Fraunhofer Institut Autonome Intelligente Systeme

Adaptation of Strategies in a Spatial IPD

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Schedule

1. Basic interaction: Prisoner’s Dilemma

2. Spatial interaction and agent dynamics

3. Spatial games with 3 strategies

4. Spatial games with 8 strategies

5. Conclusions

Basic Interaction: Prisoner's Dilemma (PD)

- ultimate goal of agent i: utility maximization
  depends on action of opponent $j \Rightarrow$ different strategies

- PD game: paradigmatic example for interaction of 2 agents

  - choice between: $\mathcal{C}$: to cooperate, $\mathcal{D}$: "to defect"
    choose $\mathcal{C}$ or $\mathcal{D}$ without knowing opponent's move

payoff matrix:

	C	**D**
C	$R=3$	$S=0$
D	$T=5$	$P=1$

- *dilemma:* $T > R > P > S \quad \Leftrightarrow \quad 2R > T + S$
  it pays more to defect against cooperators, but global utility is maximized for cooperators

Important: number of encounters n_g

- $n_g = 1$: one-shot 2-person PD: $\mathcal{D}$ is ESS
- $n_g \geq 2$: *iterated PD*
  differences only if memory of $n_m \geq 1$ steps
- $n_m = 1$: *strategy* $\Rightarrow$ 3-bit binary string $[I_0|I_cI_d]$
  I_0 : initial decision (*deterministic game*)
  I_c, I_d: code response of agent i to previous move of agent j
  - $I_c = 1$, if j was $\mathcal{C}$ before and i is $\mathcal{C}$ now
  - $I_c = 0$, if j was $\mathcal{C}$ before and i is $\mathcal{D}$ now
  - $I_d = 1$, if j was $\mathcal{D}$ before and i is $\mathcal{D}$ now
  - $I_c = 0$, if j was $\mathcal{D}$ before and i is $\mathcal{C}$ now

  - results in $s = 8$ different strategies

s	Strategy	Acronym	Bit String
0	suspicious defect	sD	000
1	suspicious anti-Tit-For-Tat	sATFT	001
2	suspicious Tit-For-Tat	sTFT	010
3	suspicious cooperate	sC	011
4	generous defect	gD	100
5	generous anti-Tit-For-Tat	gATFT	101
6	generous Tit-For-Tat	gTFT	110
7	generous cooperate	gC	111

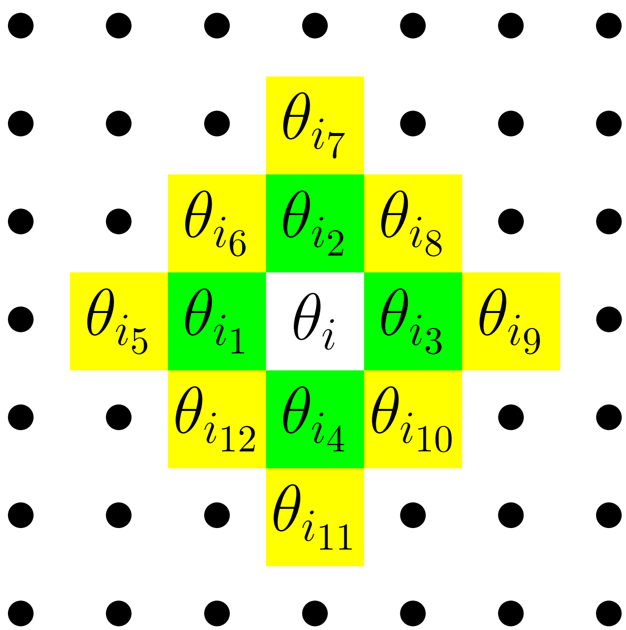
➤ suspicious strategies (s=0,1,2,3): initial defection
generous strategies (s=4,5,6,7): initial cooperation
rigid strategies (s=0,3,4,7): agents always behave the same

- known result for 2-person IPD:
  (g)TFT most successful strategy ($n_g \geq 4$)
- Now: N agents with heterogeneous strategies and local interaction

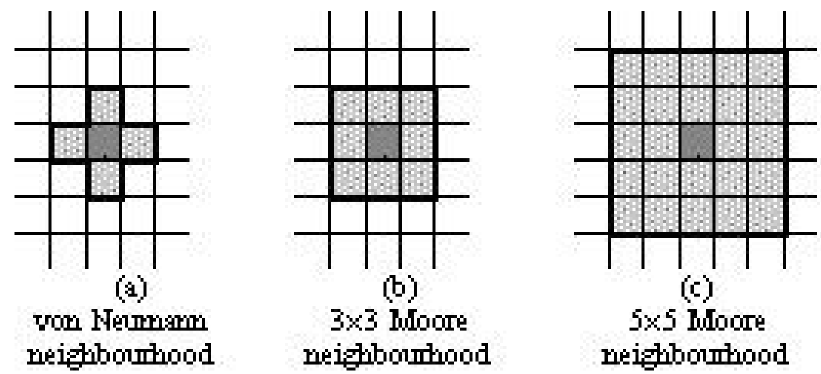
Questions:

- meaning of “lunatic” strategies?
- imitation behavior and $n_g \leq 4$: which strategies survive?
- role of spatial hererogeneity $\Rightarrow$ local interaction?
  spatial domains of prevailing strategies?
- non-stationary dynamics?

Spatial Interaction of Agents



- cell i with different states θ_i
- interaction with neighbors j



History: v. Neumann, Ulam (1940s), Conway (1970), Wolfram (1984), ...

Socio/Economy: Sakoda (1949/1971), Schelling (1969), Albin (1975), ...

Agent Dynamics

- *microscopic description:* agent i (position: $\boldsymbol{r}_i$)
  - internal degree of freedom $\theta_i \in \{0, 1, ..., 7\} \Rightarrow$ strategy
- *local* interaction of agent i with its 4 nearest neighbors
  - decompositon of *5-person* game into 4 independent, simultaneous *2-person* games, interaction: 2-person IPD
- *dynamics:* adopt strategy of most successful agent $j^{\star} = \arg\max_{j=0,...,m} a_{i_j}$ in neighborhood m

$$\theta_i(G+1) = \theta_{i_{j^{\star}}}(G)$$

  - *deterministic game*, time step in *generations* G

- total payoff of agent i

$$a_i(\theta_i) = \sum_{j=1}^{n-1} a_{\theta_i \theta_{i_j}} = \sum_{s} a_{\theta_i s}(n_g) \cdot k_i^s \; ; \quad k_i^s = \sum_{j=1}^{n-1} \delta_{s\,\theta_{i_j}}$$

  - $a_{\theta_i s}(n_g) \Rightarrow 8 \times 8$ payoff matrix dependent on n_g
  - assumption: strategy can be observed/deduced

- global variables
  - frequencies of strategies: $f_s(G) = \frac{1}{N} \sum_{i=1}^{N} \delta_{\theta_i s}$
  - average payoff per agent $\bar{a}$:

$$\bar{a} = \frac{1}{N} \sum_{i=1}^{N} a_i(\theta_i) = \sum_{s} f_s(G) \cdot \bar{a}_s \; ; \quad \bar{a}_s = \frac{\sum_i a_i(\theta_i) \delta_{\theta_i s}}{\sum_i \delta_{\theta_i s}}$$

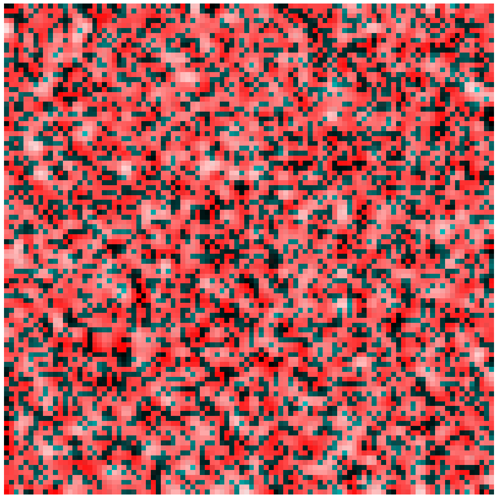
Example: Spatial Game with Three Strategies

$[110]$: TFT ("tit for tat") $\Rightarrow$ cooperative as long as opponent is cooperative

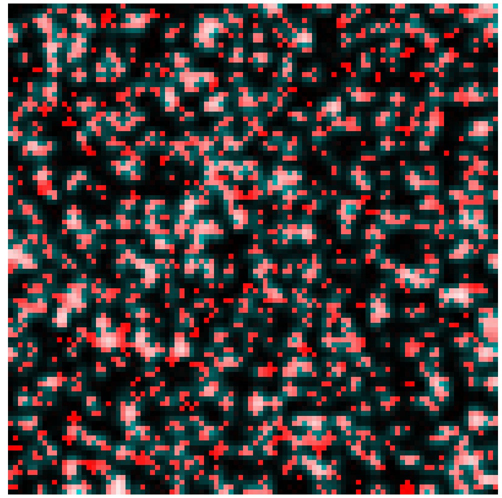
$[000]$: always defective

$[001]$: anti-TFT $\Rightarrow$ defective to cooperators, cooperative to defectors

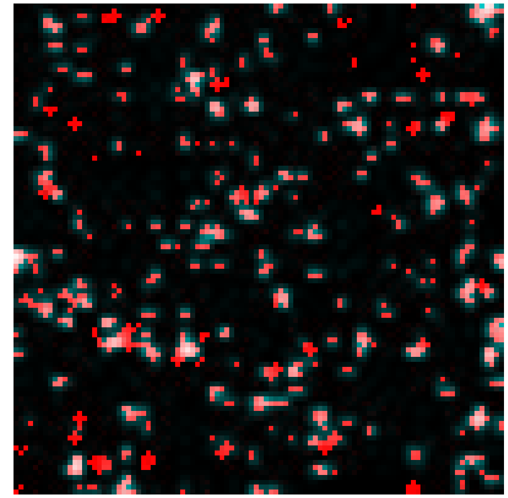
- random initial distribution, $f^s(0) = 1/3$, $N = 100 \times 100$
- results of computer simulations: $n_g = 2$ $n_g = 3$



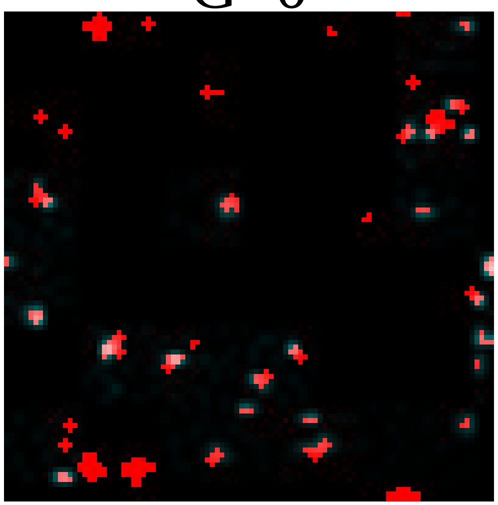
G=0



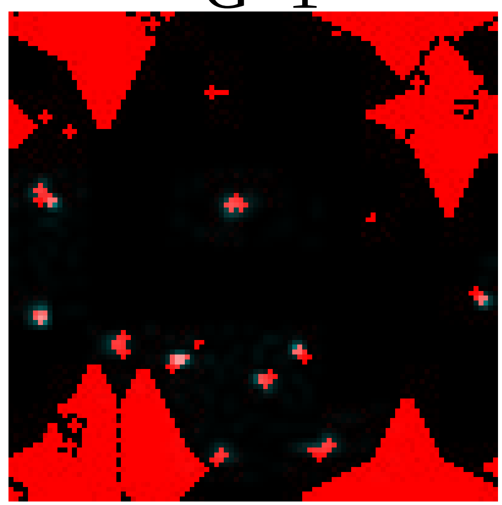
G=1



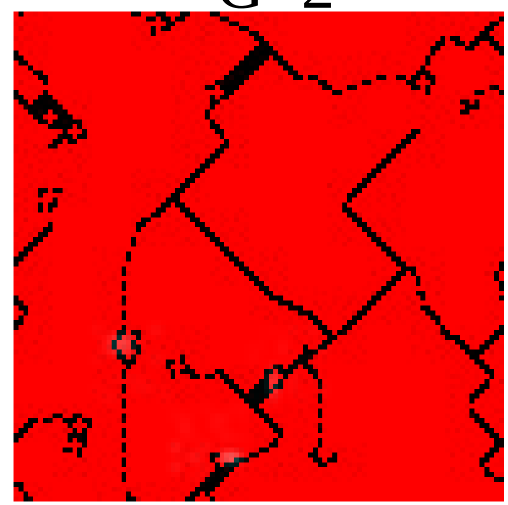
G=2



G=4



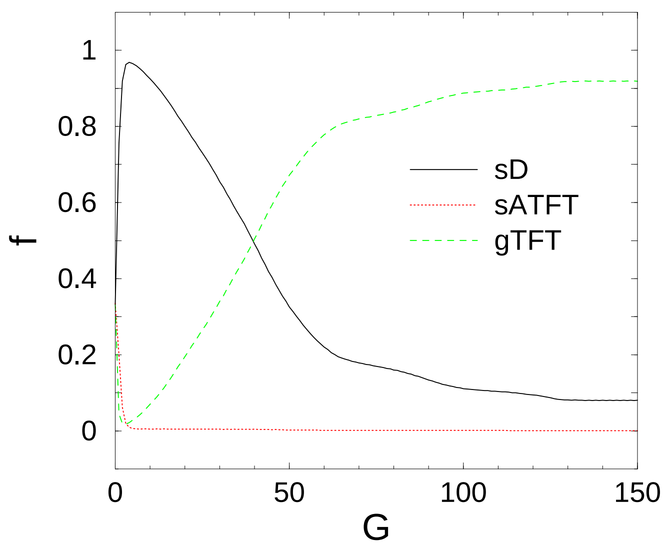
G=22

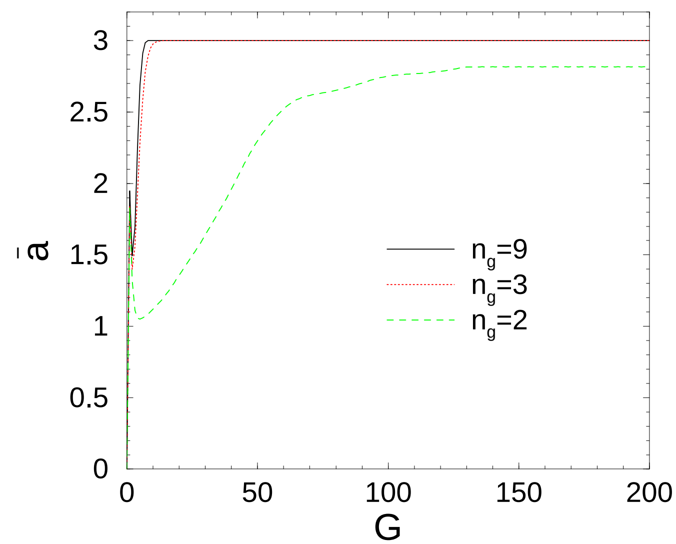


G=150

Results:

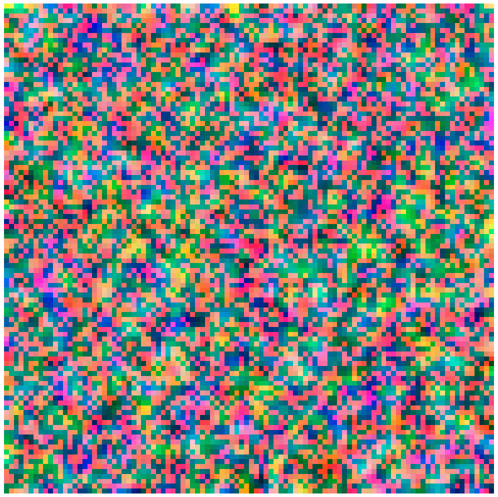
- *early stage:* steep decline of (partially) cooperative agents ([110], [001]) ⇒ survive in small clusters
- *late stage:* overwhelming rollback of cooperation ⇒ TFT takes over, majority
  - reason: defectors have "killed" anti-TFT



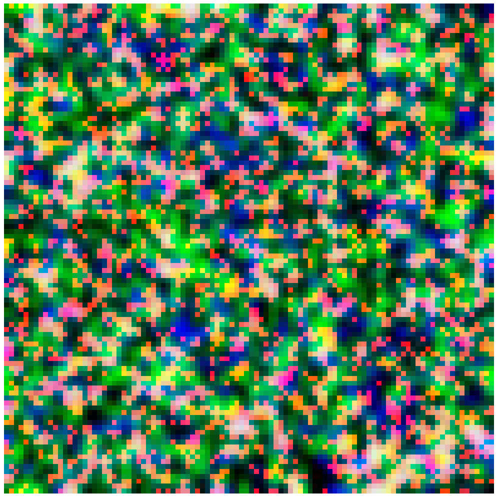


Evolution of More Diverse Strategy Patterns

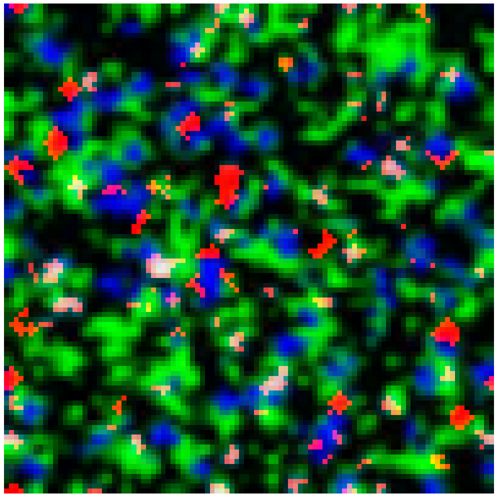
- random initial distribution of strategies, $n_g = 2$
- Questions
  - *Will (g)TFT prevail again?*
  - Is a stationary distribution of strategies reached?
- results of computer simulations: run 1 run 2



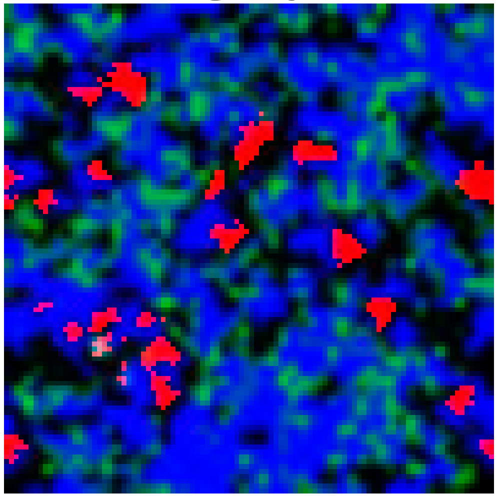
G=0



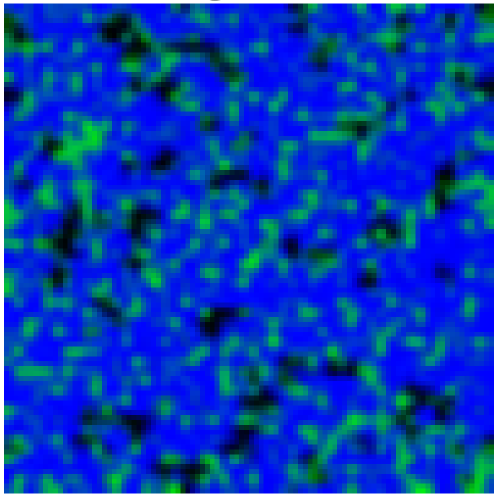
G=1



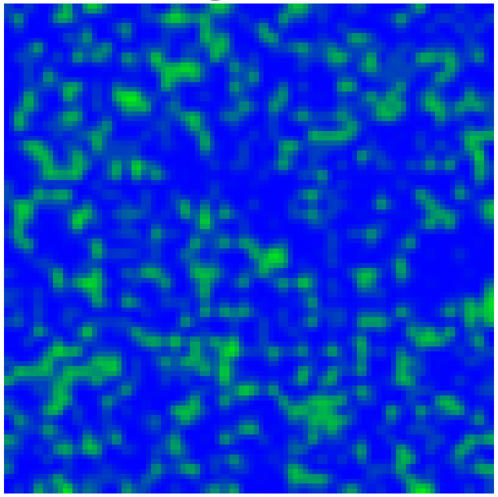
G=4



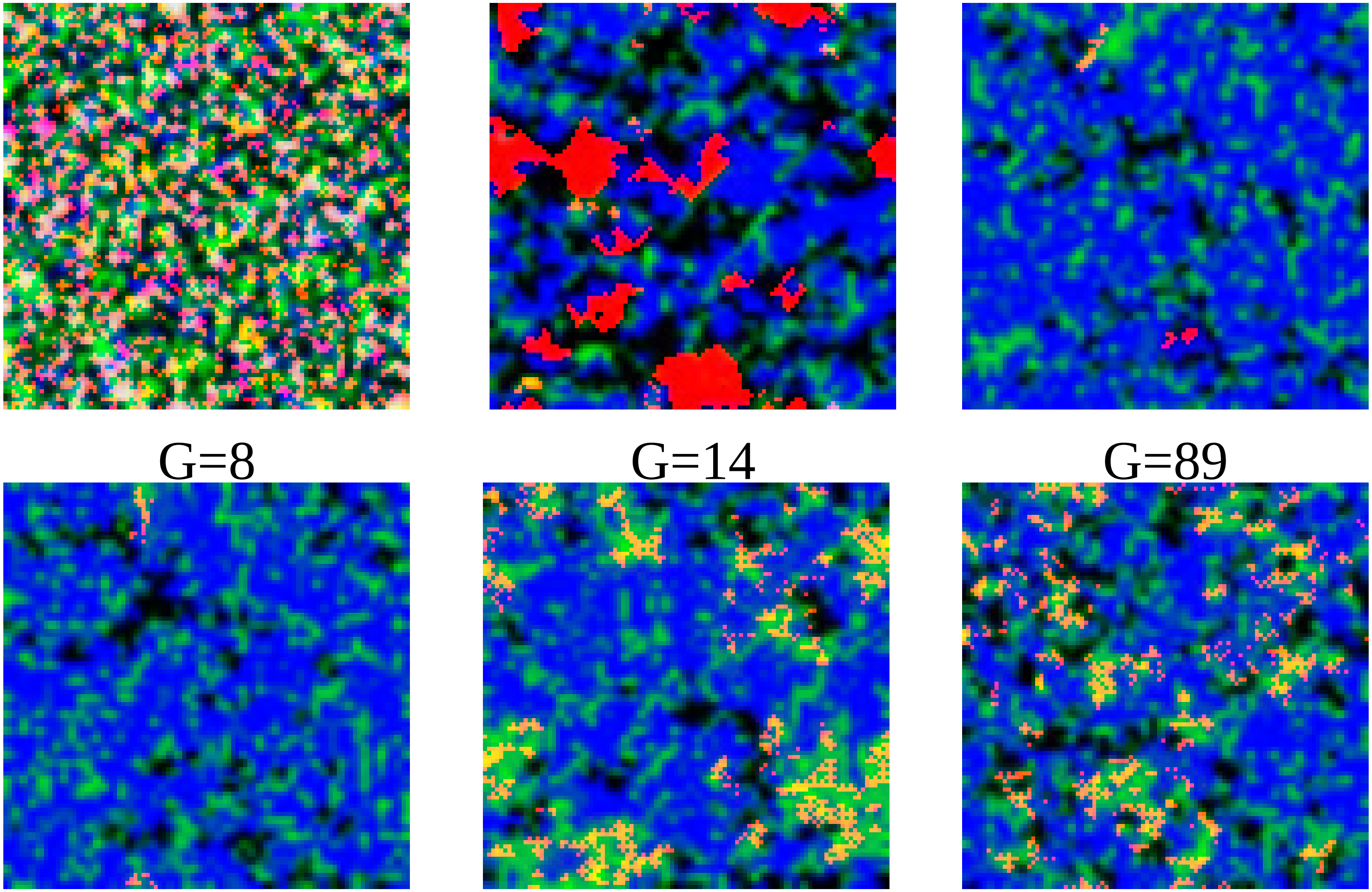
G=30



G=100



G=402

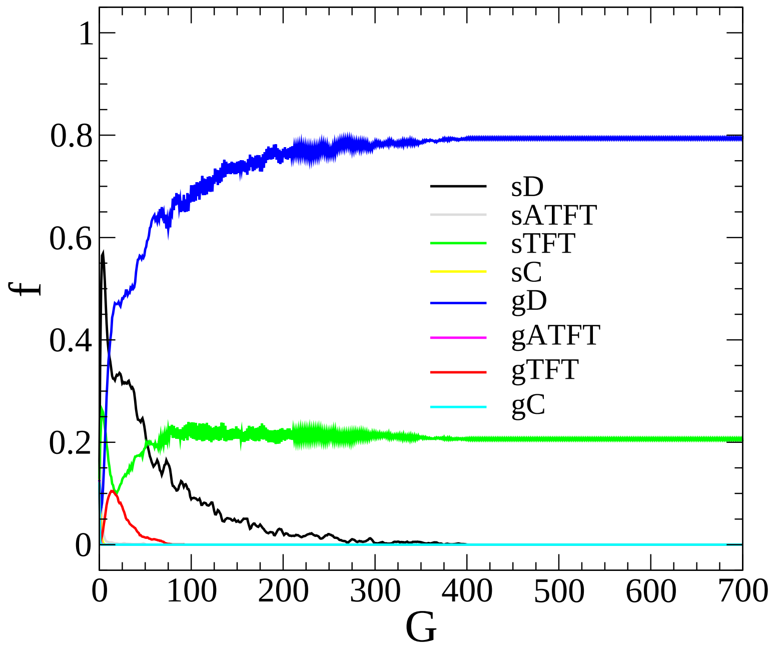


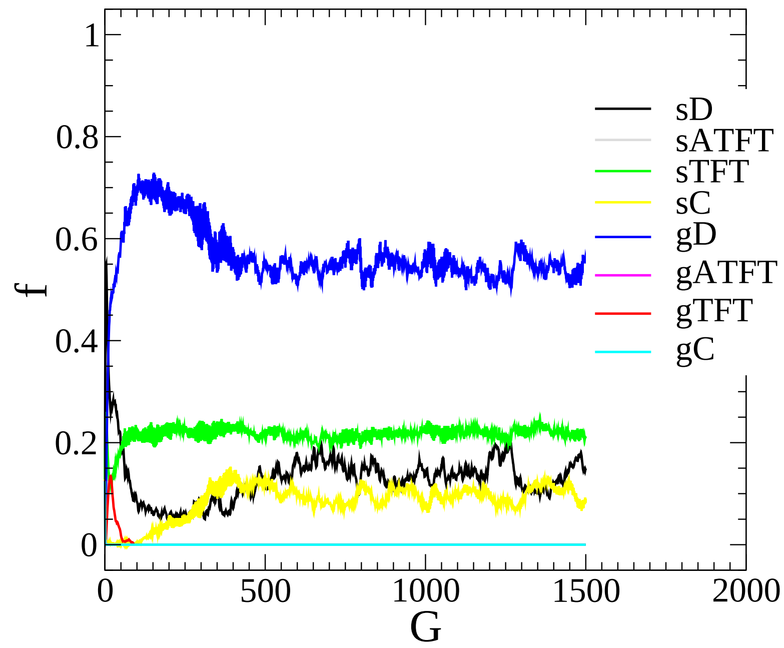
G=8 G=14 G=89

G=100 G=300 G=1500

Results (for the "same" setup and $n_g = 2$)

- *early stage:* steep decline of all generous strategies (except gD)
- *late stage:* gD global winner (different from gTFT)
- two different attractors for the global dynamics:
  - *stationary* coexistence of *two* strategies:
    gD, sTFT (small clusters) – i.e. sD, sC both *dissappear*
  - *non-stationary* coexistence of *four* strategies:
    gD, sTFT, sD, sC – i.e. sD, sC both *survive*
    (attractor less often reached)





- *conclusion* (valid for $n_g = 2$): increase of *heterogeneity* in agents' strategies and *local* interaction $\Rightarrow$ complex (sometimes non-stationary) IPD dynamics

- *to do:* detailed analysis of attractor size and stability

Conclusions

- *heterogeneous* agents: play different strategies dependent on (i) past experience ($n_m = 1$), (ii) local neighborhood
- spatial multi-agent system, *local* interaction: 2-person IPD
  $\Rightarrow$ agents: $\mathcal{C}$ or $\mathcal{D}$ with $n_m = 1 \Rightarrow$ 8 strategies
  $\Rightarrow$ investigate spatio-temporal evolution of heterogeneity
- outcome (for $n_g = 2$) depends on initial strategy mix:
  - spatial coexistence of different strategies (large domains, small clusters, ...)
  - (g)TFT may prevail only under special conditions
  - different (stationary and non-stationary) “defective” attractors dominated by gD

- *global transition* into *cooperation* becomes possible, **IF**
  - appropriate payoff structure ⇒ T, S, R, P

    F.S., L. Behera, H. Mühlenbein, *Advances in Complex Systems* **5** (2002) 269-299
  - repeated interaction ⇒ critical $n_g > 2$

    L. Behera, F.S., H. Mühlenbein, forthcoming
- relation to *social dynamics*: role of locality and heterogeneity ⇒ non-trivial results
- relation to *evolutionary optimization*: maximization of private (local) utility vs. overall (global) utility ⇒ frustration