delta t$. Further simulations could also include major cities in the neighborhood as addditional centers of demand.

Our simulations of the Berlin area are carried out for the time intervall 1910 to 1920. Therefore, we have to take into account, that, due to the previous evolution, already depletion zones of the free space exist, which surround the existing clusters. The total consumption of free space is, of course, equal to the total amount of occupied space, known from empirical data. Since we assume, that the B species, which reflects the demand, is diffusing until it reaches the A species, the existing depletion zones resulting from the $A + B$ reaction, can be simply described by a circle which surrounds the existing clusters. The radii of these depletion circles, R_{di}, can be estimated by: $R_{di} = \sqrt{N_i/4\pi a_0}$, where N_i is the pixel size of a particular urban cluster and a_0 is the mean density of the free space, given above. Outside these depletion zones, we assume again an equal distribution of free space: $a(r > R_{di}, t) = a_0$. The initial configuration for the simulation of the Berlin area is given in Fig. 7.

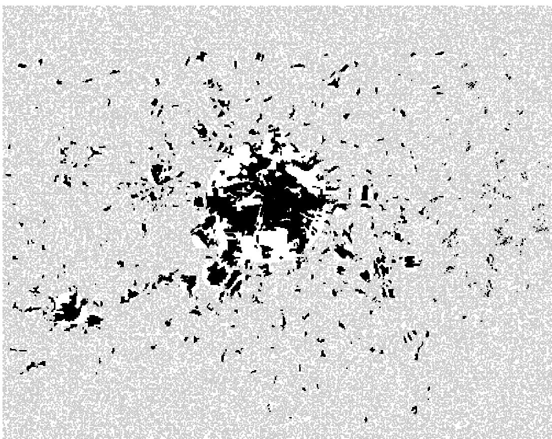

Figure 7: Initial situation for the computer simulations: (black) build-up area of Berlin, 1910, (white) depletion zones of free space

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4.3 Results of Computer Simulations

The computer simulations are carried out as stochastic simulations, which means that every growth unit - after it is released from the reaction $A + B \rightarrow C_0$ - has two probabilities, w_m and w_p, to move or to precipitate, respectively. Both depend on the local attraction field, as also described in eqs. (10), (11), in the following manner:

$$w_m \sim \lambda \, \nabla h(r,t) + \sqrt{2k_B T \beta}\,; \quad w_p \sim h(r,t)/h_{max}(t) \tag{15}$$

The attraction field $h(r,t)$ is calculated according to eq. (9), and is permanently updated during the simulated urban evolution. Fig. 8 shows a spatial section of the initial attraction field; the location of the related urban clusters can be seen in Fig. 7. Obviously, the suburbs in the Berlin area create their own attraction field which may effect the probability of precipitation for the incoming growth units.

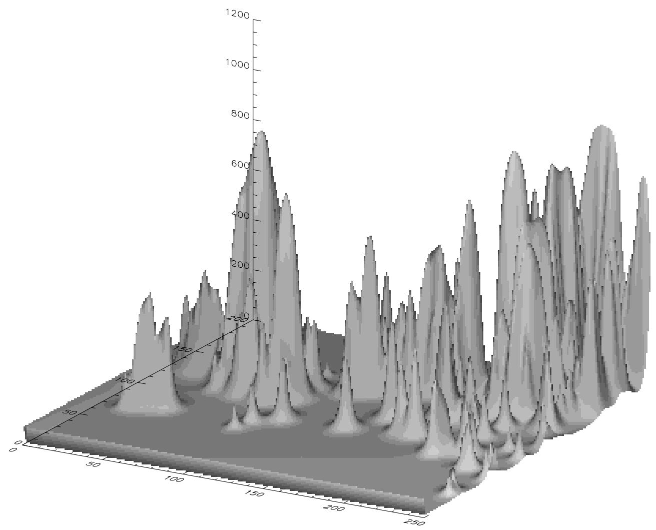

Figure 8: Calculated attraction field $h(r,t)$ for the initial situation: Berlin area 1910. The spatial section, which is identical with the lower left part in Fig. 7, shows in the middle the area of Potsdam with the adjacent suburb areas of Berlin on the right-hand side.

During the simulation, B particles are initiated with a constant rate in the center of Berlin. These particles have to diffuse through the depletion zones to meet some A particles in order to create the growth units C_0. The consumption of A leads to an expansion of the depletion zones; hence the growth units are released in always larger distance from the center of demand. During their diffusion towards the maxima of the attraction field, there is an increasing probability that the growth units precipitate before they reach the inner urban centers. Eventually, this may result in a stop of the inner urban growth.

Fig. 9 proves that, as a result of the dynamics described above, the inner part of the settlement area of Berlin does not remarkable grow, althrough the demand for build-up is only initiated in the center. Instead, we find a major growth in those suburban areas which have a suitable "mixture" of attractivity *and* free space available. Compared to the real evolution in the time interval considered, the simulated growth of the Potsdam area (the lower left part of Fig. 9) is not sufficient - this is due to the fact, that we only considered a demand in the center of Berlin. As explained above, further simulations could also consider a spatial distribution of demand in different centers.

Figure 9: Simulated growth of the Berlin area 1910 - 1920. (black) build-up area 1910, (grey) simulated growth area

To conclude our simulations, we want to point out that the A-B-C model of the urban cluster growth is able to reflect major features of the urban growth, such as (i) the depletion of free space, (ii) the burn-out of urban growth in the inner parts of the settlement and the movement of the growth zones into outer regions, and (iii) the attraction of existing settlement areas which effects the further growth of the aggregate. How much these features account for the urban evolution, depends on a couple of parameters which have to be adjusted accordingly. This could be possible e.g. by estimations based on time series of the previous urban growth. However, within a physical approach we are not able to derive a theory for most of these parameters; a suitable explanation, e.g. for the creation and the spread-out of the urban attraction, can be only found in collaboration with town-planners, regional developers, economists etc.

5 Conclusions

In this chapter, some structural aspects of urban cluster growth and their relation to physical models have been investigated. Our interest in this field is lead by the insight that the growth of urban aggregations is one of the essential problems of mankind. Althrough urbanization is a very complex phenomenon driven by a variety of reasons, we are convinced that there might be some simple rules which describe the growth of settlements and which could be easily used for a raw estimation of the future evolution. In order to describe urban aggregations by means of physical models, some restrictions have to be considered. We restrict ourselves only to structural aspects of the aggregation, in particular no internal structure of the settlement and no valuation of the build-up area are taken into account.

The structural analysis of urban aggregates has already proved some fractal properties of the clusters. However, urban aggregates do not exist as one connected (percolated) cluster, but as a distribution of separated clusters of different sizes. As we have shown, the distribution of cluster sizes may approach a PARETO-like distribution in the course of time. This allows an estimation of the developmental potential with respect to the morphology of the settlement.

The dynamics of the urban cluster distribution can be described by a master equation which considers the formation of new clusters and the growth of existing clusters with respect to their size. However, the further growth due to the same rule eventually leads to a collapse of the urban aggregate. To avoid this collapse, a decrease of the urban growth in the central region occurs which is accompanied by an increasing growth in outer regions. The "burn-out" of growth and the shift of growth zones can be modeled by an $A-B-C$ reaction-diffusion model which reflects (i) the depletion of free space, (ii) the movement of growth zones, and (iii) the attraction of existing settlements which feeds back to the future urban growth.

We want to point out again that the kinetic models derived in this chapter are physical models, which focus on the *structural* properties of urban aggregations. This restriction, on the other hand, allows us to derive simple methods suitable for an estimation of the future urban growth. This forecast, of course, is based on the conviction that self-organization plays an important role in urban evolution, and that the structure generating forces could be featured within a physical approach.

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References

BATTY, M. (Generating urban forms from diffusive growth, Environment and Planning A 23, 511-544): 1991.

BATTY, M.; LONGLEY, P.A. (*Fractal Cities*, London: Academic Press): 1994.

BECKER, S.; BRENNER, K.; FRANKHAUSER, P.; HUMPERT, K.; KULL, U.; SCHAUR, E. (Selbstorganisation urbaner Strukturen, in: *Die Architektur des Komplexen*, ARCH+ 121, pp. 57-68): 1994.

FRANKHAUSER, P. (*Beschreibung der Evolution urbaner Systeme mit der Mastergleichung*, Dissertation, Universität Stuttgart): 1991.

FRANKHAUSER, P. (Fractal Analysis of Urban Structures, Proc. 7th Colloquium on Theoretical and Quantitative Geography, Stockholm): 1991.

FRANKHAUSER, P. (Fractal Properties of Settlement Structures, Proc. 1st Intern. Seminar on Structural Morphology, Montpellier-La Grande Motte, pp. 357-368): 1992.

FRANKHAUSER, P. (*La Fractalite des Structures urbaines*, Paris: Anthropos): 1994.

FRANKHAUSER, P.; SADLER, R. (Fractal Analysis of Urban Structures, in: *Natural Structures - Principles, Strategies and Models in Architecture and Nature*, Proc. Internat. Sympos. SFB 230, vol. II, p. 57-65): 1992.

GÜNTHER, R.; SHAPIRO, B.; WAGNER, P. (Physical Complexity and Zipf's Law, J. Theor. Phys. **31**, 525-543): 1992.

HAKEN, H. (*Synergetics. An Introduction. Nonequilibrium Phase Transitions in Physics, Chemistry and Biology*, Berlin: Springer): 1978.

HELBING, D.; HILLIGES, M.; MOLNAR, P.; SCHWEITZER, F.; WUNDERLIN, A. (Strukturbildung dynamischer Systeme, in: *Die Architektur des Komplexen*, ARCH+ 121, pp. 69-75): 1994.

HUMPERT, K., BRENNER, K. (Das Phänomen der Stadt als fraktale Struktur, in: *Das Phänomen der Stadt. Berichte aus Forschung und Lehre*, Städtebauliches Institut, Universität Stuttgart, pp. 223-269): 1992.

SCHAUR, E. (*Non-Planned Settlements: Characteristic Features - Path Systems, Surface Subdivision*, IL 39, Universtität Stuttgart): 1991.

SCHIMANSKY-GEIER, L.; SCHWEITZER, F.; MIETH, M. (Interactive Structure Fromation with Brownian Particles, in: F. Schweitzer (ed.): *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, vol. I, London: Gordon and Breach): 1996.

SCHWEITZER, F. (Selbstorganisation von Wege- und Transportsystemen, in: K. Teichmann, J. Wilke (ed.): *Prozeßund Form "Natürlicher Konstruktionen"*, Berlin: Ernst & Sohn, pp. 163-169): 1996.

SCHWEITZER, F.; SCHIMANSKY-GEIER, L. (Clustering of active walkers in a two-component reaction-diffusion system, Physica A 206, 359-379): 1994.

SCHWEITZER, F.; STEINBRINK, J. (Die Berliner Bebauungsstruktur, ARCH+ 122, p. 34): 1994.

SCHWEITZER, F.; STEINBRINK, J. (Urban Cluster Growth: Analysis and Simulation of the Rank-Size Cluster Distribution, submitted for publication): 1995.

SCHWEITZER, F.; STEINBRINK, J. (Analysis and Computer Simulations of Urban Cluster Distributions, in: K. Humpert, S. Becker, K. Brenner (eds.): *From Nördlingen to Los Angeles: Fractal Laws of Urbanization*, London: Gordon and Breach, in press): 1996.

WEIDLICH, W. (Physics and Social Science - the Approach of Synergetics, Physics Reports 204, 1-163): 1991.

WHITE, R.; ENGELEN, G. (Cellular automata and fractal urban form: a cellular modelling approach to the evolution of urban land-use patterns, Environment and Planning A 25, 1175-1199): 1993.

WHITE, R.; ENGELEN, G. (Urban System Dynamics and Cellular Automata: Fractal Structures between Order and Chaos, Chaos, Solitons & Fractals 4/4, 563-583): 1994.