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Active Brownian Particles: Artificial Agents in Physics

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Abstract

Individual–based models are considered as a modern and flexible tool to describe self-organization in complex systems. A physical approach is provided by active Brownian particles with the ability to generate a self-consistent field which in turn feeds back to their behavior. Different examples elucidate the broad variety of applications, ranging from physico-chemical pattern formation, to self-assembling networks and ensemble search strategies. The consideration of biological features, such as replication or internal energy depots, extents the description of active Brownian particles towards an articifical agents model.

1 Introduction

In the science of complexity, today different variations of artificial agent models are applied to simulate adaptive behavior, ranging from ecology to engineering and to artificial life (DeAngelis and Gross (1992), Langton (1994), Varela and Bourgine (1992), Maes (1991), Meyer et. al (1991)). In these *individual–* or *particle–based models*, the elements of the system are treated, in a very general sense, as *artificial agents*, relatively autonomous entities which have a set of different rules to interact with each other. Which of the rules applies for a specific case, may also depend on local variables, which in turn can be influenced by the (inter)action of the artificial agents.

Individual-based models are not restricted to the social and life sciences, they are also useful in physics in cases where only small particle numbers govern the structure formation. Here, partial differential equations are not sufficient to describe the behavior of the system. The specific pattern obtained in the asymptotic regime, is path-dependent which means it is intrinsically determined by the history of its creation and irreversibility and early symmetry breaks play a considerable role. Hence, a stochastic description is needed which considers fluctuations in the system.

2 Equations for the Active Brownian Particles

The model featured in this paper, is based on Brownian particles, named after the Britisch botanist Robert Brown, who in 1827 discovered the erratic motion of small particles immersed in a liquid. Due to Langevin, the motion of a Brownian particle can be described by the stochastic differential equations:

$$\frac{dr}{dt} = v \quad ; \quad \frac{dv}{dt} = -\gamma v + \sqrt{2\,\varepsilon\,\gamma} \quad \xi(t) \tag{1}$$

Here $r(t)$ is the position of the particle at time t, and $v(t)$ is its velocity permanently changed by impacts of some surrounding liquid which is modelled by the random function $\xi(t)$. γ is the friction coefficient, and ε is the intensity of the stochastic force $\xi(t)$, which is assumed to be Gaussian white noise:

$$\langle \xi(t) \rangle = 0 \quad ; \quad \langle \xi(t)\xi(t') \rangle = \delta(t - t'). \tag{2}$$

As Einstein pointed out, in the long-time limit, $t \gg \gamma^{-1}$, there is a relation between the intensity ε and the macroscpic diffusion coefficient D_n of the Brownian particles:

$$D_n = \varepsilon/\gamma = k_B T/\gamma \tag{3}$$

Obviously, simple Brownian motion is not sufficient to originate structures. Therefore, we introduce *active Brownian particles* (Schimansky-Geier et. al (1995), (1996)) which are Brownian particles with the ability to generate a self-consistent field, which in turn influences their further movement and physical and chemical behavior. This non-linear feedback between the particles and the field generated by themselves results in an interactive structure formation process on the macroscopic level.

Within a discrete approximation, the particles can be described as Active Walkers – a term, introduced by Freimuth and Lam (1992). Recently, Active Walker models have been used to simulate a broad variety of pattern formations in complex systems (Kayser et. al (1992), Lam and Pochy (1993), Schweitzer and Schimansky-Geier (1994), (1996), Lam (1995), Schweitzer et. al (1996)).

We consider an ensemble of N active Brownian particles which are treated as individuals $i = 1, ..., N$. In the following, a constant particle number is assumed, but *birth- and death processes* of active Brownian particles could be considered as well (Schimansky-Geier et. al (1995), (1996)). For $N = const.$, we introduce the canonical N-particle distribution function $P(r_1, ..., r_N, t)$, which gives the probability to find the N active Brownian particles in the vicinity of the space coordinates $r_1, ..., r_N$ at time t. The equation of motion for particle i is given by the following Langevin equation:

$$\frac{dr_i}{dt} = v_i \quad ; \quad \frac{dv_i}{dt} = -\gamma v_i + \alpha_i \left.\frac{\partial h(r,t)}{\partial r}\right|_{r_i} + \sqrt{2\,\varepsilon_i\,\gamma}\,\xi(t). \tag{4}$$

which, in the Einstein-Smoluchowski limit, can be reduced to

$$\frac{dr_i}{dt} = \frac{\alpha_i}{\gamma} \left. \frac{\partial h(r,t)}{\partial r} \right|_{r_i} + \sqrt{\frac{2\varepsilon_i}{\gamma}} \xi_(t) \tag{5}$$

The Langevin equation considers the response of the particles to the *gradient* of the field $h(r,t)$. Further, two "individual" parameters appear in the equation:

(i) the individual intensity of the noise ε_i, which is related to the temperature (eq. 3) and represents the stochastic influences. ε_i is a measure of the individual *sensitivity* s_i of the particle: $s_i \propto 1/\varepsilon_i$.

(ii) the individual response to the field, α_i, which represents the strength of deterministic influences, resulting from the gradient of the field.

Both individual parameters may depend also an internal parameter, θ_i, which represent internal degrees of freedom for the particle. The relation between α_i and ε_i has some impact on the behavior of a particle: if both the response to the field and the sensitivity are low, the particle nearly behaves as a random particle. On the other hand, a strong response or a high sensitivity may result in a decrease of stochastic influences, and the particle pays more attention to the field, which guides its motion.

The response of the particle to the field, α_i may depend on the situation to be described, e.g.

(i) attraction to the field, $\alpha_i > 0$, or repulsion, $\alpha_i < 0$

(ii) response only if the local value of the field is above a certain threshold h_0: $\alpha_i = \Theta[h(r,t) - h_0]$, with $\Theta[y]$ being the Heavyside function: $\Theta = 1$, if $y > 0$, otherwise $\Theta = 0$.

(iii) response only if the particle has a specific internal value θ_0: $\alpha_i = \delta[\theta_i - \theta_0]$. Here, θ is assumed as a discrete variable, with δ being the delta function

(iv) response to the field depends on the direction of movement: this case might describe e.g. the situation of biological creatures, which usually have a certain angle of perception

The non-linear feedback between the particles and the field is given by the fact, that the particles can locally change the field with an individual rate q_i which may also depend on the internal parameter, θ_i. Further, it is assumed that the field can diffuse with a diffusion coefficient D_h and can decay with a rate k_h. Hence, for the field $h(r,t)$, we obtain a linear evolution law:

$$\frac{\partial}{\partial t} h(r,t) = \sum_{i=1}^{N} q_i \, \delta(r - r_i(t)) - k_h \, h + D_h \, \Delta h. \tag{6}$$

In order to summarize the model, the interaction between the active Brownian particles can be described as a *non-linear and indirect communication process* (Schweitzer (1996)), which all particles

are involved in. Communication is based on the exchange of information, and therefore needs a medium; direct communication can be considered as a special case of indirect communication. In the model of active Brownian particles, this medium is described as a space and time dependent field $h(r,t)$. With respect to the particles, the communication consists of three processes:

- "writing": the particles locally generate information by contributing to the field
- "reading": the particles locally receive information by measuring the gradient of the field
- "acting": the particles locally change the direction of their movement based both on the response to the information received and to erratic circumstances

By means of the field $h(r,t)$, the information generated is stored and distributed through the system via diffusion. On the other hand, the information can also fade out, expressed by a decay of the field. Communication can be considered here as a special type of *global coupling* between the particles, which feeds back to their individual actions.

3 A Simple Case: Identical Particles

For a first insight into the model, we now restrict to the simplest case, where all particles are assumed to be indentical ($\alpha_i = \alpha, \varepsilon_i = \varepsilon, q_i = q = const.$), and do not have internal degrees of freedom (Schweitzer and Schimansky-Geier (1994), (1996)). Then, the (macroscopic) density of the active Brownian particles, $n(r,t)$, can be obtained by:

$$n(r,t) = \int \sum_{i=1}^{N} \delta(r - r_i(t))\, P(r_1, ..., r_N, t) dr_1 ... dr_N \tag{7}$$

In the mean-field limit, from the Langevin equation (eq. 4) for the active Brownian particles the following Fokker-Planck equation can be derived:

$$\frac{\partial}{\partial t} n(r,t) = \frac{\partial}{\partial r} \left\{ -\frac{\alpha}{\gamma} \frac{\partial h(r,t)}{\partial r} n(r,t) + D_n \frac{\partial n(r,t)}{\partial r} \right\} \tag{8}$$

Further, in the mean-field limit eq. (6) becomes a linear deterministic equation:

$$\frac{\partial}{\partial t} h(r,t) = q\, n(r,t) - k_h\, h + D_h\, \Delta h. \tag{9}$$

For the coupled equations (8) and (9) a homogeneous solution $n_0 = N/S, h_0 = n_0 q/k_h$ exist; with S being the area of the surface the particles are assumed to move on. This homogeneous state becomes instable, if

$$\alpha\, q\, n_0 > \varepsilon\, (k_h + \kappa^2\, D_h) \tag{10}$$

where κ is the wave number of a fluctuation. This condition gives a relation for the parameters, α and ε, which can be used to distinct between different types of particle motion in the mean field limit (cf. Fig. 1).

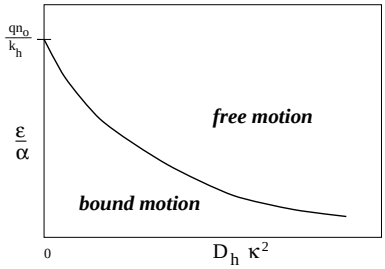

Figure 1: Diagram indicating the transition from a free motion of the active Brownian particles to a bound motion, in dependence on the "sensitivity" $(1/\varepsilon)$ and the "response" (α). D_h is the diffusion constant and k_h the decay rate of the field.

Here, "free motion" means that the particles more or less ignore the attraction of the field, thus behaving like random particles which move around. On the other hand "bound motion" means that the particles in the average follow the gradient of the field, which restricts their movement to the maxima of the field. Hence, the particles will be trapped in certain areas. In the example of Fig. 2, the particles eventually concentrate at some locations forming groups (or aggregates).

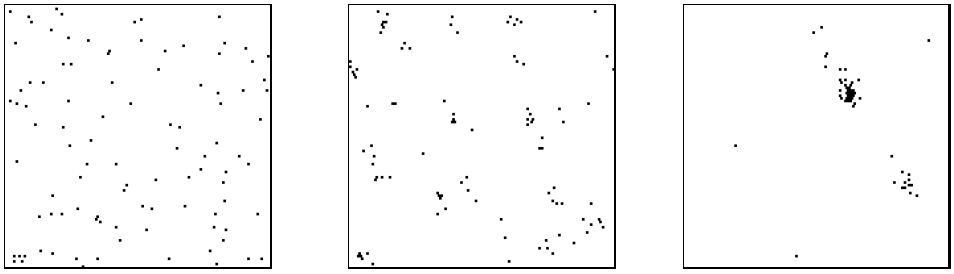

Figure 2: Snapshots of the position of 100 active Brownian particles moving on a triangular lattice (size: $S = 100 \times 100$). Time in simulation steps: (left) $t = 10$, (middle) $t = 1.000$, (right) $t = 50.000$ (from Schweitzer and Schimansky-Geier (1994))

In order to describe processes of structure formation or collective behavior, the parameter range of the "bound motion" seems to be the more interesting one, with applications e.g. in biological aggregation (Stevens and Schweitzer (1996)). In the other limit, all cooperative actions become impossible either because of an overrunning diffusion, or by an overcritical temperature which means a subcritical sensitivity. Althrough this insight results from a mean field analysis, it might basically hold also for the case where we have particles with "individual" parameters, as discussed in the following sections.

4 Particles with Internal Degrees of Freedom

4.1 Example: Travelling Patterns

Let us now assume that the action of the active Brownian particles may depend on an internal parameter θ_i. In the first example (Schimansky-Geier et. al (1996)), θ_i should be either 0 or 1. Further, we assume that only particles in the internal state $\theta_i = 1$ are able to contribute to the field, $h(r,t)$. Additionally, both kinds of particles can undergo a transition into the opposite state, by changing their internal parameter. The total number of particles should be constant. The example can be summarized using the following symbolic reactions:

$$\overset{\beta}{\longrightarrow} C_1 \overset{k(T)}{\longrightarrow} C_0 \overset{\beta}{\longrightarrow} \qquad N = N_0 + N_1 = const. \tag{11}$$

Here, C_i denotes an individual particle C with the internal state i. β is the transition rate from state 0 to 1. As eq. (11) indicates, an outflux of C_0 particles is compensated by an influx of C_1 particles. This situation is similar to a cross flow reactor. As an example, we investigate an exothermic reaction within this reactor (Yakhin et. al (1994)), hence, the contribution of the C_1 particles to the field can be specified as follows:

$$q_i(\theta = 1) = \eta\, k(T) = \eta\, k_0\, \exp(T\,/\,(T_0 + T)), \;\; q_i(\theta = 0) = 0 \tag{12}$$

Here, η is the heat released during one reaction, and $k(T)$ is the temperature dependent reaction rate for the transition from state 1 to 0. The field $h(r,t)$ the particles contribute to, can in this example be identified as a temperature field: $h(r,t) \to T(r,t)$ which obeys an equation similar to eq. (6):

$$\frac{\partial}{\partial t} T(r,t) = \sum_{i=1}^{N} q_i(\theta = 1)\, \delta(r - r_i(t, \theta = 1)) - k_T\, h + \chi\, \Delta T. \tag{13}$$

The decay of the field results from the coupling to a thermal bath outside the reactor, the diffusion of the field is replaced by heat conduction. In this example, the non-linear feedback between the motion of the particles and the field is not given by the response to the gradient of the field, but by the intensity of the fluctuations, ε_i which is related to the temperature (eq. 3). Hence, the resulting Langevin equation for the motion of the C_0 and C_1 particles reads in the one-dimensional case as follows:

$$\frac{dr_i(\theta)}{dt} = v \quad ; \quad \frac{dv_i(\theta)}{dt} = -\gamma\,(v_i(\theta) - v_0) + \sqrt{2\,\varepsilon_i\,\gamma}\,\xi(t). \tag{14}$$

In the cross flow reactor, in the average, all particles move with velocity v_0 relative to the temperature field $T(r,t)$, and can undergo the transitions specified in eq. (11), which may locally increase

the temperature. As the result, a travelling periodic pattern in the temperature field occurs, as shown in Fig. 3.

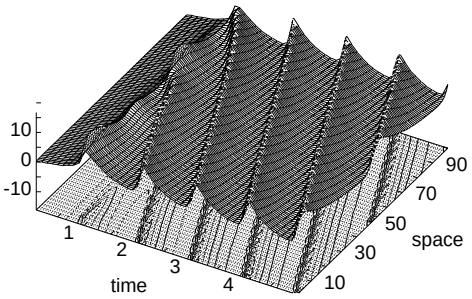

Figure 3: Space-time plot of traveling periodic structures which occur in the temperature field. The simulation is carried out with only 5000 particles and is very stable and fast. The results are in good agreement with those obtained by Yakhin et. al (1994) (from Schimansky-Geier et. al (1996)).

This example gives the opportunity for some notes in favour of the proposed particle based algorithm. Usually reaction-diffusion problems are solved by integrating the related equations on a lattice. Hence, the system of partial differential equations corresponds to a large number of coupled ordinary differential equations. The time step required for the integration, is mainly determined by the nonlinearities of the equations. Considering for example eq. (8), the allowed time step Δt should be less than $(\nabla n(r,t))^{-2}$ if we suppose that $n(r,t)$ and $h(r,t)$ are of the same order. As large gradients comes into play, the time step should be decreased according to the *square* of ∇n. On the other hand, if we solve the corresponding Langevin equations (eq. 4), the gradient appears only in a linear manner and, therefore much larger time steps are allowed for the integration. Hence, a simulation of a large number of particles do not necessarily cause larger simulation times, since in the considered example the equations are in fact linearized.

4.2 Example: Network Formation

As a second example for active Brownian particles with an internal parameter, we discuss the formation of links between a set of nodes (Schweitzer and Tilch (1996)). The nodes $(j = 1, ..., z)$ are located on a surface at the positions r_{nj}. A number of n_+ nodes should be characterized by a positive potential, $V_j = +1$, while $n_- = z - n_+$ nodes have a negative potential, $V_j = -1$. It is the (twofold) task of the particles, first to discover the nodes and then to link nodes with an opposite potential, this way forming a self-organized network between the set of nodes. This task is quite complicated since the nodes do not have any long-range interaction with the particles, like attractive or repulsive forces. Their effect is restricted to their location, r_{nj}.

In this example, the internal parameter of the particles, θ_i can take three different values $0, -1, +1$. Particles with the internal state 0 are considered as neutral particles which do not contribute to the field $h(r,t)$. Initially, $\theta_i(t_0) = 0$ yields for every particle. Further, θ_i could be changed only, if

a particle during its motion hits one of the nodes. Then it takes over the value of the potential of the respective node, V_j, which means $\theta_i = const.$, if $V_j = \theta_i$, and $\theta_i \to V_j$, if $V_j \neq \theta_i$.

The self-consistent field $h(r,t)$ generated by the active Brownian particles is, in the considered case, a chemical field consisting of two components, A and B. Which of these chemicals will be produced by the particle i, depends on the actual value of the internal parameter, θ_i. The production rate, $q_i(\theta_i, t)$, is defined as follows:

$$q_i(\theta_i,t) = \frac{\theta_i}{2} \quad [\quad (1+\theta_i) q_A^0 \, \exp\{-\beta_A \, (t - t^i_{n+})\} \\ - \quad (1-\theta_i) q_B^0 \, \exp\{-\beta_B \, (t - t^i_{n-})\}] \tag{15}$$

q_A^0, q_B^0 are the initial production rates and β_A, β_B are the decay parameters for the production of chemical A or B. Respectively, t^i_{n+}, t^i_{n-} are the times, when the particle i hits either a node with a positive or a negative potential.

The chemical field generated by the particles is assumed again to obey a recation equation, as given in eq. (6), but diffusion is not considered here ($D_h = 0$). The field should influence the movement of the particles according to the Langevin eq. (4). However, since the chemical field consists of two components A, B, we assume that the internal state of the particles determines which of the components affect the particle's movement:

$$\theta_i = 0 \to h = 0 \, ; \; \theta_i = (+1) \to h = h_B(r,t) \, ; \; \theta_i = (-1) \to h = h_A(r,t) \tag{16}$$

Here, $h_A(r,t)$, $h_B(r,t)$ mean the local concentration of the chemicals A and B, respectively.

The result of computer simulations, based on the model described above, is shown in Fig. 4. We can observe the formation of a network, where the connections between the nodes exist as a two-component chemical field generated by the active Brownian particles. This self-assembling network is created very fast and remains stable after the initial period. Patterns, like the network shown, are intrinsically determined by the history of their creation. Irreversibility and early symmetry breaks play a considerable role in the determination of the final structure. An extension of the model which also considers biological features has been applied to simulate the trunk trail formation in ants connecting a nest to different food sources (Schweitzer et. al (1996)).

5 Ensemble Search with Active Brownian Particles

In this section, some suggestions are made to use active Brownian particles for search processes. Here, we consider a potential $U(r)$ which *cannot* be changed by the particles, however, they have to find the minima of this potential during their search. For the simple case of Brownian particles moving in a potential $U(r,t)$, the related Langevin equation is, in the overdamped limit, given by eq. (5), considering $h(r,t) \to U(r,t)$ and $\alpha_i = 1, \varepsilon_i = \varepsilon$. The search process has to ensure that the

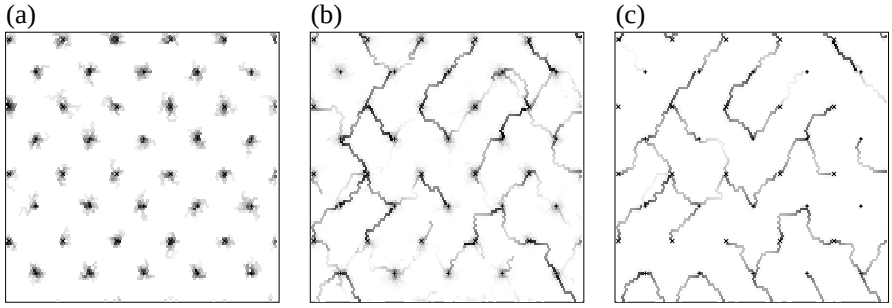

Figure 4: Time series of the evolution of a network after (a) 10, (b) 100, (c) 1000 simulation steps. The network was generated by 5000 particles on a 100×100 lattice with 40 nodes (from Schweitzer and Tilch (1996))

occupation probability for the minima of the potential will increase during the search, and indeed it can be shown by means of the related mean field equation (8), that the probability distribution for the Brownian particles in the stationary limit reads: $p^0(r) \sim \exp\left(-U(r)/T\right)$, which means that the particles in the asymptotic regime can be found in the minima of the potential.

However, as known e.g. from frustrated optimization problems, in many practical situation a large quantity of evenly matched minima exist. Here, the search strategy should avoid a total locking of the searchers in the minima found so far in order to guarantee further search. In different optimization routines, like the simulated annealing approach, this is realized by a temporary increase of the temperature, which keeps the particles moving. On the other hand, this could also result in the loss of appropriate minima which have to be found again. Hence, the proper adjustage of the temperature of the searchers is a major problem in optimization.

In the model of active Brownian particles, this dilemma could be solved using an individual search temperature for every particle. Particles which already found a minimum, should be very sensitive during the search, expressed in a very low ε_i, wheras particles which are far away from the minima, should increase their mobility to search around further. The idea of an adjustable sensitivity has been successfully applied to active random walkers simulating ants, which search for unknown food sources (Schweitzer et. al (1996)). Here, we may suggest $\varepsilon_i \propto U(r_i)$, where $U(r_i)$ is the current value of the potential at the location of the particle i.

Moreover, an additional coupling between the particles can be assumed, which, in terms of communication, means an instant information of all particles about the best minimum found so far. In this case, only the searcher with the best minimum will stay in rest, wheras the other particles are forced to search further on. Using the global coupling, the Langevin equation for the active Brownian searchers can be modified to:

$$\frac{dr_i}{dt} = v_i \; ; \; \frac{dv_i}{dt} = -\gamma v_i + \left.\frac{\partial U(r,t)}{\partial r}\right|_{r_i} + \sqrt{2\left(\varepsilon_i(U_i) - \varepsilon_{min}\right)\gamma}\,\xi(t).$$

$$\varepsilon_{min}(t) = const.\ \min\{U_i(t)\} \tag{17}$$

Here $\varepsilon_{min}(t)$ is defined by the minimum of all potential values at the current positions of the particles. Noteworthy, every searcher counts on two informations: (i) a *local* information, provided by the local value of the potential and its gradient, (ii) a *global* information, provided by $\varepsilon_{min}(t)$, which means an additional coupling between all particles.

Another possibility for a global coupling between the searchers is adopted in evolutionary searching strategies. In the so called Darwin strategy (Boseniuk et. al (1987)), the ensemble of N searchers is divided into different subpopulations, $x_i = N_i/N$, each characterized by a fitness E_i. As for biological species, *replication* and *mutation* of the members of the subpopulations are allowed. In the average, only subpopulations with a fitness above the mean fitness, $E_i > \langle E \rangle$, grow; therefore the replication rate is assumed proportional to the fitness. For the search problem considered, the fitness E_i of the subspecies i can be chosen to be the negative of the potential U_i indicating that the subspecies which has found the better minimum in the potential landscape, also has the higher replication rate. Then the average replication rate $\langle E \rangle$ is given by

$$\langle E \rangle = -\langle U \rangle = -\frac{1}{N} \sum_{i=1}^{N} U_i x_i(t), \quad N = \sum_{i=1}^{N} x_i(t) \tag{18}$$

Due to the mean value $\langle U \rangle$, there exist a global coupling between the different subpopulations.

The second element of the Darwin strategy, mutation, means that the searchers by chance can be transferred into a state with a better or worse fitness. The mutation rates A_{ij} are usually assumed to be symmetric, since there are no directed mutations. However, in the so called mixed Boltzmann-Darwin strategies (Asselmeyer and Ebeling (1996)), the transition probabilities of the Metropolis algorithm are used instead:

$$A_{ij} = A^0_{ij} * \begin{cases} 1 & \text{if } U_i < U_j \\ \exp\left(-(U_i - U_j)/T(t)\right) & \text{if } U_i \geq U_j \end{cases} \tag{19}$$

The prefactor A^0_{ij} is symmetrical ($A^0_{ij} = A^0_{ji}$), it defines a set of possible states j which can be reached from state i. Usually, only small steps for the change of the current state are allowed. The temperature $T(t)$ may decrease during the search by a certain rule, e.g. by a power law. Noteworthy, the temperature is assumed here as a *global* parameter, valid for all particles, whereas for the active Brownian searchers discussed above an individual temperature is assumed.

To conclude the dynamics of the subpopulations, the basic equation for the mixed Boltzmann-Darwin strategy reads as follows:

$$\frac{dx_i}{dt} = \kappa(\langle U \rangle - U_i)\, x_i + \sum_{j \neq i} [A_{ij}\, x_j - A_{ji}\, x_i] \tag{20}$$

with the transition matrices A_{ij} obtained from eq. (19). By changing the parameters κ and T in the range $0 \leq \kappa \leq 1,\ 0 < T \leq \infty$, we may interpolate between the two limit cases (i) Boltzmann strategy ($\kappa = 0,\ T > 0$) and (ii) Darwin strategy ($\kappa = 1,\ T \to \infty$).

6 Active Brownian Particles with an Internal Energy Depot

In addition to the replication, discussed in the previous section, metabolism is generally considered a characteristic feature of biological entities. Active Brownian particles do not need metabolism, moreover, they can take for granted the energy to move, since it is provided by the thermal noise in the system. However, on the way from the basically physical model of active Brownian particles towards the more detailed description of artificial agents, it might be useful also to consider the restrictions for individuals which result from a limited supply of energy.

Whereas in the previous sections always a passive (Brownian) motion of the particles has been assumed, here the case of active motion is considered Ebeling et. al (1996). Active motion occurs under energy consumption and requires metabolic activity and a supply of energy. If the distribution of energy is inhomogeneous, the individuals have to move and to search for energy sources. During these search periods, the supply of energy from internal sources is crucial. This implies that the individual is able (i) to store energy in internal depots, (ii) to convert energy from internal energy depots into energy of motion.

The model of active Brownian particles is now extended by considering an *internal energy depot*, $e_i(t)$, for every particle, which may be altered due to three different processes:

(i) take-up of energy from external sources (food) with an influx $b_i(r)$,

(ii) loss of energy (metabolic processes) which is assumed to be proportional to the internal energy, c_i being the rate of energy loss,

(iii) conversion of internal into kinetic energy, proportional to the depot energy, $d_i(v)$ being the conversion rate, which may depend on the velocity.

The resulting balance equation for the energy depot is then given by

$$\frac{d}{dt}e_i(t) = b_i(r) - c_i\ e_i(t) - d_i(v)\ e_i(t) \tag{21}$$

A simple non-linear ansatz for the conversion rate, $d_i(v)$ which also satisfies the condition $d(v = 0) = 0$ reads $d(v) = d_2 v_i^2;\ d_2 > 0$. If it is assumed that the active Brownian particles move in a potential $U(r)$ which is not affected by them, the equation of motion is now given by the Langevin equation:

$$\dot{v_i} + \gamma v_i + \bigtriangledown U(r) = d_2\, e_i(t)\, v_i\ + \sqrt{2\varepsilon_i \gamma}\, \xi(t) \tag{22}$$

Here, the first right-hand side term of eq. (22) reflects the acceleration of motion due to the conversion of internal energy into kinetic energy as an additional driving force.

In order to give an example for the motion of the active Brownian particles on a two dimensional surface, we assume that the potential should represent a home: $U(x_1, x_2) = a(x_1^2 + x_2^2)/2$, which

provides a certain attraction to the particles. The funcion for the energy influx $b(r)$ is zero everywhere, except at a specific area, which stands for the "food source", modeled as a circle on the surface. The individuals are *not* attracted by the food source due to long-range attraction forces. Fig. 5 provides an example for the stochastic motion of an individual. The gain of energy inside the circle determines how far the particle can reach out from the "home" again. After an initial period of stabilization, we see a quasi-stationary movement between the "food" and the "home".

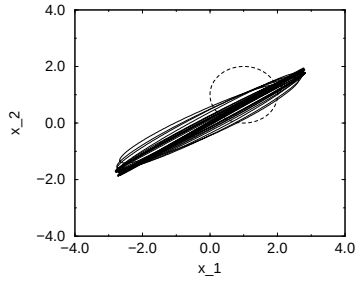

Figure 5: Trajectories in the x_1, x_2 space for the stochastic motion of the active Brownian particle. The circle indicates the area of energy supply.

In the example given, no additional coupling between the active Brownian particles exist. This could be provided based on the energy consumption of the particles. In this case the source, which provides energy (or food) to the particles, should be identified as the field $h(r,t)$ which mediates the communication between the particles. The take-up of energy leads to a local depletion of the field, on the other hand, we assume that the food also grows again with a certain rate $g(r,t)$, but it may not diffuse. Thus, the resulting equation for the field reads:

$$\frac{\partial}{\partial t}\, h(r,t) = g(r,t) - \sum_{i=1}^{N} b_i(r)\, \delta(r - r_i(t)). \tag{23}$$

It should be realistic that the active Brownian particles which need the supply of energy for metabolism and movement, are attracted by the maxima of the source field. In this case the Langevin eqs. (22) and (4) have to be combined, and the equation of motion reads finally:

$$\frac{dr_i}{dt} = v_i\,; \frac{dv_i}{dt} = -\gamma v_i + \frac{\partial}{\partial r}\{\alpha_i\, h(r,t) \;-\; U(r,t)\}\Big|_{r_i} + d_2\, e_i(t)\, v_i \;+ \sqrt{2\,\varepsilon_i\,\gamma}\,\xi(t). \tag{24}$$

In a generalized manner, the term $\{\alpha_i\, h(r,t) - U(r,t)\}$ represent an *environmental potential* for the community of the active Brownian particles, which considers both environmental changes originated by the individuals and changes which may result from external influences. For the latter one, different possibilities can be discussed, ranging from "simple" periodic changes (such as "day and night" cycles) to complex couplings to neighboring communities.

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