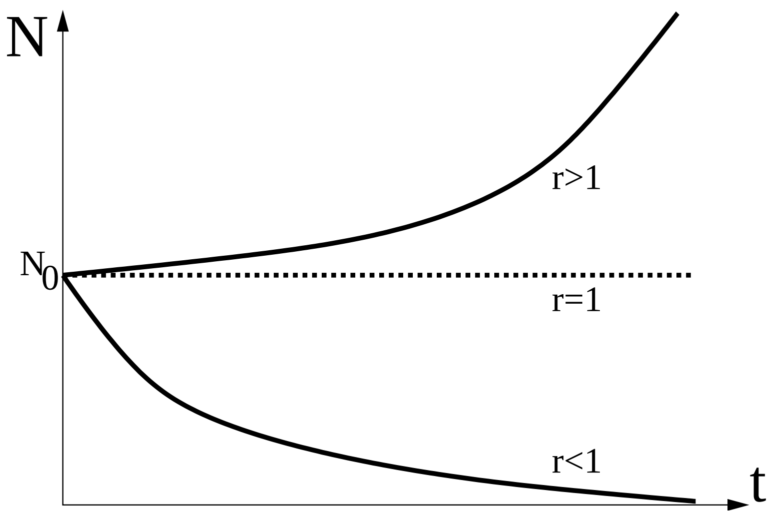
Macroscopic Growth Dynamics

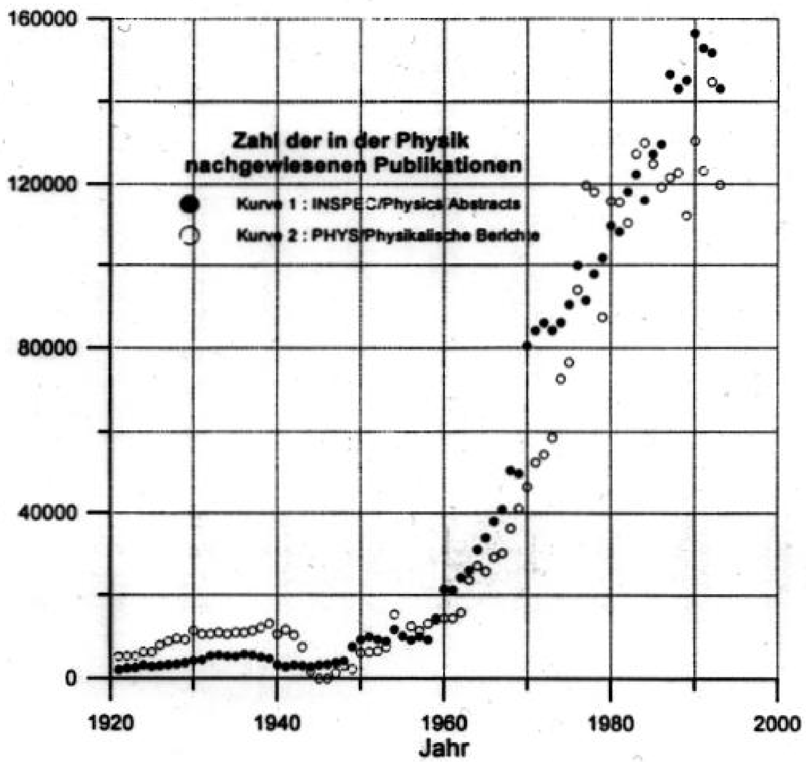
- quantity $N(t)$ (GNP, income, firm size ...)

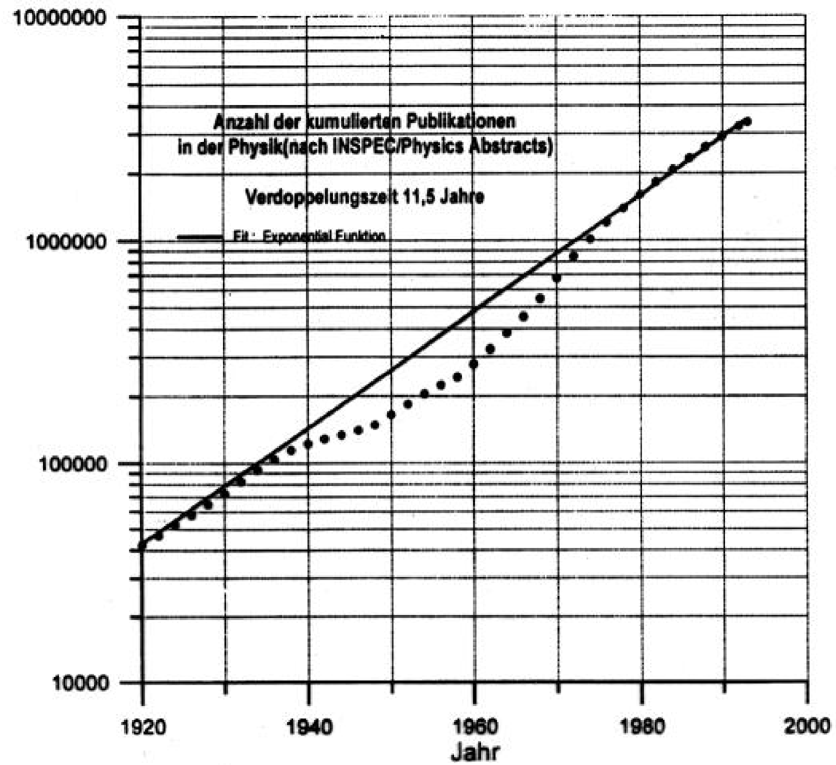
$$\frac{dN}{dt} = f(N, u, t)$$

- most simple case: $f(N) = b\ N$
  - iterative: $N_{t+1} = (b+1)N_t$; $\Delta t = 1$, integration: $N(t) = N_0 r^t$; ($r = b+1$)



Exponential growth indeed occurs:





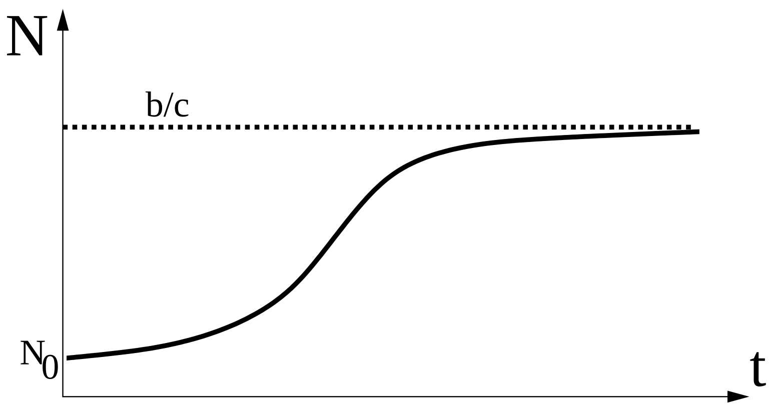
Limits to Growth

- Pierre Francois Verhulst (* 1804 in Brussels): 1845
  1973: Report for the Club of Rome's Project on the Predicament of Mankind by Donella H. Meadows (Meadows & Meadows '04)
- system can sustain only a maximum N_{max}
  reasons: depletetion of ressources, competition, ...

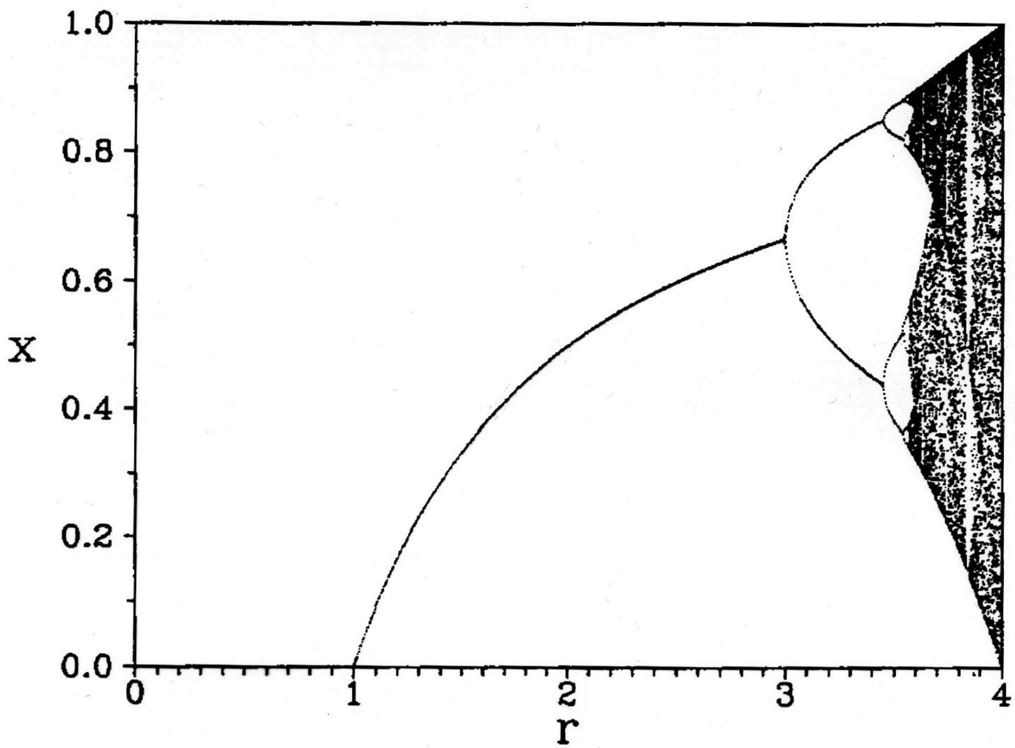
$$f(N) = bN - cN^2 = cN(b' - N)\ ; \quad b' = b/c = N_{max}$$

$$N(t) \Rightarrow t = \frac{1}{b'} \ln \frac{N}{b' - N}$$

$\Rightarrow$ logistic growth, saturation effects $\Rightarrow$ cause innovation



- iterative dynamics: $N_{t+1} = (b+1)N_t - cN_t^2$
  with $\lambda = b+1$, $N_t = (\lambda/c)\, x_t \quad \Rightarrow \quad x_{t+1} = \lambda\, x_t(1-x_t)$
- logistic map: initial value $x_0 \in (0, 1,)$, control parameter λ



- predictability $x(t \to \infty)$?? $\Rightarrow$ complex dynamics, deterministic chaos

Evolon Model of Cooperative Growth

- so far: $\dot{N} = cN(b' - N)$
- more elaborated: cooperativity exponents

$$\frac{dN}{dt} = c\, N^{\boldsymbol{k}}\, (b' - N)^{\boldsymbol{l}}$$

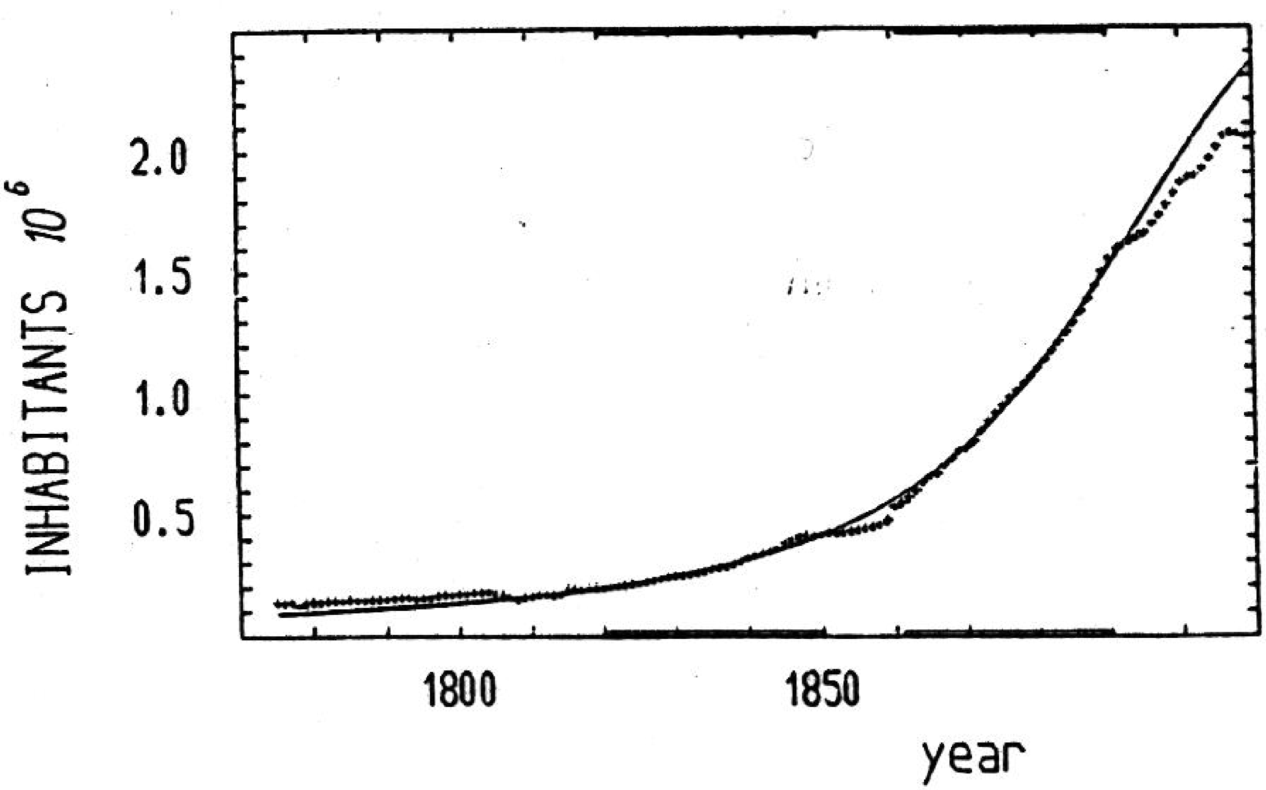
phenomenological approach, holds for (relatively) autonomous growth processes (no “pertubations”) (Albrecht & Mende ’91)

- **k**: (“extensive”) growth, ***l***: (“intensive”) saturation

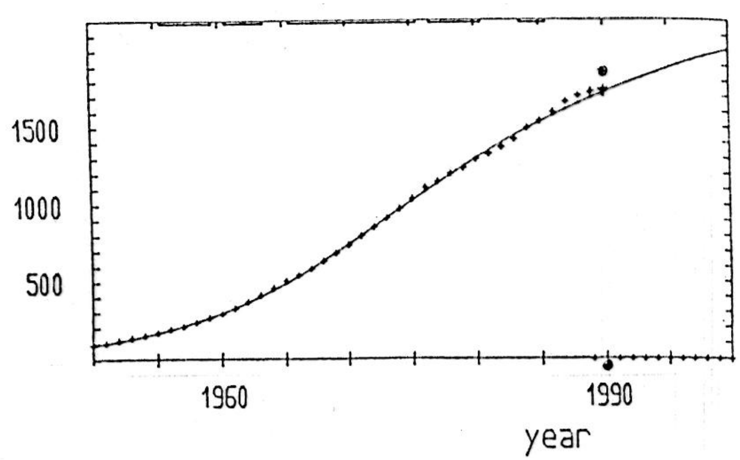
$k = l = 1$:	logistic growth
$k, l < 1$:	parabolic growth with saturation try to reach a given target by known methods applied repeatedly (substitution processes, ...)
$k, l > 1$:	hyperbolic growth with saturation no predetermined targets, evolutionary processes innovations

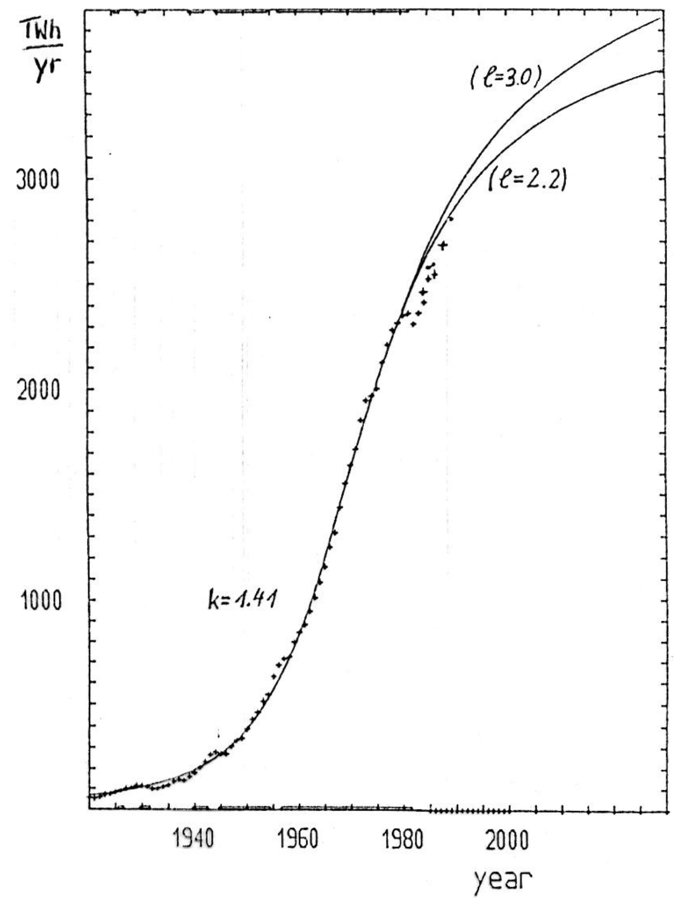
Examples (Albrecht & Mende '91)

- growth of population of Berlin: 1860-1910
  $k = 1.6$, $l > k$ (after 1890: coalescence)



- electric power production
  former Soviet Union after 1956: $k = 1.03$, $l = 2.2$
  USA after 1925: $k = 1.41$, $l > 1.8$





Further Examples:

- parabolic processes: targeted electricity production ($k \approx$ 0.65-0.68)
  USSR, FRG, GDR for 1946-1955 (reconstruction)
  Germany 1900-1920 (substitution)
  USA 1902-1929 (substitution)
- hyperbolic processes:
  population of 10 largest German cities for 1871-1970 ($k =$1.6, $l >$ 1.6)
  precision of time measurement 1200-1970 ($k = 1.12$)

$l > k$ rule:

- cooperativity increases during saturation period
  $\Rightarrow$ additional cooperativity of subsystems while adapting to growth limiting factors

“Derivation” of $cN^k(b' - N)^l$??

- coupling of successive exponential growth processes ("exponential tower") (Mende ...)

$$\begin{array}{llrl} \text{subprocess } n & : & \dot{N}_n = & [c_n N_{n-1}] N_n \\ \text{subprocess } n-1 & : & \dot{N}_{n-1} = & [c_{n-1} N_{n-2}] N_{n-1} \\ \ldots & & & \ldots \\ \text{subprocess } 1 & : & \dot{N}_1 = & [c_1 N_0] N_1 \end{array}$$

- N_0 =const., $c = c_{n-1} = c_{n-2} = ... = c_1$

$$\frac{dN}{dt} = \frac{dN_n}{dt} = cN^k \; ; \quad k = 1 + \frac{c}{c_n}$$

- *saturation:* substitution $N^\star = b' - N$
- multiplicative coupling of growth and saturation
- $k < 1$: $c_n > 0$, $c < 0 \Rightarrow$ one driving process, but inhibiting subprocesses $\Rightarrow$ parabolic growth
- $k > 1$: $c_n > 0$, $c > 0 \Rightarrow$ cooperation of subprocesses to support growth

Generalization of Growth Dynamics

- $N \Rightarrow N_i \; (i = 1, ..., z)$ – e.g. size (income) of company i

$$\frac{dN_i}{dt} = f_i(N_1, N_2, ..., N_z, u, t) + \text{stochastic forces}$$

- expansion

$$\frac{dN_i}{dt} = \epsilon_i N_i + \sum_j^z \gamma_{ij} N_i N_j + \sum_k^z \sum_j^z \mu_{ijk} N_i N_j N_k + \cdots$$

- Lotka-Volterra equation

$$\dot{N}_i = N_i \left[a_i - \sum b_{ij} N_j \right]$$

b_{ii} : inter“species” competition, b_{ij}: intra“species” competition
positive, negative feedbacks
exystence of unstable (“buisiness”) cycles, ...

Formation of Hierarchies

- complex system ("company"): various organizational hierarchies
  global aim: increase of "productivity" (utility, fitness, ...)

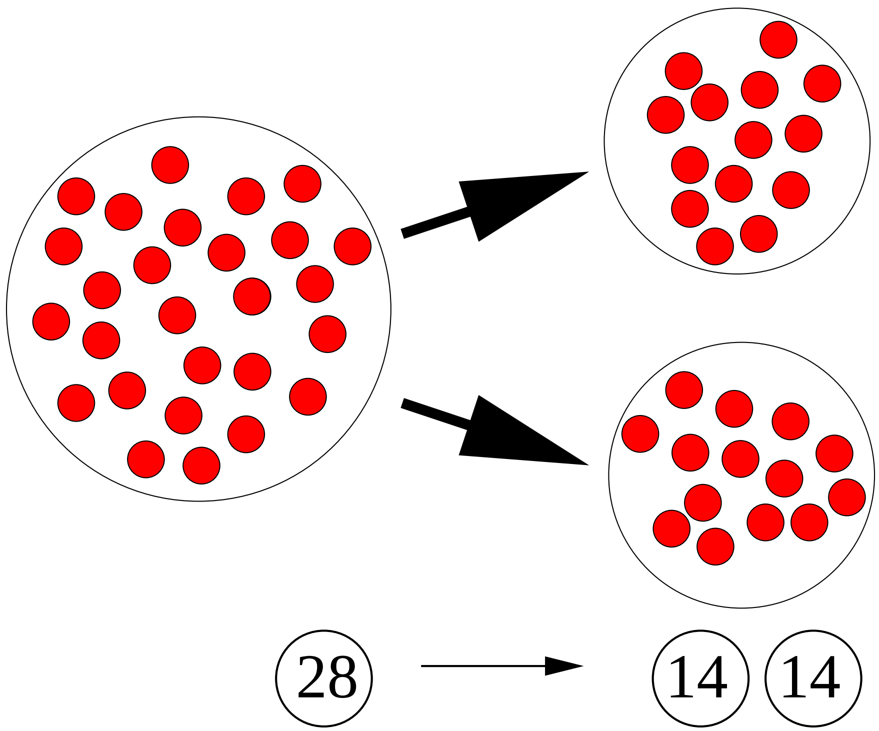
simple, but illustrative model of Drossel '99:

- productivity of 1 unit at (lowest) level 1: $P_1(1)$ (negligible)
  productivity of N interacting units at (lowest) level 1: $P_1(N)$
  - *increases* with interaction possibilities: $P \sim N(N-1)$
  - *decreases* with costs of interaction (e.g. transportation costs)
    if system size increases linearly with N, then $P \sim -N(N-1)\ N$

$$P_1(N) = \left[g_1 N - c_1 N^2\right](N-1)$$

  - maximum size: $P_1(N) \to 0$: $N_{\max} = g_1/c_1$

- maximum productivity per unit: $P_1(N)/N \longrightarrow \max$
  $\Rightarrow$ optimal size $N_{\rm opt} = (g_1 + c_1)/2c_1$
- reasons to split into $N = N' + N''$ if $P_1(N) < P_1(N') + P_1(N'')$
  most profitable split for total productivity: $N' \approx N''$
  $\Rightarrow$ critical (“split”) size: $N_{\rm crit} = 2(g_1 + c_1)/3c_1 = 4/3\ N_{\rm opt}$



- iterative process:

  N units $\rightarrow$ groups with I_1 units
  $\rightarrow$ supergroups with I_2 groups
  $\rightarrow$ groups of supergroups with I_3 supergroups ...

- productivity at level k *increases* with
  - number *and* productivity of groups at level $k-1$: $P_k \sim I_{k-1}P_{k-1}$
  - interaction possibilities between "supergroups" at level k: $P_k \sim I_k^2$
- costs at level k *increase* with system extension and number of interactions: $P_k \sim -I_k\, I_k^2$
  - increase also with number of groups at level $k-1$: $P_k \sim -I_{k-1}$

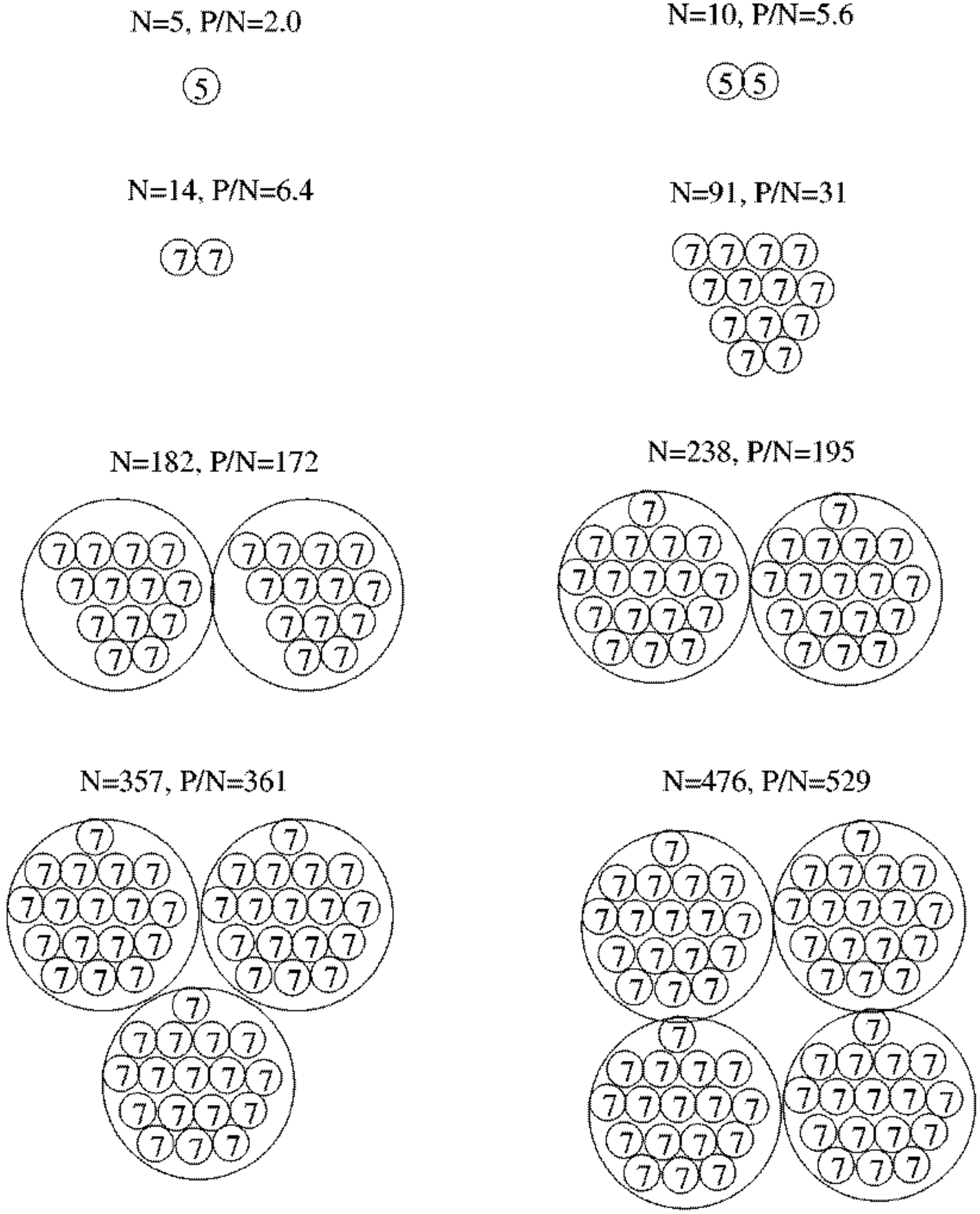
$$P_k = \Big[(g_k I_{k-1} P_{k-1})\; I_k - (c_k I_{k-1})\; I_k^2\Big] I_k \;; \quad N = I_1 I_2 ... I_k$$

- if $g_2, ..., g_k \sim c_1/g_1,\; c_k \sim g_1^{2k-3}/c_1^{2k-4}$: $\quad P_k \sim g_1 N^2$ (!!)
  - productivity of a single large group with *no* interaction costs
  - formation of hierarchies: efficient way of reducing interaction cost

- so far: optimization of *global* productivity
  - realistic system: cannot probe all possible configurations
    no breaking/rearrangement of large number of connections
  - growing systems: more likely follow established pathways
    $\Rightarrow$ search for "local" optima (rather than global optima)
    $\Rightarrow$ different set of growth rules lead to high productivity

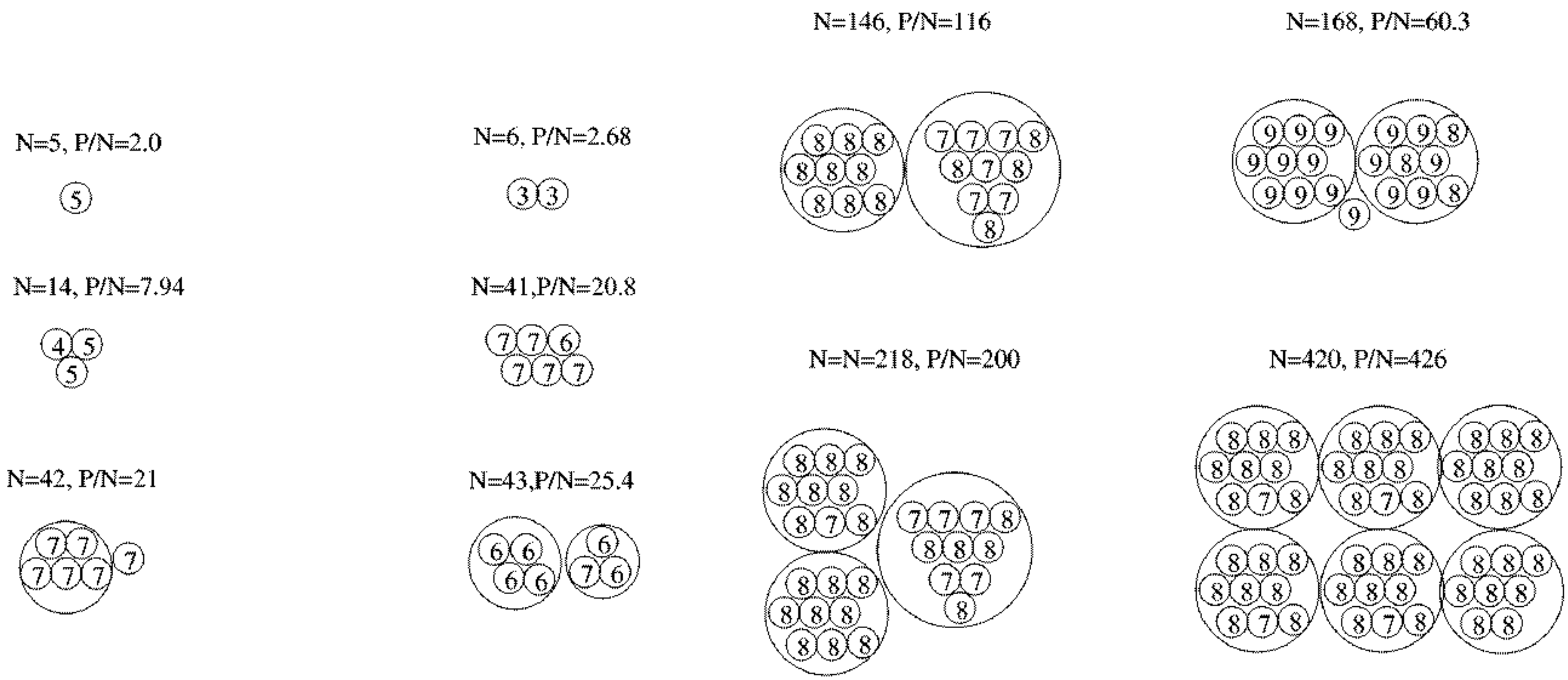
Example 1: (Drossel '99)

1. simultaneous formation of isolated groups until P_1/N decreases
2. groups interact $\rightarrow$ formation of supergroups until P_2/N decreases
3. supergroups interact $\rightarrow$ formation of super-supergroups until P_3/N decreases
4. groups at level $k-1$ can still grow further if this increases productivity at level k


N=5, P/N=2.0
N=10, P/N=5.6
N=14, P/N=6.4
N=91, P/N=31
N=182, P/N=172
N=238, P/N=195
N=357, P/N=361
N=476, P/N=529

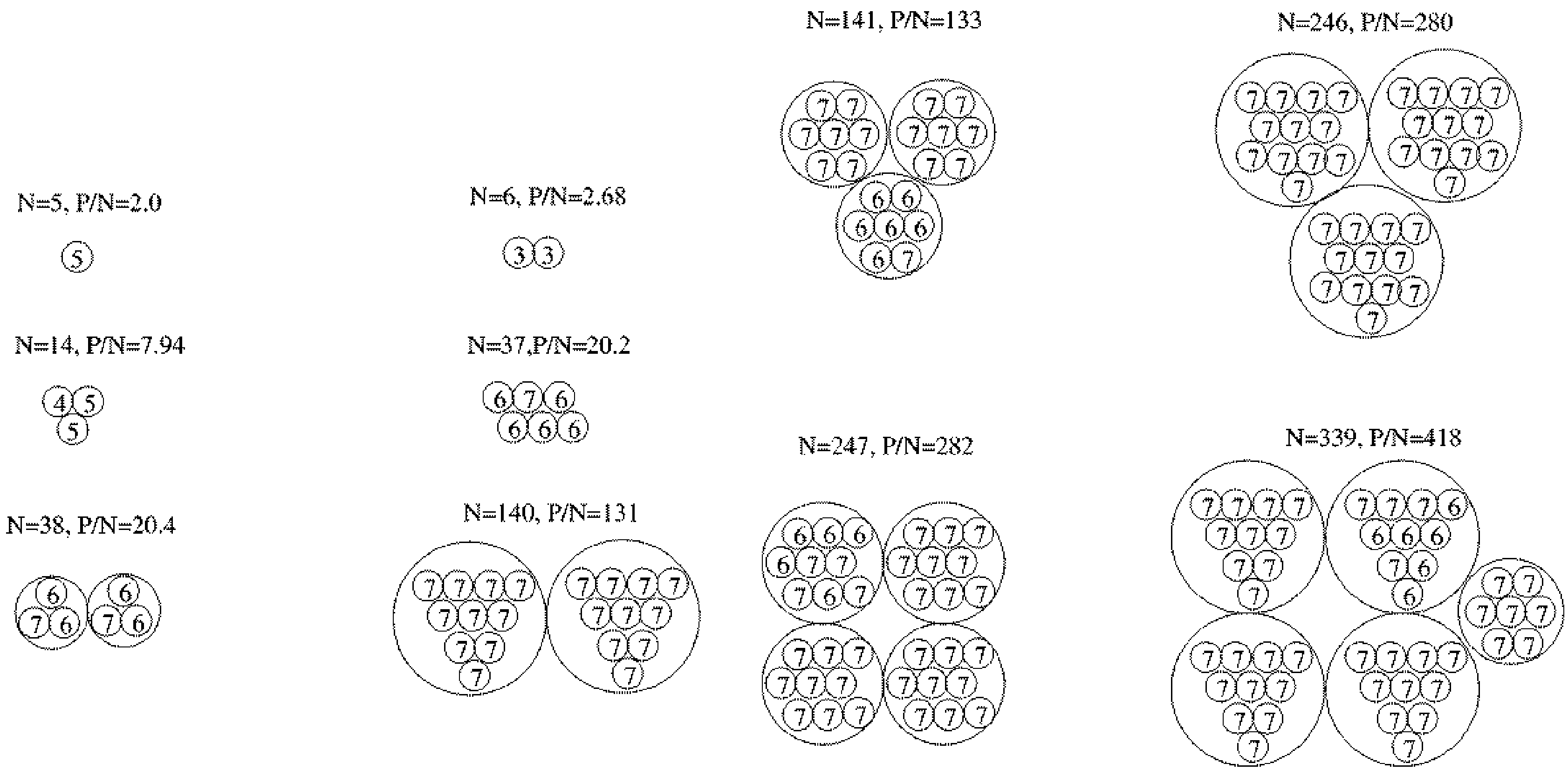
Example 2: (Drossel '99)

1. level 1: add new units $\longrightarrow$ (i) join existing groups if this increases productivity, OR (ii) form a new group with one of the units in other groups, as long as productivity increases
2. migration of units from other groups to newly formed one, as long as productivity increases
3. level k: split of groups from supergroups to form new supergroups, and migration of groups to other supergroups, as long as productivity increases


N=5, P/N=2.0
N=6, P/N=2.68
N=146, P/N=116
N=168, P/N=60.3
N=14, P/N=7.94
N=41,P/N=20.8
N=N=218, P/N=200
N=420, P/N=426
N=42, P/N=21
N=43,P/N=25.4

Example 3: (Drossel ’99)

1. add new units to a group until productivity decreases
2. split into two groups that grow until productivity decreases
3. rearrangement into three groups that grow until productivity decrease
4. split into two supergroups that grow until productivity decreases


N=5, P/N=2.0
N=14, P/N=7.94
N=38, P/N=20.4
N=6, P/N=2.68
N=37,P/N=20.2
N=140, P/N=131
N=141, P/N=133
N=247, P/N=282
N=246, P/N=280
N=339, P/N=418

Result:

- complexity emerges: formation of hierarchies to optimize two contradicting requirements (benefit vs. costs of interaction)

Extensions:

- *heterogeneous agents:* no identical units, groups, ...
- explicite time dependence: "aging of groups"
- explicite dependence on distance, costs of migration
- dynamics of entry/exit: "birth" dependent on local conditions, "death" of units, groups, ...
- dependence on resources

Some References:

The Limits to Growth: The 30 Year Global Update by Randers Meadows, Donella H. Meadows, Chelsea Green Publishing Company (June 2004), 368 pages

Albrecht, K.- F. and Mende, W.: Cooperative growth processes in complex social systems, in: W. Ebeling, M. Peschel and W. Weidlich (eds.) Models of Selforganization in Complex Systems, MOSES, Akademieverlag, Berlin, 1991, p. 288-295.

Albrecht, K.- F.: Problems of Modelling and Forecasting on the Basis of Phenomenological Investigations, Ecological Modelling, 63 (1992), p. 45-69.

Mende, W., Albrecht, K.- F. Electric Energy Production as an Evolution Process, in: Voigt-Mühlenbein-Schwefel (eds.) Evolution and Optimization ’89, Akademieverlag, Berlin, 1990, p. 177-189.

Barbara Drossel, Simple model for the formation of a complex organism, Phys. Rev. Lett. 82, 5144 (1999)