Fraunhofer Institut Autonome Intelligente Systeme

Self-Assembing of Networks in an Agent-Based Model

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Self-Organization of Networks

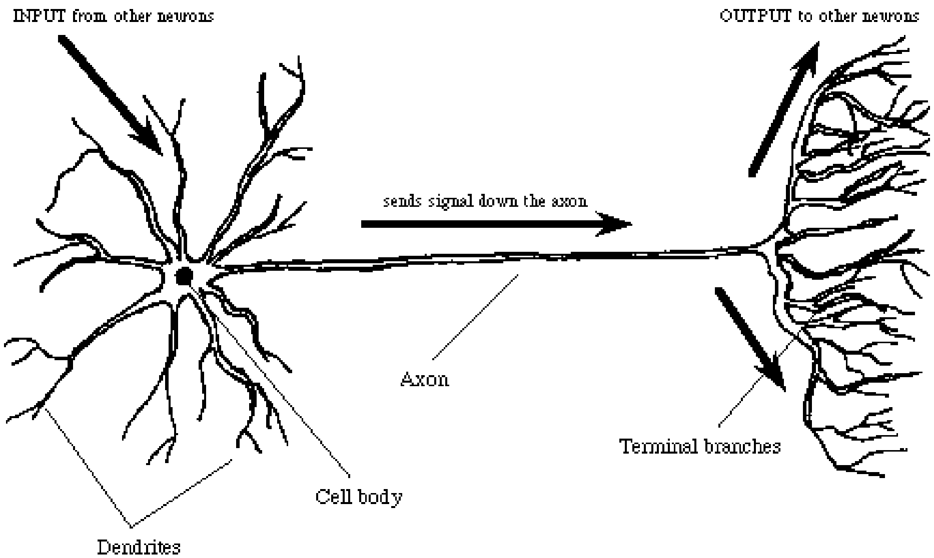
Problems

- *emergence* of network structures
  $\Rightarrow$ self-organized formation of links (different from drawing lines between a set of nodes)
- *co-processing*: discover new nodes, link them to the network *without* external guidance (information)
- *adaptivity:* keep the network updated $\Rightarrow$ connect/disconnect nodes, repair links, ...

$\Rightarrow$ ambitious aims ! natural paradigms?

Self-Wiring of Neural Structures

- neuron grows from retina to optic tectum of the brain
  - no prior information about destination node
  - navigation in unknown environment
  - generation of information to establish pathways



Video

Jerry Silver *et al.*, J Molecular and Cellular Neuroscience (MCN) Vol. 6:5 (1995) 413-432, 433-449 http://nervegrowth.com/ Webpages/Movie.html

Questions

- generation of relevant information?
  - results from interaction of agents
  - reamplification/selection of relevant information
- collective interaction?
  - based on (indirect) communication
  - memory effects? dissemination of information?
- formal modeling? $\Rightarrow$ multi-agent system
  - challenge: link agent-based (microscopic) model to analytical (macroscopic) model
  - allows prediction of collective behavior

Brownian Agent

different state variables: u_i^k

➤ external state variables: $\boldsymbol{r}_i$, $\boldsymbol{v}_i$, ...

➤ *internal degrees of freedom*:

- energy depot e_i: capability of action
- $\theta_i \in \{-1, 0, +1\}$: different types of action/response
  $\Rightarrow$ situation-dependent behavior

➤ combines features of *reactive* and *reflexive* agents

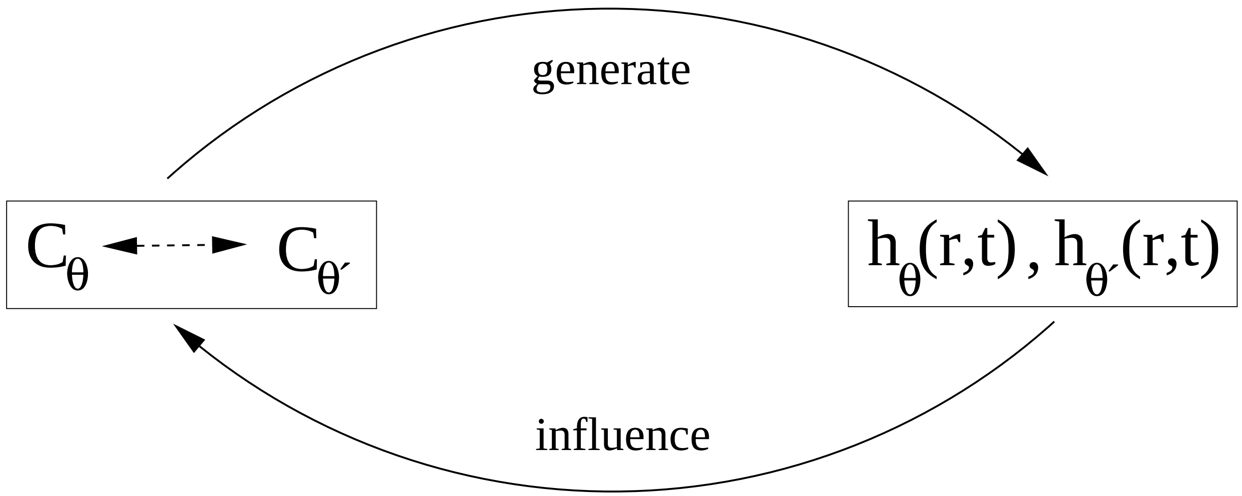
- changes of u_i^k: *deterministic* and *stochastic* influences

$$\frac{du_i^k}{dt} = f_i^k(..., u_i, u_j, ..., \sigma_1, ..., \sigma_n, t) + \sqrt{2\varepsilon_i}\, \xi_i(t)$$

- $f_i(\underline{u}, \underline{\sigma}, t)$ considers:
  - nonlinear interactions with other agents $j \in N$
  - control parameters, external conditions $(\sigma_1, ..., \sigma_n)$
  - explicite time dependence $\Rightarrow$ *eigendynamics*
- stochastic term: $\sqrt{2\varepsilon_i}\, \xi_i(t)$: summarizes all effects on smaller time/length scales ("other" influences)

Adaptive Landscape

- $f_i(\underline{u}, \underline{\sigma}, t) \Longrightarrow$ multi-component scalar field $h_\theta(\boldsymbol{r}, t)$



- *non-linear feedback* $\Rightarrow$ indirect *communication*
- *communication medium*: adaptive landscape $h_\theta(\boldsymbol{r}, t)$

Spatio-temporal communication field

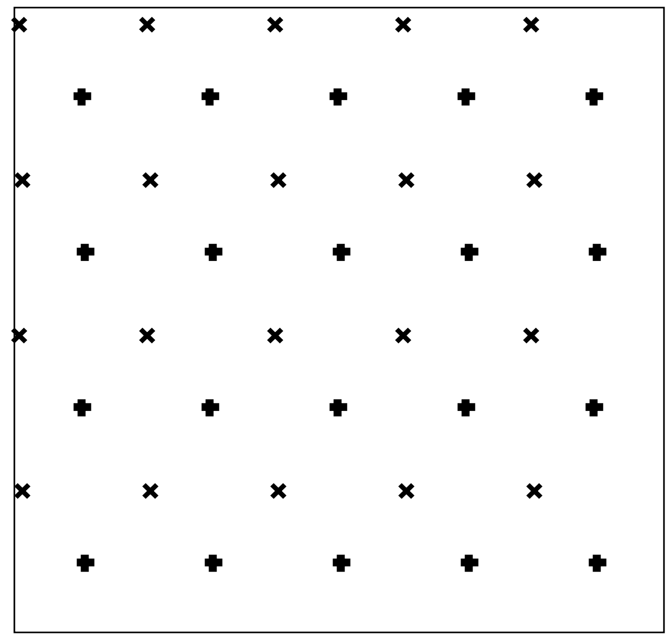
$$\frac{\partial}{\partial t} h_\theta(\boldsymbol{r},t) = \sum_{i=1}^{N} s_i\, \delta_{\theta,\theta_i}\, \delta(\boldsymbol{r}-\boldsymbol{r}_i) \;-\; k_\theta h_\theta(\boldsymbol{r},t) \;+\; D_\theta \Delta h_\theta(\boldsymbol{r},t)$$

- multi-component scalar field reflects:
  - existence of *memory* (past experience)
  - *exchange of information* with *finite* velocity
  - influence of *spatial distances* between agents
    $\Rightarrow$ *weighted* influence (space, time)

Example: Self-Wiring of Networks

- medicine: growth of neurons, neural nets
- engineering: self-assembling of circuits

- *task:* connect a set of "unknown" nodes **without** external guidance
- self-organized networks: adaptivity, self-repairing



Brownian Agents

- two state variables: position r_i
  internal parameter $\theta_i \in \{-1, +1\}$ (transitions possible)
- state dependent production rate

$$s_i(\theta_i, t) = \frac{\theta_i}{2} \Big[(1+\theta_i)\, s^0_{+1}\, \exp\{-\beta_{+1}\,(t - t^i_{n+})\} - (1-\theta_i)\, s^0_{-1}\, \exp\{-\beta_{-1}\,(t - t^i_{n-})\} \Big]$$

- two-component field

$$\frac{dh_\theta(\boldsymbol{r}, t)}{dt} = -k_\theta\, h_\theta(\boldsymbol{r}, t) + \sum_{i=1}^{N} s_i(\theta_i, t)\, \delta_{\theta,\theta_i}\, \delta(\boldsymbol{r} - \boldsymbol{r_i}(t))$$

- dynamic equation for r_i:

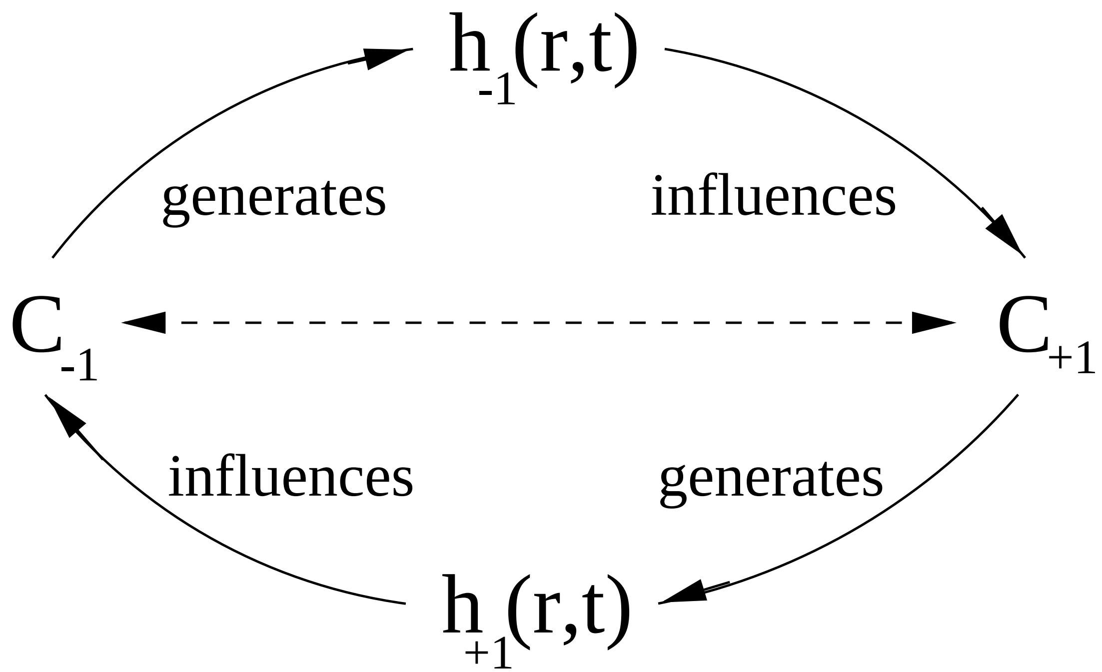
$$\frac{d\boldsymbol{r}_i}{dt} = \frac{1}{\gamma_0}\boldsymbol{\nabla}_{\boldsymbol{i}} h^e(\boldsymbol{r},t) + \sqrt{\frac{2k_BT}{\gamma_0}}\boldsymbol{\xi}_i(t)$$

$$\boldsymbol{\nabla}_{\boldsymbol{i}} h^e(\boldsymbol{r},t) = \frac{\theta_i}{2}\Big[(1+\theta_i)\,\boldsymbol{\nabla}_{\boldsymbol{i}} h_{-1}(\boldsymbol{r},t) - (1-\theta_i)\,\boldsymbol{\nabla}_{\boldsymbol{i}} h_{+1}(\boldsymbol{r},t)\Big]$$

- dynamic equation for θ_i:

$$\Delta\theta_i(t) = \sum_{j=1}^{z}(V_j - \theta_i) \int \delta\Big(\boldsymbol{r}_j^z - \boldsymbol{r}_i(t)\Big)\, dr$$

Non-linear feedback:



➤ **Result:** self-assembling of networks Film

Estimation of Network Connectivity

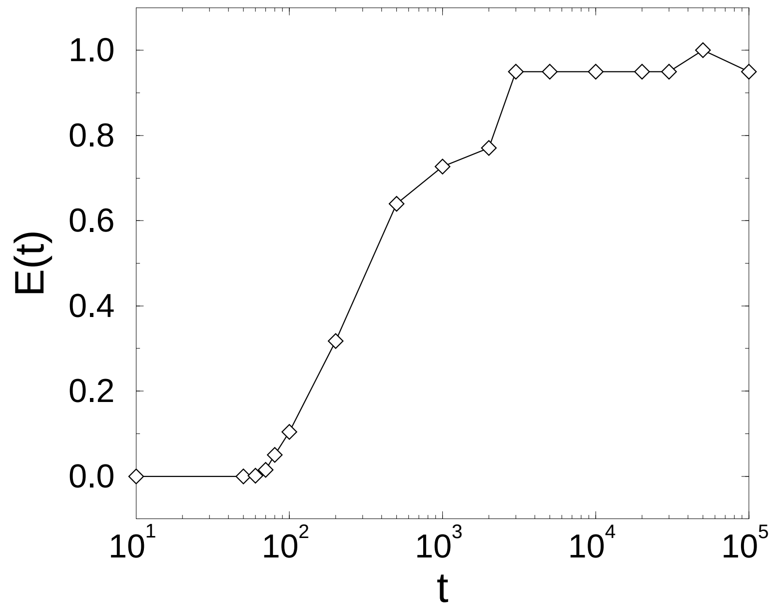
- *local connectivity*:

$$E_{lk} = \begin{cases} 1 & \text{if nodes } k \text{ and } l \text{ are connected by a path } a \in A, \\ & \text{along which } \hat{h}(a,t) > h_{thr} \\ 0 & \text{otherwise} \end{cases}$$

- *global connectivity*:

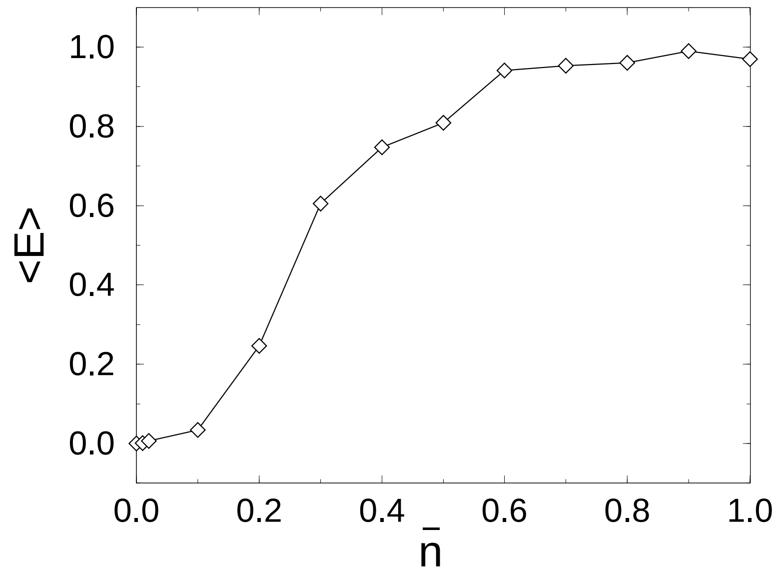
$$E = \frac{\sum_{k=1}^{z} \sum_{l>k}^{z} E_{lk}}{\sum_{k=1}^{z} \sum_{l>k}^{z} 1} = \frac{2}{z(z-1)} \sum_{k=1}^{z} \sum_{l>k}^{z} E_{lk}$$

Results:



- initial period ($t < 10^2$): no connections
  transient period ($10^2 < t < 10^4$): network establishes
  saturation period ($t > 10^4$): no further links

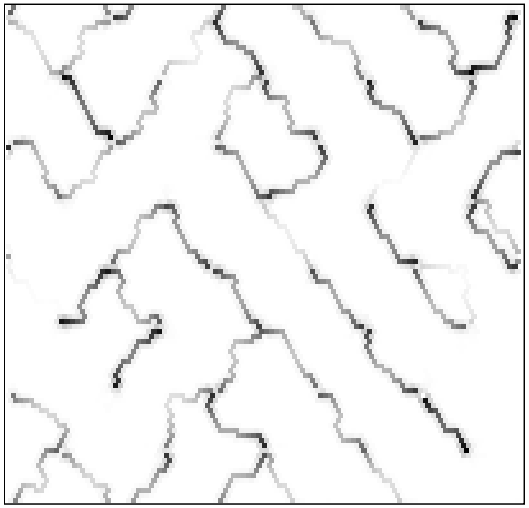
Dependence on agent density $\bar{n} = N/A$



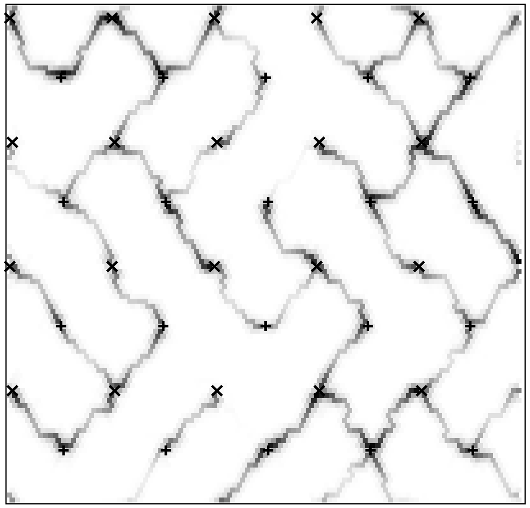
➤ *screening effect* concentrates all agents on established links

Critical Temperature

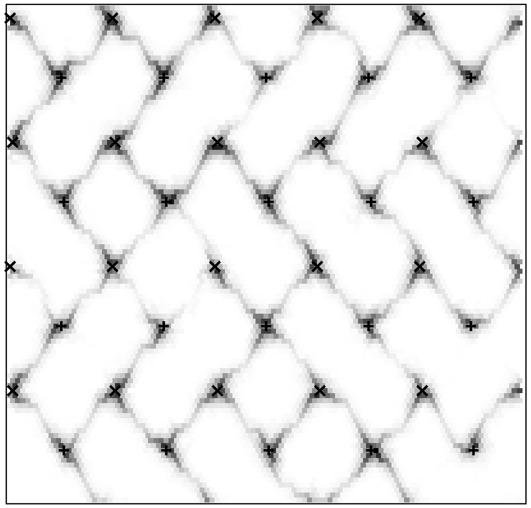
- T: measure of fluctuations $\Rightarrow$ response to field vs. mobility
- structure formation possible only for $T < T^c = \frac{\alpha}{2} \frac{\bar{s}\,\bar{n}}{k_B\,k_h}$



T = 0.2 T_c

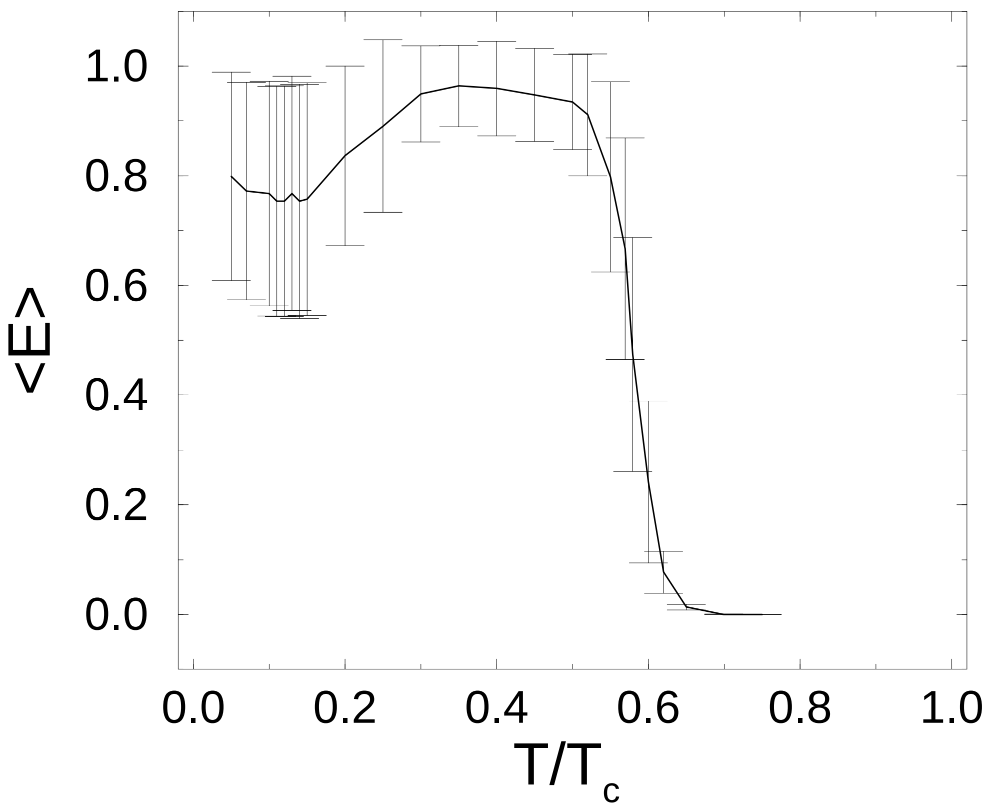


T = 0.4 T_c

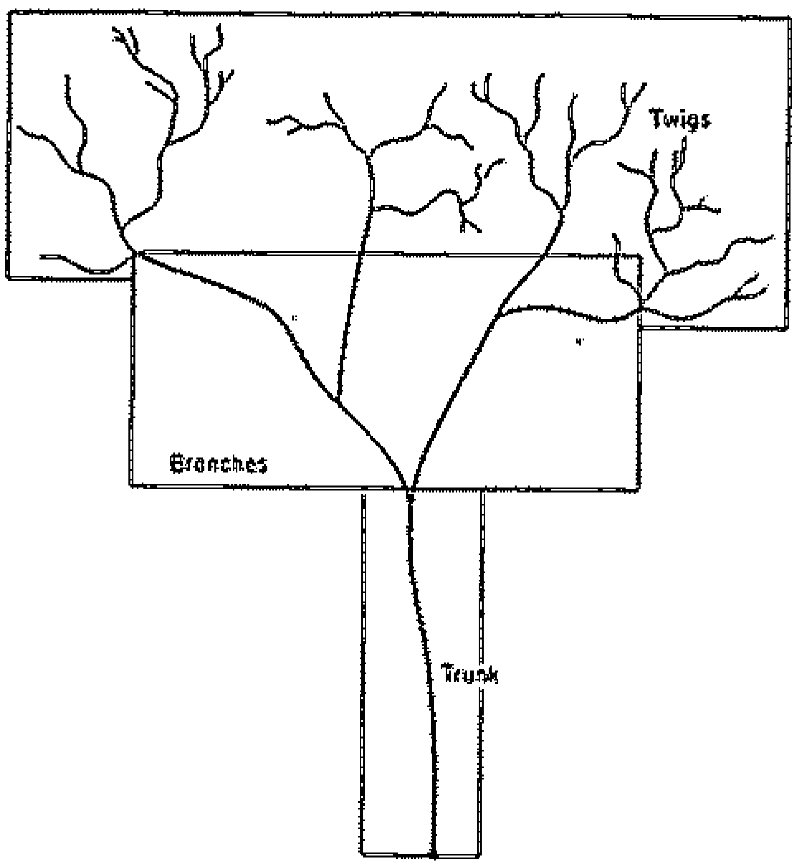


T = 0.6 T_c

- *optimal range of temperature* $0.3 \leq T/T^c \leq 0.5$



Example: Foraging Route of Ants



Schematic representation of the complete foraging route of *Pheidole milicida*, a harvesting ant of the southwestern U.S. deserts. Each day tens of thousands of workers move out to the dendritic trail system, disperse singly, and forage for food.

Hölldobler, B. and Möglich, M.: The foraging system of *Pheidole militicida (Hymenoptera: Formicidae), Insectes Sociaux* **27/3** (1980) 237-264

Brownian Agents

- state variables
  - position r_i
  - $\theta_i \in \{-1, +1\}$ (found food or not)
  - $\omega_i \in \{0; 1\}$ (scouts, recruits)
  - sensitivity η_i

- **Result:** exploitation of food sources Film

Insights for Social and Economic Systems?

- What are our unknown "food sources"?
  - markets, suppliers/customers, ..., scientists, ...
    $\Rightarrow$ to be detected, connected, exploited, ...
  - formation of economic or social networks
- Role of information?
  - created while exploring the state space
  - information about "success" breaks symmetry
  - access to relevant information most important
    at the right time at the right place, "networking"
- Non-linear feedback?
  - herding effect: imitation to reduce the risk
  - reamplification: success is contagious

Conclusions

- simple model shows self-organized establisment of networks between arbitrary nodes
- *Brownian Agents*: response to local information generated by themselves (non-linear feedback)
- basic dynamics: generalized Langevin approach $\Rightarrow$ allows use of statistical physics for complex structure formation
- broad range of possible applications in collective dynamics physico-chemistry, biology, evolutionary optimization, economics, social sciences, ...

Frank Schweitzer: *Brownian Agents and Active Particles. Collective Dynamics in the Natural and Social Sciences*. With a foreword by J. Doyne Farmer, Springer 2003