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# **Räumliche Selbstorganisation in der Standortverteilung**

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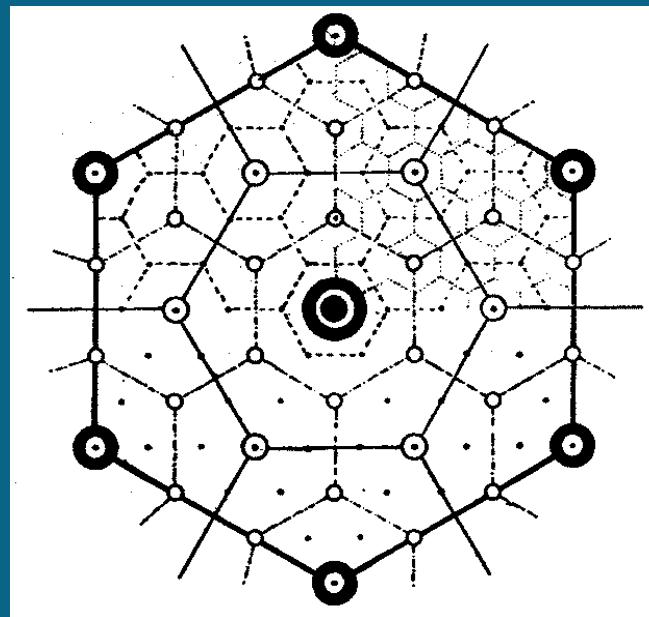
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5. Schlußfolgerungen

# Central Place Theory



Walter Christaller: *Die zentralen Orte in Süddeutschland.*

Eine ökonomisch-geographische Untersuchung über die Gesetzmäßigkeit der Verbreitung und Entwicklung der Siedlungen mit städtischen Funktionen, Jena: Fischer, 1933  
(Reprint: Darmstadt: Wissenschaftliche Buchgesellschaft, 1980)

English translation by C.W. Baskin:

*Central Places in Southern Germany*, London: Prentice Hall, 1966



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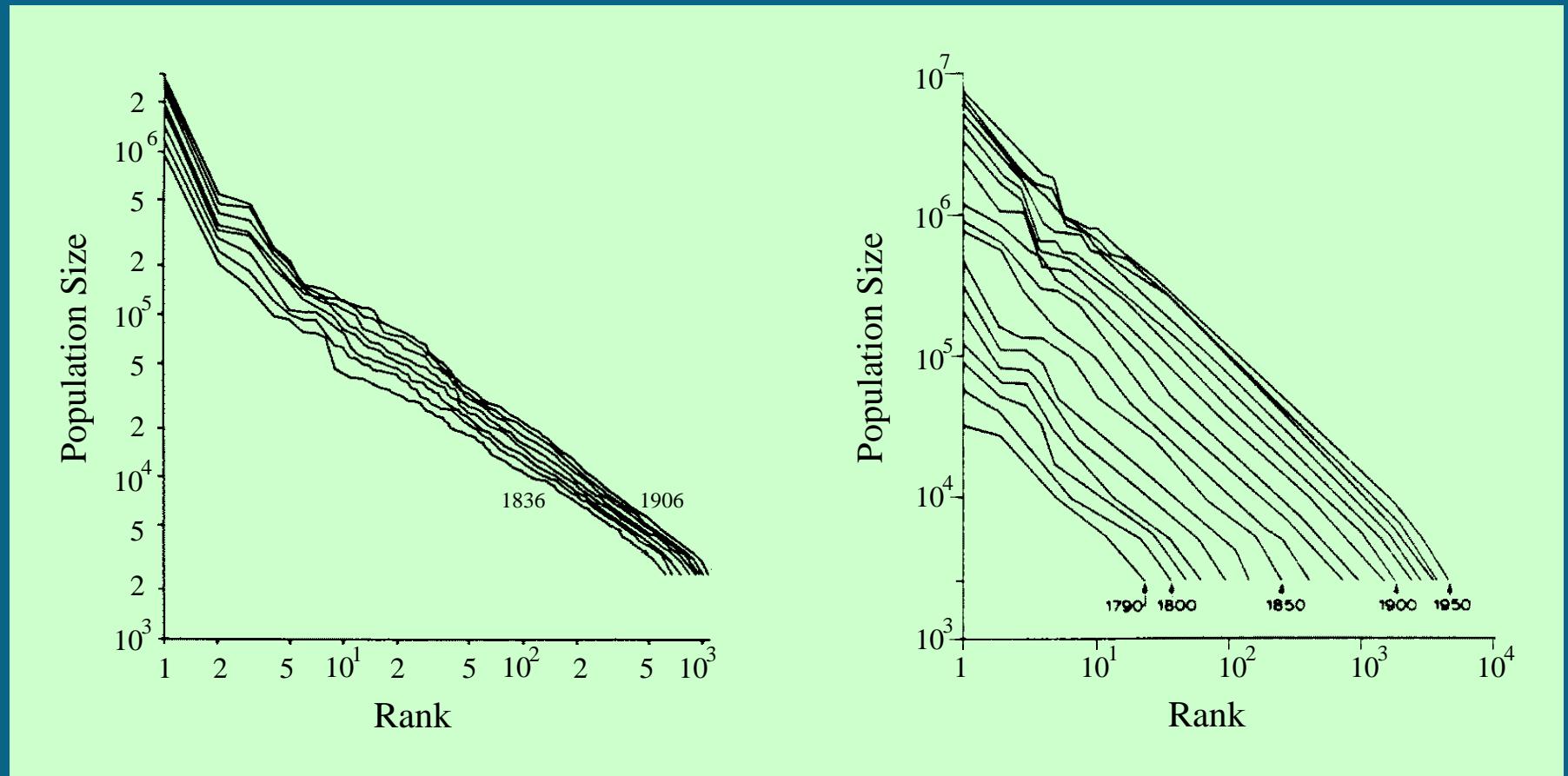
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Evolutionary perspective:

$\Rightarrow$  *Dynamical principles* which generate the hierarchical and spatial distribution *bottom up*

## Hierarchical Rank-Size Distribution

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Population of cities in different countries in the course of time:  
France (years 1836-1906)      USA (years 1790-1950)

## Pareto-Zipf Distribution:

$$n_k(t) = n_1(t) k^{-q(t)}$$

$n_k(t)$ : population of the settlement with rank  $k$

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- ◊ may result from various dynamical assumptions

## **Spatial Distribution of Locations**

empirical evidence:

## Spatial Distribution of Locations

empirical evidence:

- ◊ characteristic distance of locations with the same rank
- ◊ coexistence of multiple locations ?
- ◊ in a critical distance

*bottom-up approach:*

economic actors  
self-organization

# Complex System

## Complex System

*“By complex system, it is meant a system comprised of a (usually large) number of (usually strongly) interacting entities, processes, or agents, the understanding of which requires the development, or the use of, new scientific tools, nonlinear models, out-of equilibrium descriptions and computer simulations.”*

Journal “Advances in Complex Systems”

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  - ★ global / indirect interactions (coupling via resources)

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⇒ coevolution, circular causality

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◊ *freedom:* define rules *and* interactions  $\Rightarrow$  pitfall

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  - ★ specific for each agent  $\Rightarrow$  enables actions, decisions



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- ◊ cooperative interaction instead of autonomous action

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- ◊ “hiring” and “firing”:  $C_0 \xrightleftharpoons[k^+]{k^-} C_1$
- ◊ migration: overdamped Langevin equation:

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{f}(\mathbf{r}_i) + \sqrt{2D} \, \boldsymbol{\xi}_i(t)$$

$\mathbf{f}(\mathbf{r}_i)$ : guiding force, *local* influence



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◊ *economic theory*: determine  $\mathbf{f}(\mathbf{r}, t)$ ,  $k^+$ ,  $k^-$

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prefactor  $A$ : represents level of productivity  
 $\beta < 1$ : decreasing returns to scale

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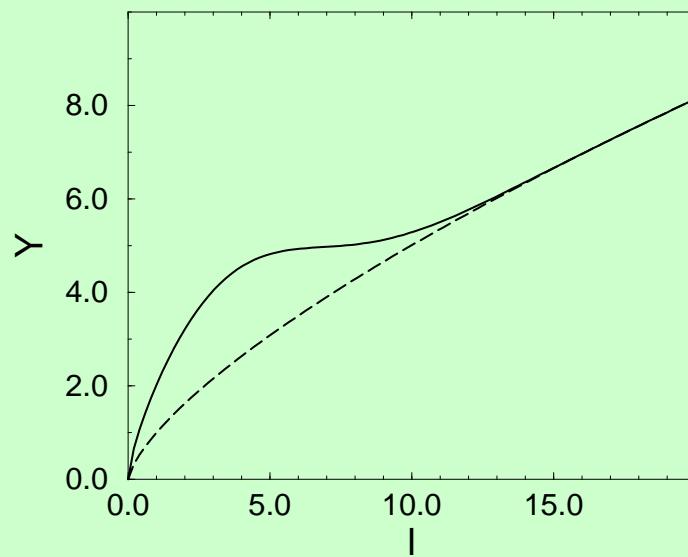
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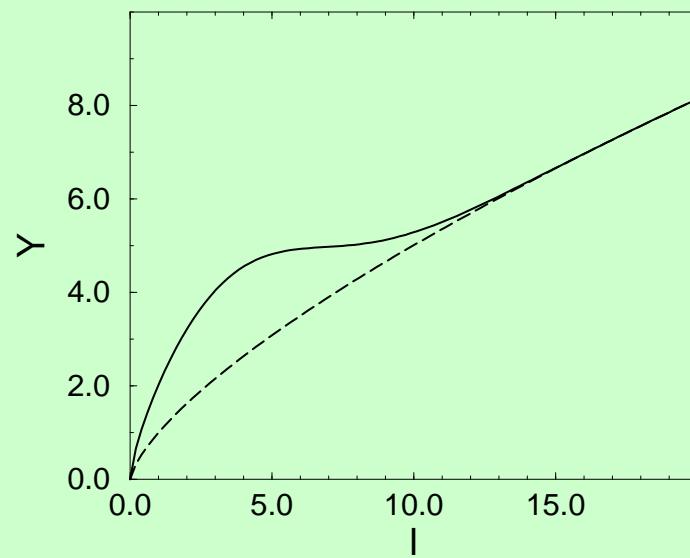
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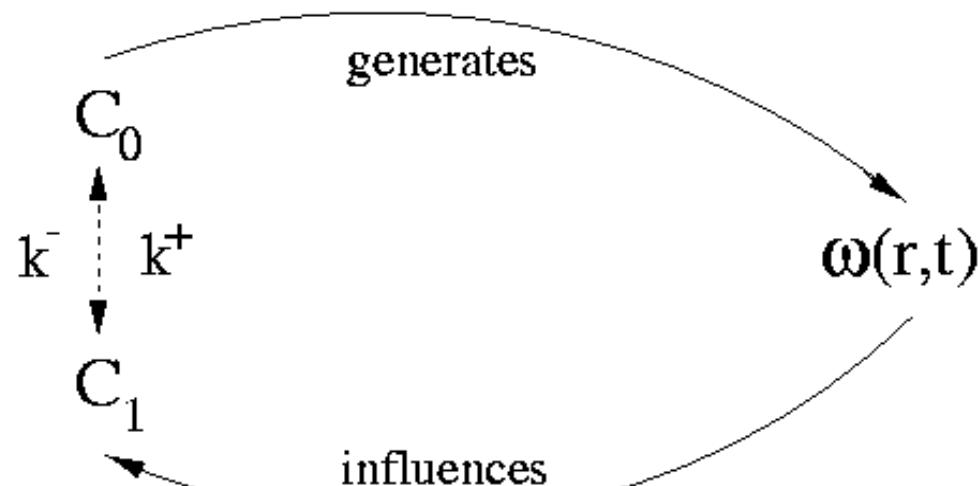
- ◊  $a_2 < 0$ : saturation effects  $\Rightarrow$  advantages of cooperative effects compensated by disadvantages of crowding

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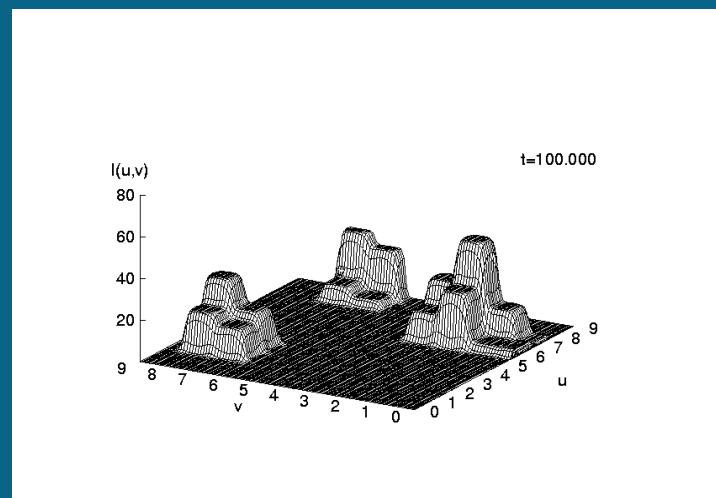
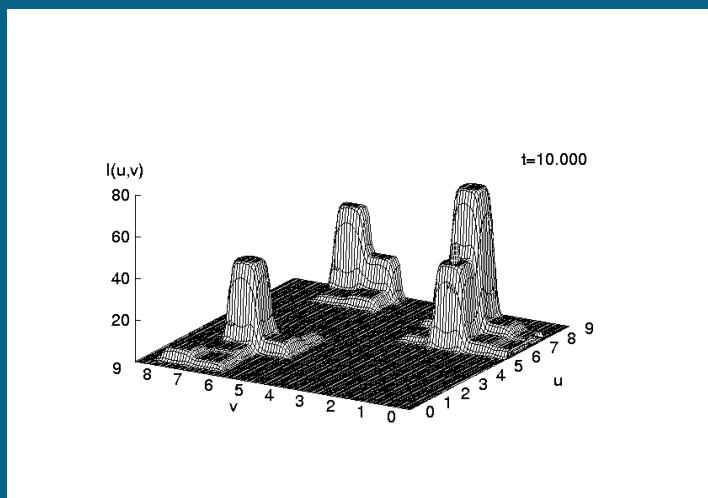
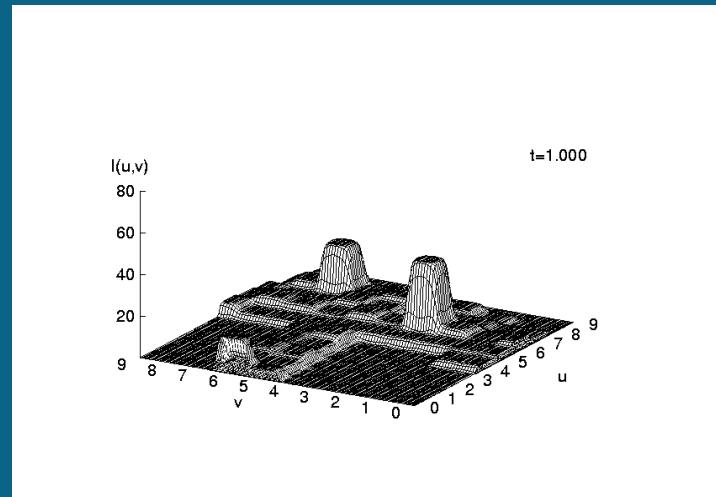
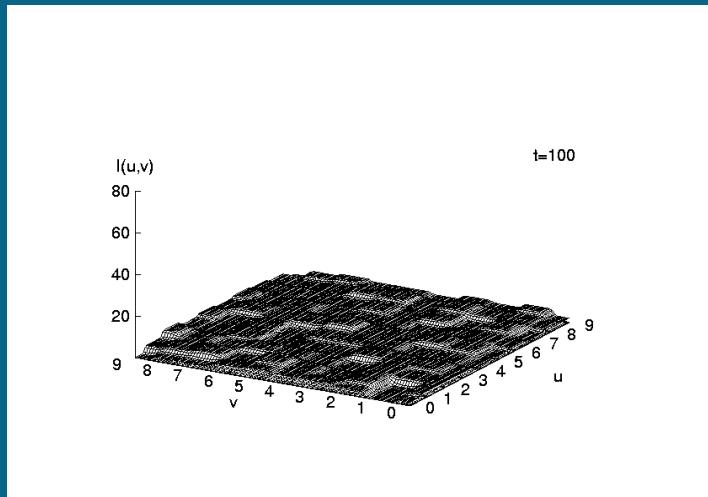
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- (i) workers are fired if  $\frac{\delta Y}{\delta l} < \omega^*$
- (ii) workers can quit their job for better opportunities

$$k^- = k^- \{l(\mathbf{r}, t)\} = \eta \exp \left\{ - \left[ \frac{\delta Y \{l(\mathbf{r}, t)\}}{\delta l} - \omega^* \right] + c \frac{\partial \omega(\mathbf{r})}{\partial r} \right\}$$

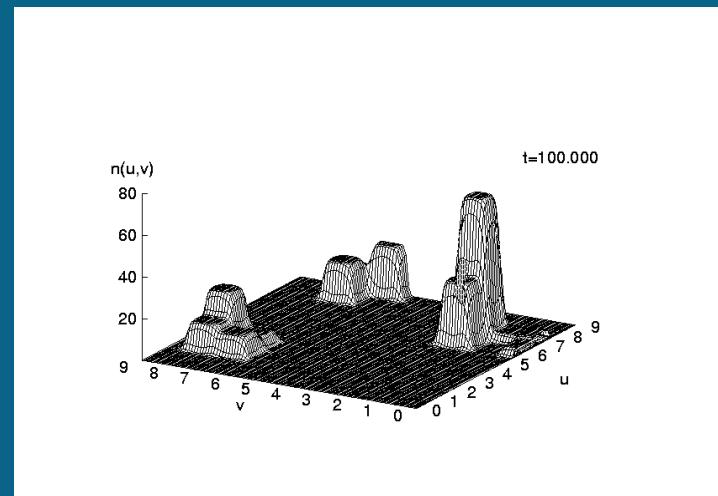
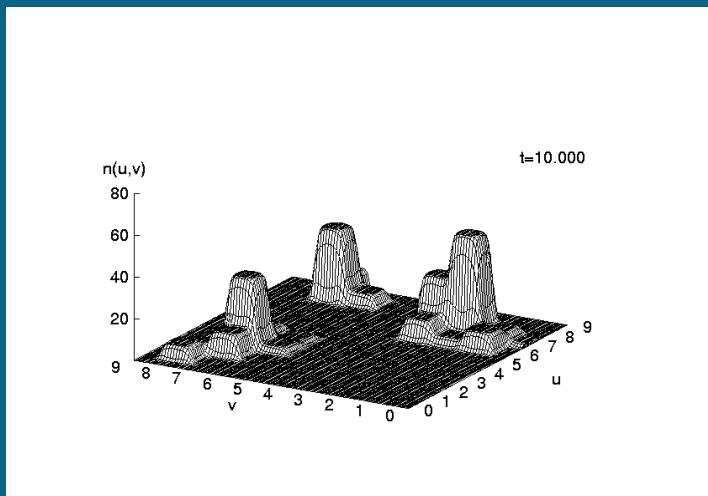
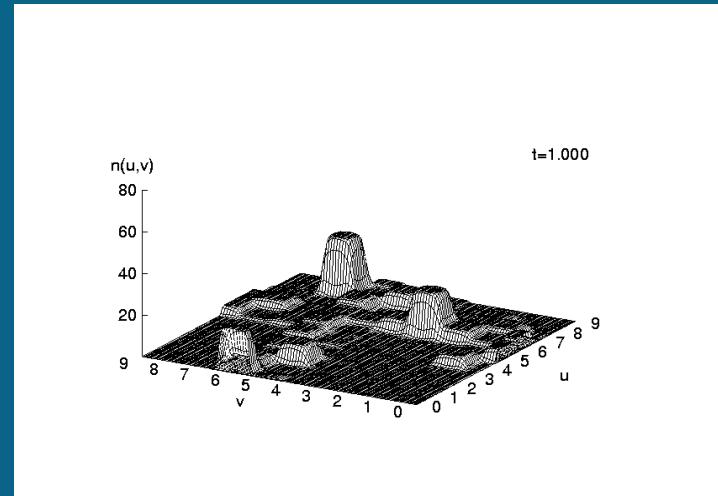
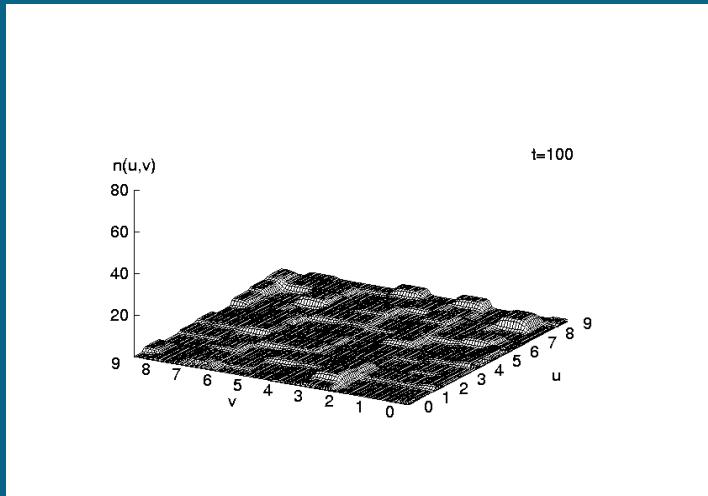
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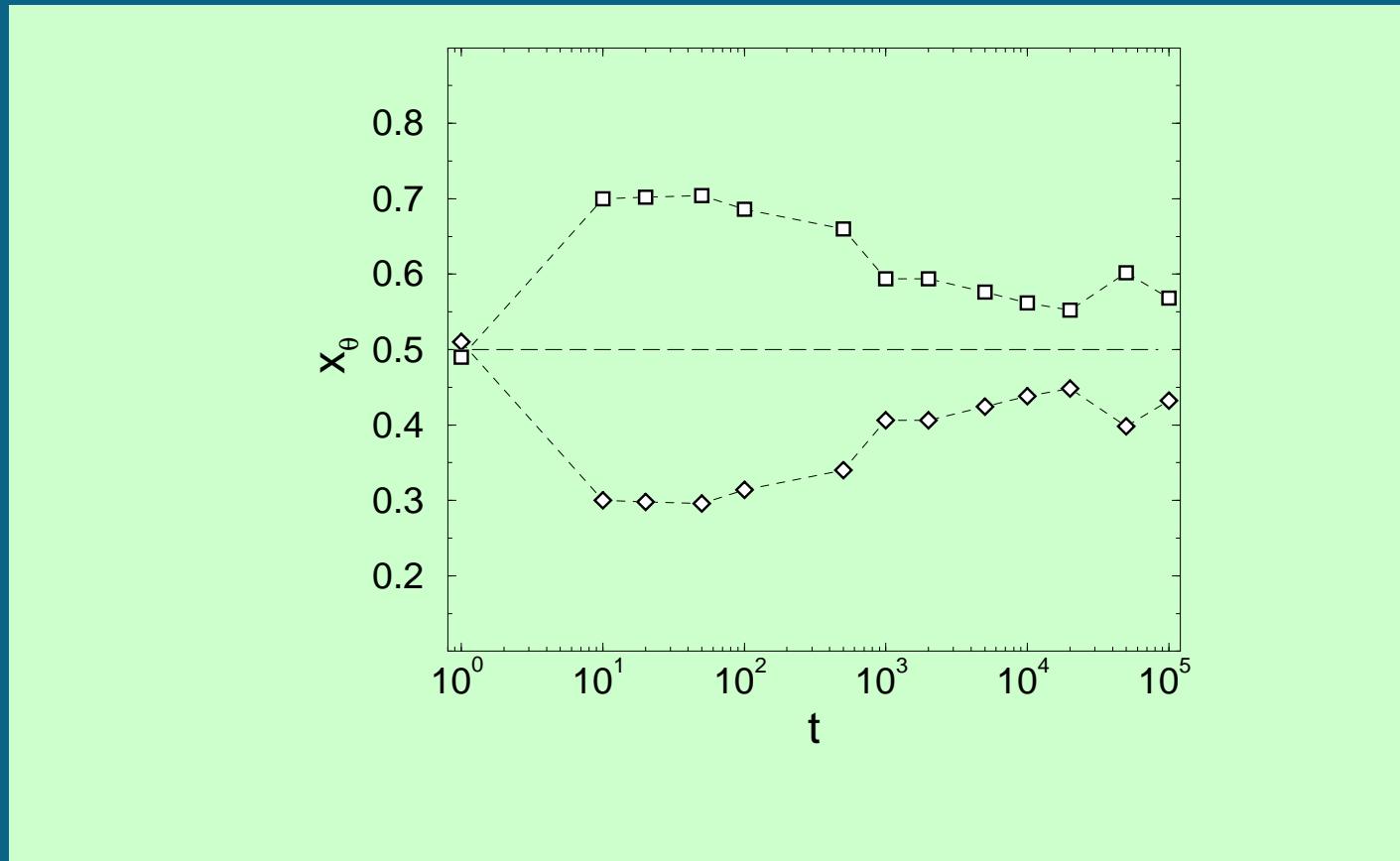


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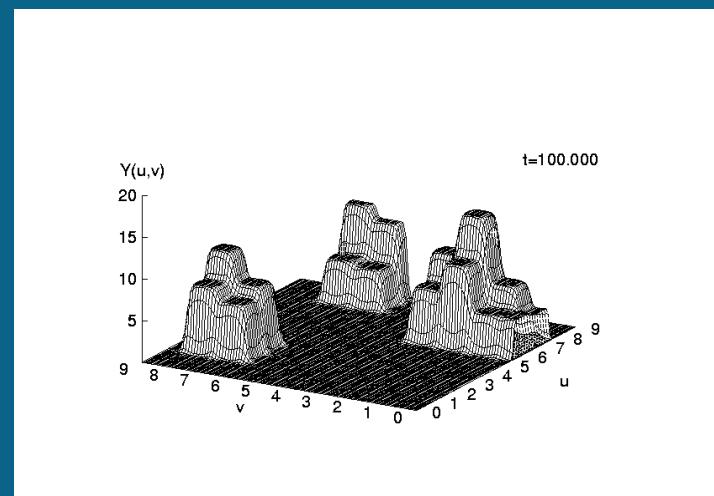
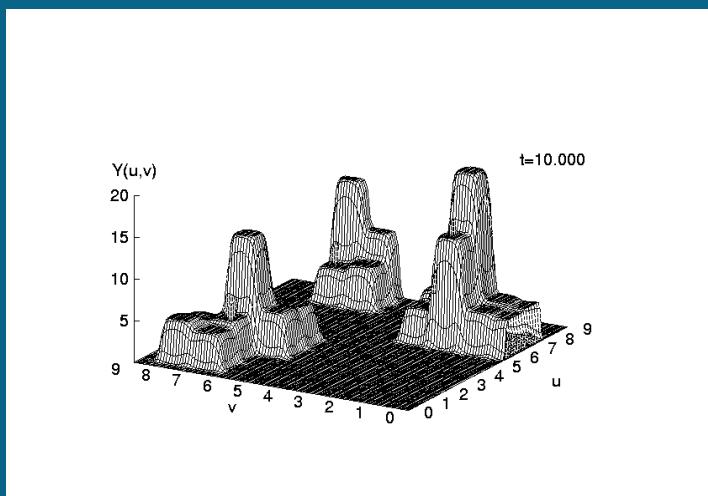
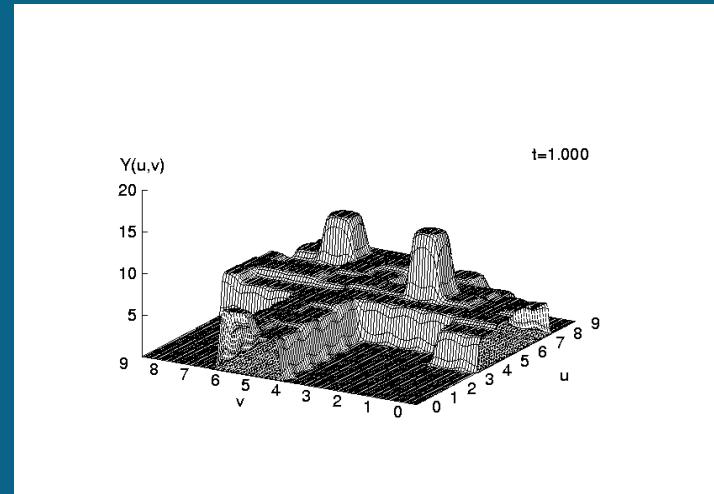
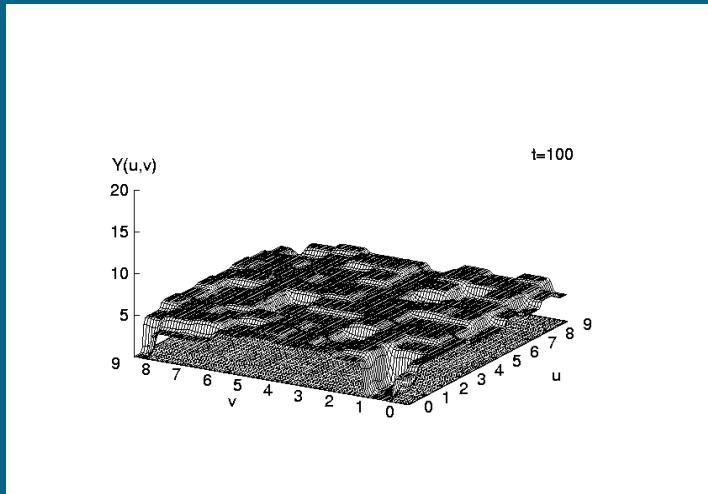
**Total share**  $x_\theta = N_\theta/N$



employed agents: ( $\square$ )

unemployed agents ( $\diamond$ )

## Spatial distribution of production



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    ⇒ stable centers, but *increase of unemployment*



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boundary conditions (semi-structured environment)



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# Self-Organization

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*Self-organization is the process by which individual subunits achieve, through their cooperative interactions, states characterized by new, emergent properties transcending the properties of their constitutive parts.*

Biebricher, C. K.; Nicolis, G.; Schuster, P.: Self-Organization in the Physico-Chemical and Life Sciences, EU Report 16546 (1995)

*Self-organization is defined as spontaneous formation, evolution and differentiation of complex order structures forming in non-linear dynamic systems by way of feedback mechanisms involving the elements of the systems, when these systems have passed a critical distance from the statical equilibrium as a result of the influx of unspecific energy, matter or information.*

SFB 230 “Natural Constructions”, Stuttgart, 1984 - 1995