To move to the next slide, just press

ENTER or RETURN

Active Motion and Coherent Motion of Brownian Particles

Frank Schweitzer

GMD Institute for Autonomous intelligent Systems (AiS)

schweitzer@gmd.de   http://ais.gmd.de/~frank/

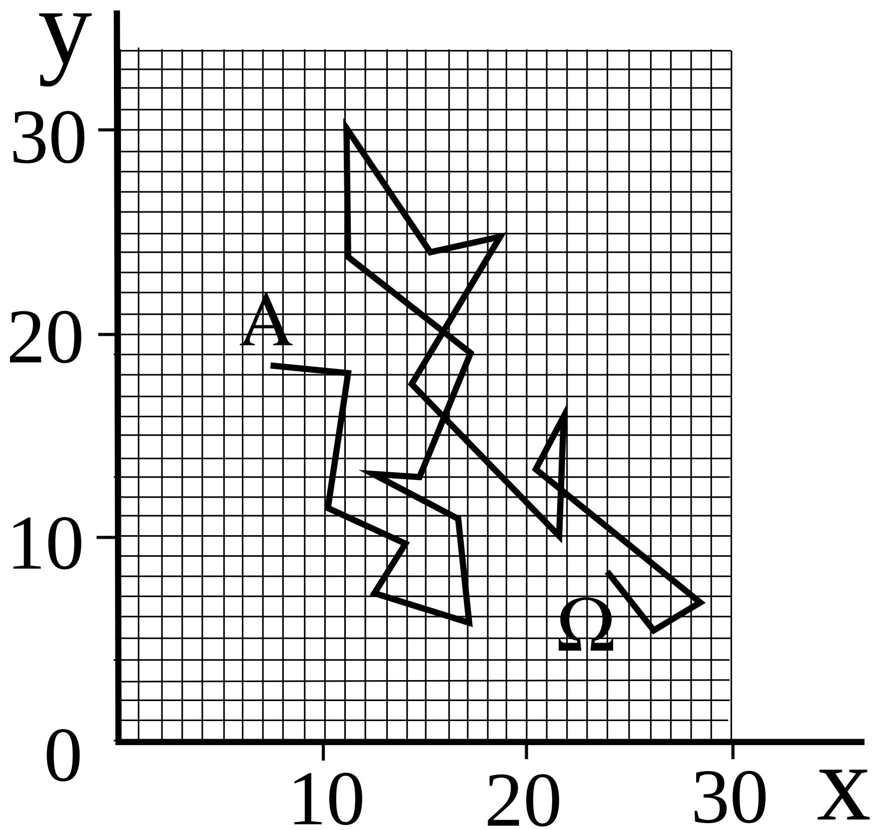
in collaboration with:

Werner Ebeling (Berlin), Benno Tilch (Stuttgart), Lutz Schimansky-Geier (Berlin)

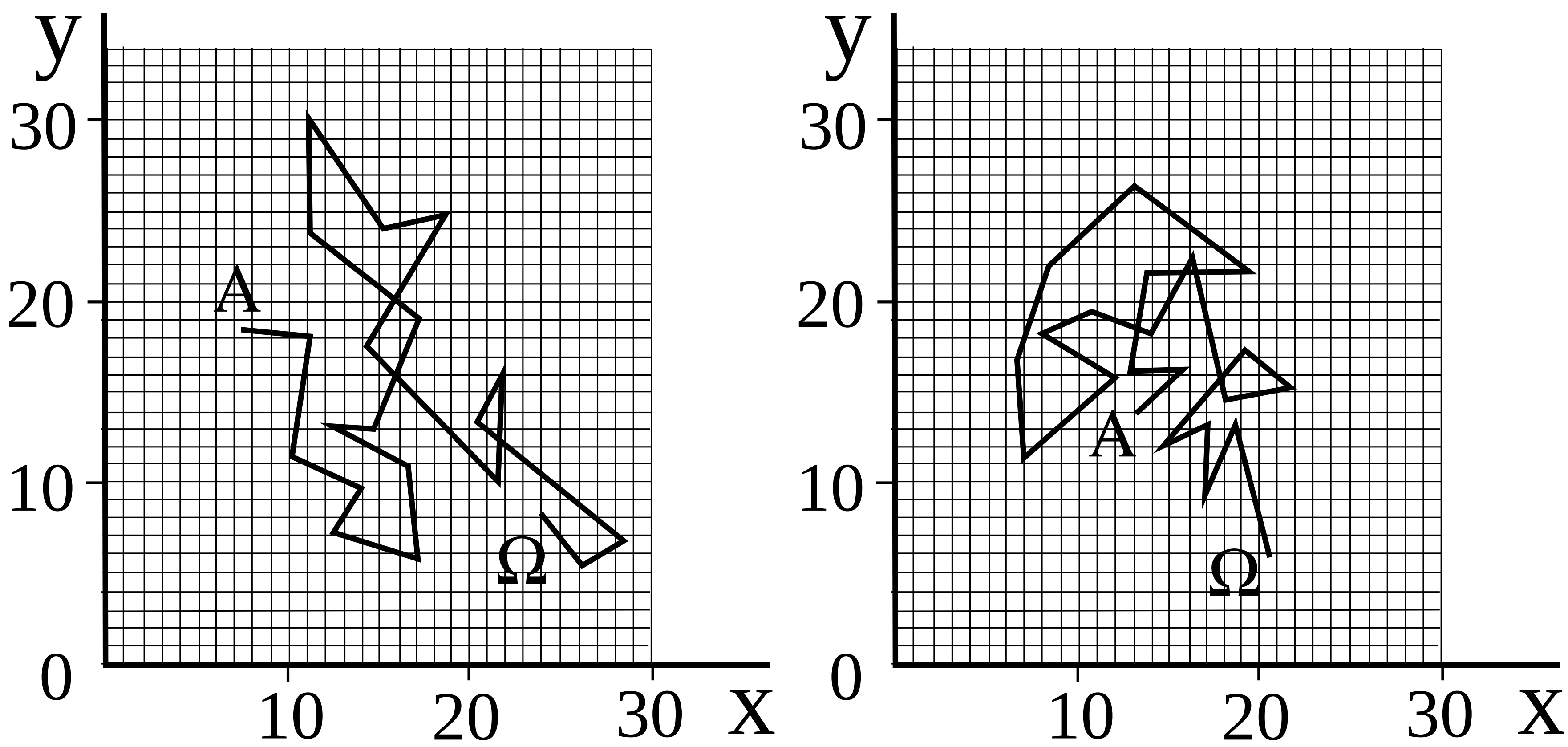
Schedule

1. Passive vs. Active Motion
2. Model of Active Brownian Particles
3. Motion under Energy Consumption
4. Coherent Motion of Interacting Particles
5. Conclusions

Observation of Brownian Motion



Observation of Brownian Motion



The position of the Brownian particle (radius 0.4 μm) is documented on a millimeter grid in time intervals $t_0 = 30$ seconds.

Passive vs. Active Motion

passive motion:

(1) *undirected motion:*

driven by thermal noise / random impacts

$\Rightarrow$ Brownian motion

Passive vs. Active Motion

passive motion:

(1) *undirected motion:*

driven by thermal noise / random impacts

$\Rightarrow$ Brownian motion

(2) *directed motion:*

driven by convection, currents, external fields

active motion:

➤ requires energy

- physico-chemical systems: particles produce gradients
- biological systems: metabolism, activities
- traffic systems: cars need to be fueled

$\Rightarrow$ *open system*: energy consumption, conversion

➤ self-driven particles, self-propelled particles:
focus on interaction (swarms, crowds, ...)

General concept of active motion:

(1) How can we turn *passive* into *active* motion?

- take-up of energy $\Rightarrow$ mechanisms of energy pumping ?
- (internal) storage of energy (depot, tank, battety)
- conversion of stored energy into kinetic energy

General concept of active motion:

(1) How can we turn *passive* into *active* motion?

- ➤ take-up of energy ⇒ mechanisms of energy pumping ?
- ➤ (internal) storage of energy (depot, tank, battety)
- ➤ conversion of stored energy into kinetic energy

(2) How can we turn *undirected* into *directed* motion?

- ➤ *deterministic* influences (intended direction of motion)
- ➤ *stochastic* influences (random events)
- ➤ *collective* interdependencies (coherent motion)

Model of Active Brownian Particles (ABP)

- moving in an “effective” or *quasi-potential* (environment):

$$U^{\star}(\boldsymbol{r},t) = U(\boldsymbol{r}) - h(\boldsymbol{r},t)$$

$U(\boldsymbol{r})$: external potential
$h(\boldsymbol{r},t)$: adaptive, can be changed by the ABP

Model of Active Brownian Particles (ABP)

➤ moving in an “effective” or *quasi-potential* (environment):

$$U^{\star}(\boldsymbol{r}, t) = U(\boldsymbol{r}) - h(\boldsymbol{r}, t)$$

$U(\boldsymbol{r})$: external potential

$h(\boldsymbol{r}, t)$: adaptive, can be changed by the ABP

➤ *internal energy depot* $e(t)$

$$\frac{d}{dt} e(t) = q(\boldsymbol{r}) - c\, e(t) - s - \eta(v^2)\, d_2 v^2\, e(t)$$

- $q(r)$: take-up of energy from the environment
- c: internal dissipation (“metabolism”)
- s: environmental changes (generation of $h(\boldsymbol{r}, t)$)
- acceleration: conversion rate d_2, efficiency $\eta(v^2)$

Equation of motion:

Langevin-type equation:

$$\dot{\boldsymbol{v}} = -\gamma_0\,\boldsymbol{v} - \frac{1}{m}\boldsymbol{\nabla}\left[U(\boldsymbol{r}) - h(\boldsymbol{r},t)\right] + \eta(v^2)\,d_2 e(t)\,\boldsymbol{v} + \sqrt{2\,D}\,\boldsymbol{\xi}(t)$$

considers:

- stochastic influences (D)
- spatial differences in $h(\boldsymbol{r},t)$ (“signal-response-behavior”)
- dissipation / acceleration of motion

Non-linear Friction Function

assumptions: $q(\boldsymbol{r}) \equiv q_0 \; ; \; \eta(v^2) \equiv 1 \; ; \; s \equiv 0$

equations of motion:

$$
\begin{aligned}
\dot{\boldsymbol{v}} + \gamma_0 \boldsymbol{v} + \nabla U(\boldsymbol{r}) &= d_2 e(t) \boldsymbol{v} + \mathcal{F}(t) \\
\dot{e}(t) &= q_0 - c\, e(t) - d(\boldsymbol{v})\, e(t)
\end{aligned}
$$

Non-linear Friction Function

assumptions: $q(\boldsymbol{r}) \equiv q_0 \ ; \ \eta(v^2) \equiv 1 \ ; \ s \equiv 0$

equations of motion:

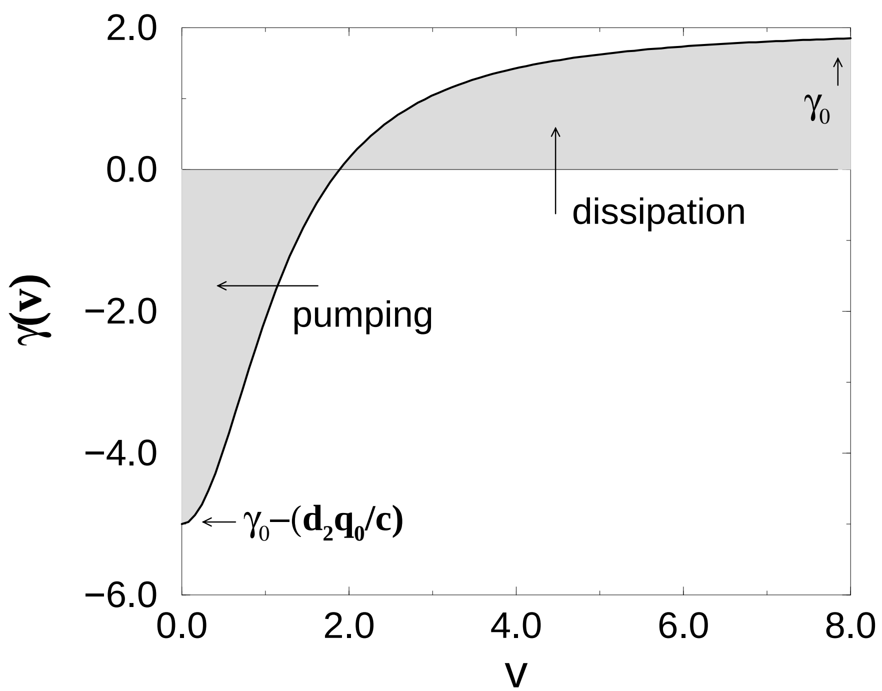
$$\begin{aligned} \dot{\boldsymbol{v}} + \gamma_0 \boldsymbol{v} + \nabla U(\boldsymbol{r}) &= d_2 e(t) \boldsymbol{v} + \mathcal{F}(t) \\ \dot{e}(t) &= q_0 - c\, e(t) - d(\boldsymbol{v})\, e(t) \end{aligned}$$

quasistationary energy depot: $\dot{e}(t) = 0$

$$e_0 = \frac{q_0}{c + d_2 v^2} \quad \Rightarrow \quad \gamma(\boldsymbol{v}) = \gamma_0 - \frac{q_0\, d_2}{c + d_2 v^2}$$

zero:

$$v_0^2 = \frac{q_0}{\gamma_0} - \frac{c}{d_2}$$



Velocity Distribution

Fokker-Planck equation for $p(\boldsymbol{v}, t)$
with $U(\boldsymbol{r})$ =const. and $e(t) \rightarrow e_0 = \dfrac{q_0}{c + d_2 v^2}$

$$\frac{\partial p(\boldsymbol{v},t)}{\partial t} = \frac{\partial}{\partial \boldsymbol{v}} \left[\left(\gamma_0 - \frac{d_2 q_0}{c + d_2\, v^2} \right) \boldsymbol{v}\, p(\boldsymbol{v},t) + D\, \frac{\partial p(\boldsymbol{v})}{\partial \boldsymbol{v}} \right]$$

Velocity Distribution

Fokker-Planck equation for $p(\boldsymbol{v},t)$
with $U(\boldsymbol{r})$ =const. and $e(t) \rightarrow e_0 = \dfrac{q_0}{c + d_2 v^2}$

$$\frac{\partial p(\boldsymbol{v},t)}{\partial t} = \frac{\partial}{\partial \boldsymbol{v}} \left[\left(\gamma_0 - \frac{d_2 q_0}{c + d_2\, v^2} \right) \boldsymbol{v}\, p(\boldsymbol{v},t) + D\, \frac{\partial p(\boldsymbol{v})}{\partial \boldsymbol{v}} \right]$$

stationary solution: $\dot{p}(\boldsymbol{v},t) = 0$

$$p^0(\boldsymbol{v}) = C' \, \left(c + d_2 v^2 \right)^{q_0/2D} \, \exp\left\{ -\frac{\gamma_0}{2D}\, v^2 \right\}$$

small v^2: power series

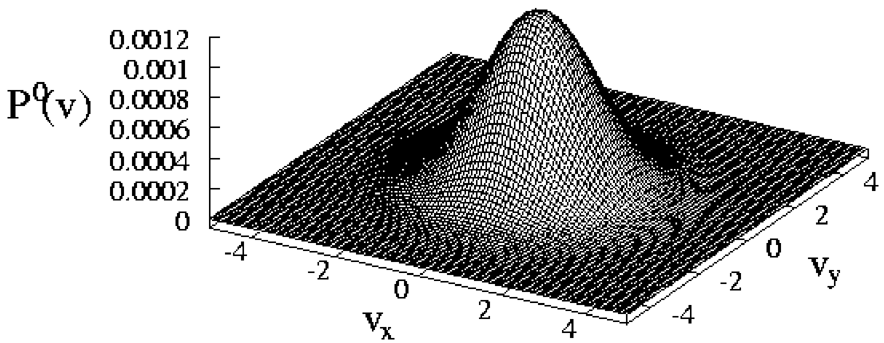
$$p^0(\boldsymbol{v}) \sim \exp\left\{-\frac{\gamma_0}{2D}\left(1-\frac{q_0 d_2}{c\gamma_0}\right)\,v^2+\cdots\right\}$$

small v^2: power series

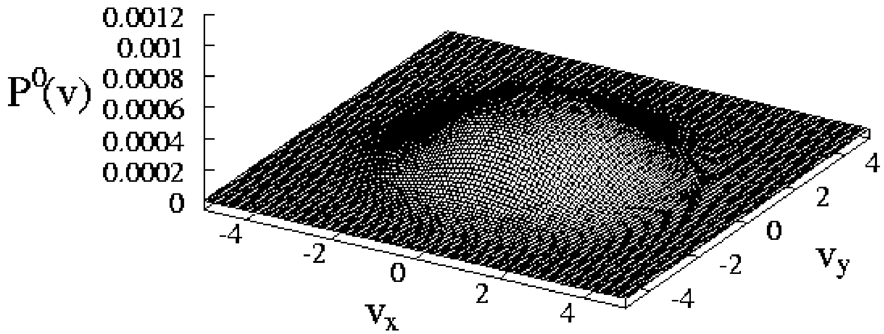
$$p^0(\boldsymbol{v}) \sim \exp\left\{-\frac{\gamma_0}{2D}\left(1-\frac{q_0 d_2}{c\gamma_0}\right)v^2+\cdots\right\}$$

$q_0\, d_2 < c\,\gamma_0$: subcritical pumping
$\Rightarrow$ Maxwellian velocity distribution

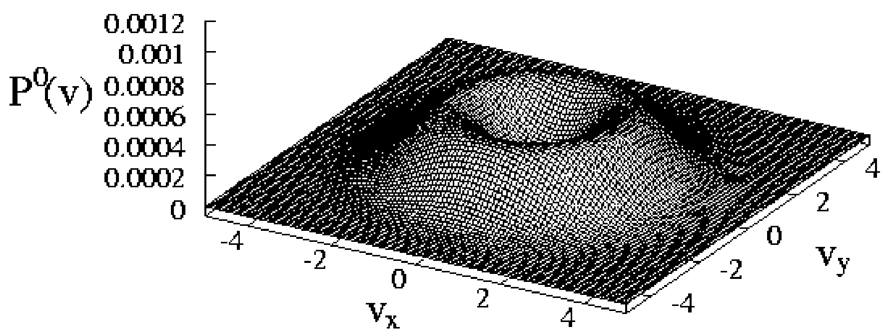
$q_0\, d_2 > c\,\gamma_0$: supercritical pumping
$\Rightarrow$ crater-like distribution



$d_2 = 0.07$



$d_2 = 0.2$

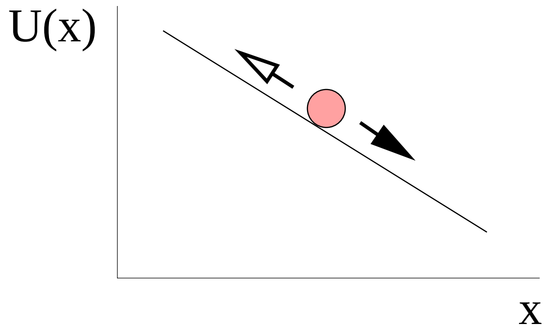


$d_2 = 0.7$

Normalized stationary solution $p^0(\boldsymbol{v})$, $\gamma_0 = 2$, $D = 2$, $c = 1$, $q_0 = 10$.

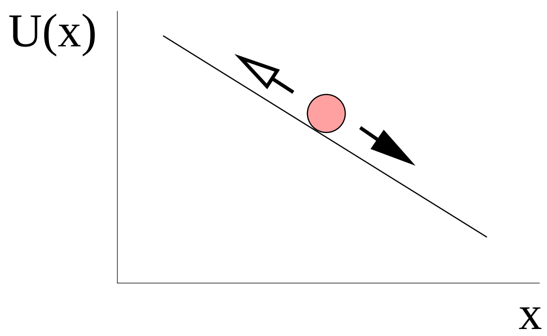
Uphill Motion

one-dimensional case: $\boldsymbol{F} = -\boldsymbol{\nabla} U$



Uphill Motion

one-dimensional case: $\boldsymbol{F} = -\boldsymbol{\nabla} U$



$$\begin{aligned} \dot{\boldsymbol{v}} &= -(\gamma_0 - d_2 e(t))\boldsymbol{v} + \boldsymbol{F} + \sqrt{2\,D}\,\boldsymbol{\xi}(t) \\ \dot{e} &= q_0 - ce - d_2 v^2 e \end{aligned}$$

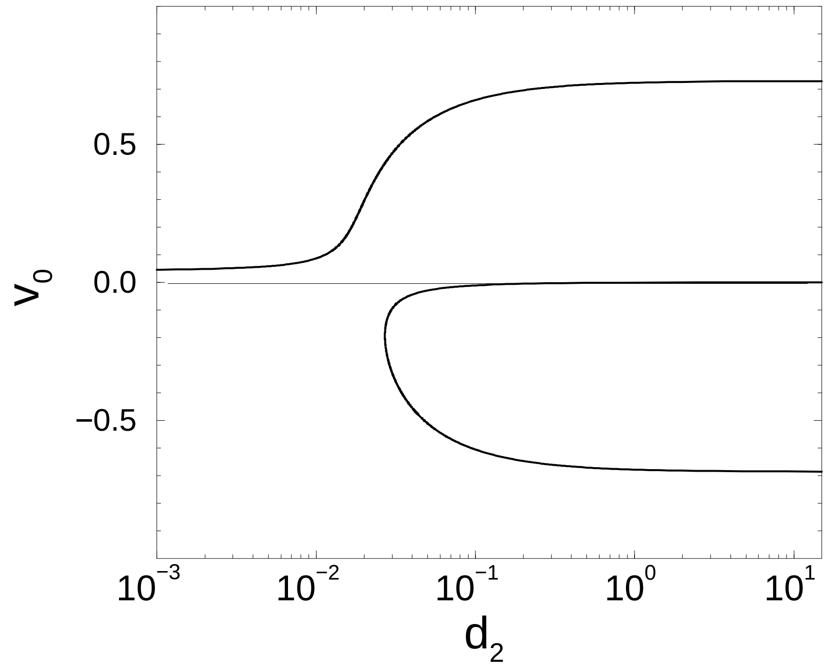
stationary solutions: ($D = 0$)

$$\boldsymbol{v}_0 = \frac{\boldsymbol{F}}{\gamma_0 - d_2 e_0} \quad ; \quad e_0 = \frac{q_0}{c + d_2 \boldsymbol{v}_0^2}$$

$$\Rightarrow \left[d_2\gamma_0\boldsymbol{v}_0^2 - d_2\boldsymbol{F}v_0 - (q_0d_2 - c\gamma_0)\right]\boldsymbol{v}_0 = c\boldsymbol{F}$$

- always existing: $\boldsymbol{v}_0(\boldsymbol{x}) \sim \boldsymbol{F}(\boldsymbol{x})$
  $\Rightarrow$ analytic continuation of Stokes' law: $\boldsymbol{v}_0 = \boldsymbol{F}/\gamma_0$
- subcritical supply of energy:
  $\Rightarrow$ *passive* motion driven by $\boldsymbol{F}$
- supercritical supply of energy: 3 solutions
  $\Rightarrow$ "high velocity" or *active* mode of motion
  $\Rightarrow$ motion *in* or *against* the direction of $\boldsymbol{F}$

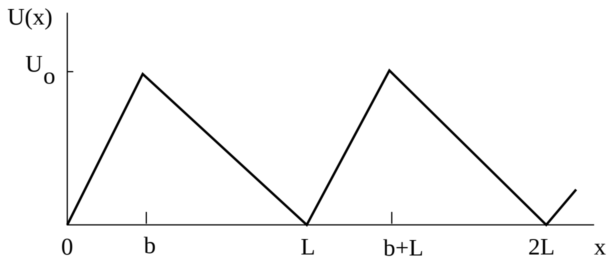
bifurcation diagram:

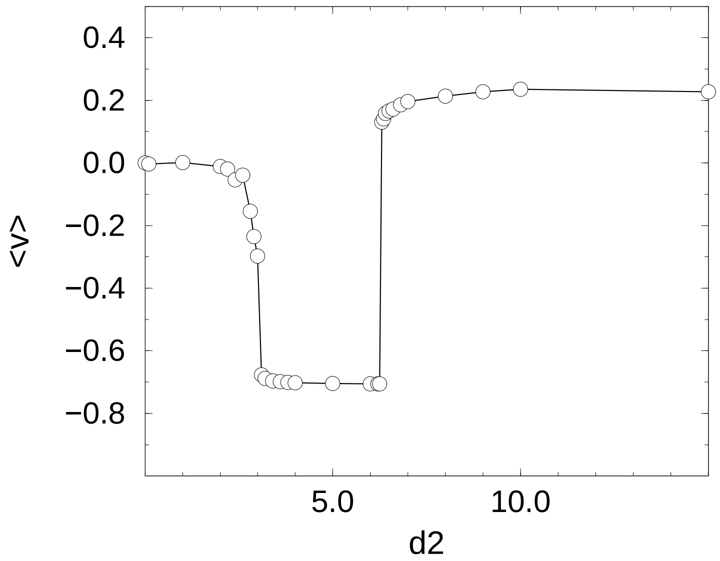


stability analysis ⇒ *stable* uphill motion ($c \longrightarrow 0$):

$$d_2^{crit} = \frac{F^4}{8q_0^3}\left(1 + \sqrt{1 + \frac{4\gamma_0 q_0}{F^2}}\right)^3$$

Motion in a Ratchet Potential



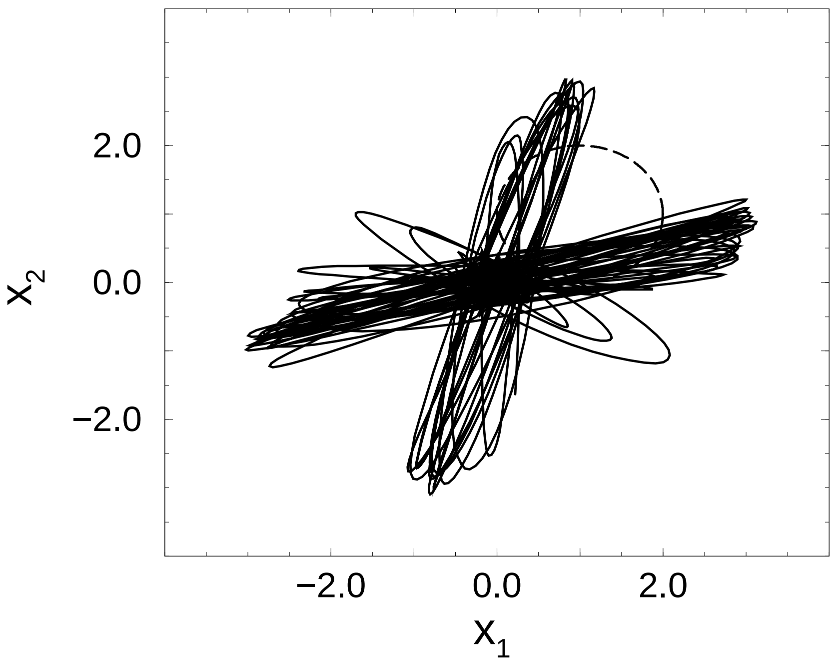


skipped here ....

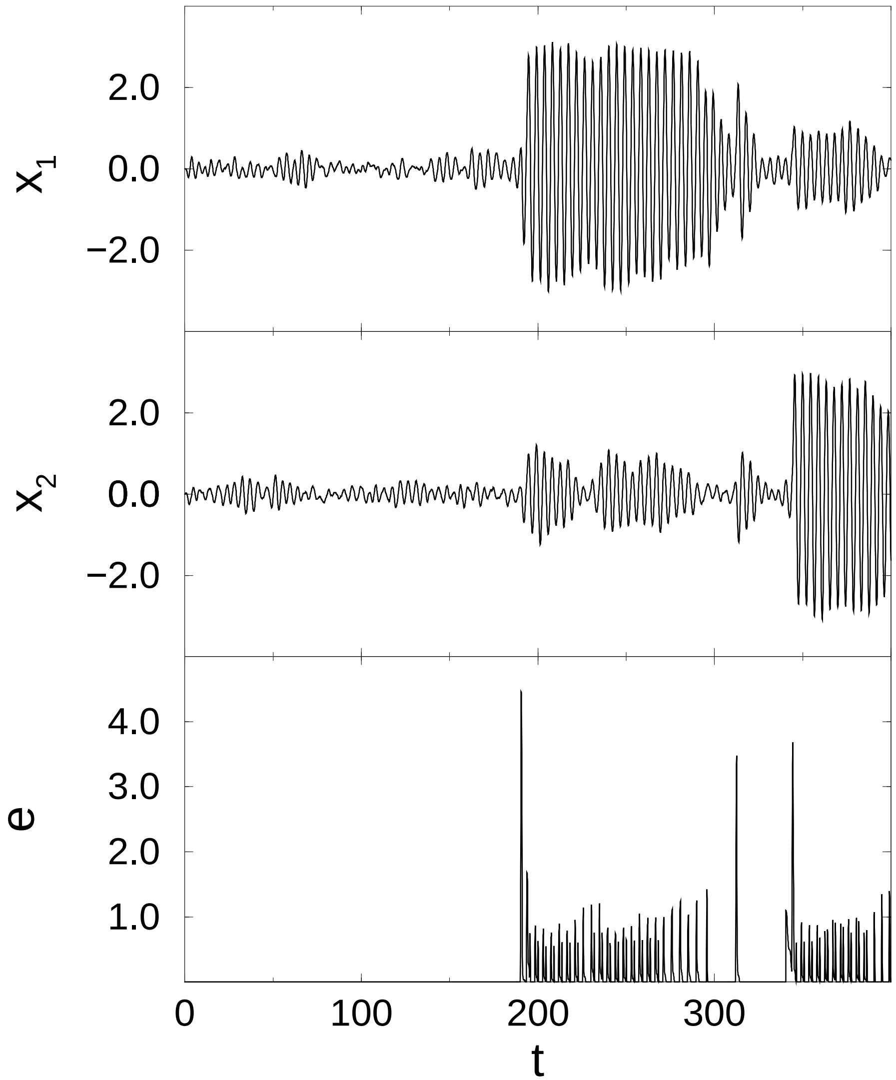
Localized Energy Sources

- two-dimensional parabolic potential:
  $U(x_1, x_2) = \frac{a}{2}(x_1^2 + x_2^2)$
- take-up of energy in a restricted area:

$$q(x_1, x_2) = \begin{cases} q_0 & \text{if } \left[(x_1 - b_1)^2 + (x_2 - b_2)^2\right] \le R^2 \\ 0 & \text{else} \end{cases}$$



- motion into the energy area becomes accelerated
- oscillatory movement with fixed direction



- intermittent type of motion: *active* $\Longleftrightarrow$ *passive*
- new cycles start with a burst of energy
- increase in d_2 abridges the cycle $\Rightarrow$ directed motion more susceptible to become Brownian motion

Swarming in a Parabolic Potential

- ensemble of N active Brownian particles
- $U(x_1, x_2) = \frac{a}{2}(x_1^2 + x_2^2)$, $\quad q(\boldsymbol{r}) = q_0 = \mathrm{const.}$
- initial conditions: $x_1(0)$, $x_2(0)$, $v_1(0)$ $v_2(0)$, $e(0) = 0$
- parameters: q_0, c, γ_0, d_2 Film

Swarming in a Parabolic Potential

- ensemble of N active Brownian particles
- $U(x_1, x_2) = \frac{a}{2}(x_1^2 + x_2^2)$, $\quad q(\boldsymbol{r}) = q_0 = \text{const.}$
- initial conditions: $x_1(0)$, $x_2(0)$, $v_1(0)$ $v_2(0)$, $e(0) = 0$
- parameters: q_0, c, γ_0, d_2 Film

result: coherent motion with slow spatial dispersion

- for $q_0 d_2 > \gamma_0 c$: two stochastic limit cycles (left- and righthand rotation)
- $$v_1^2 + v_2^2 = \boldsymbol{v}_0^2 = \frac{q_0}{\gamma_0} - \frac{c}{d_2} = \text{const.}$$
- $$x_1^2 + x_2^2 = r_0^2 = \frac{v_0^2}{a} = \text{const.}$$

Interacting Active Brownian Particles

- “effective” potential (environment):

$$U^{\star}(\boldsymbol{r},t) = U(\boldsymbol{r}) - h(\boldsymbol{r},t)$$

- $h(\boldsymbol{r},t)$: interaction “potential” (scalar field)

Interacting Active Brownian Particles

- “effective” potential (environment):

$$U^\star(\boldsymbol{r},t) = U(\boldsymbol{r}) - h(\boldsymbol{r},t)$$

- $h(\boldsymbol{r},t)$: interaction “potential” (scalar field)
  example: reaction-diffusion dynamics

$$\frac{dh(\boldsymbol{r},t)}{dt} = s \sum_{i=1}^{N} \delta(\boldsymbol{r} - \boldsymbol{r_i}(t)) \; - k_0\, h(\boldsymbol{r},t) \; + \; D_0\, \boldsymbol{\Delta} h_\theta(\boldsymbol{r},t)$$

- non-linear feedback: Langevin dynamics

$$\frac{d\boldsymbol{v}_i}{dt} = -\gamma_0 \boldsymbol{v}_i + \left.\frac{\partial h(\boldsymbol{r},t)}{\partial \boldsymbol{r}}\right|_{\boldsymbol{r}_i} + \sqrt{2D}\, \boldsymbol{\xi}_i(t)$$

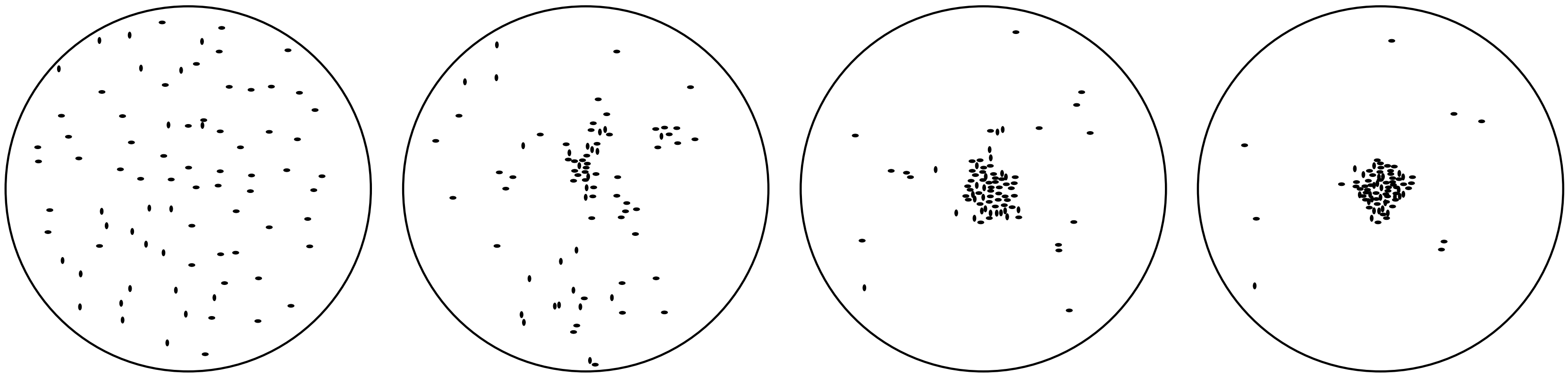
for $U(\boldsymbol{r}) = \mathrm{const.}$, $d_2 \equiv 0$

Application: biological aggregation

cells, slime mold amoebae, myxobacteria generate a *chemical field* to communicate

Application: biological aggregation

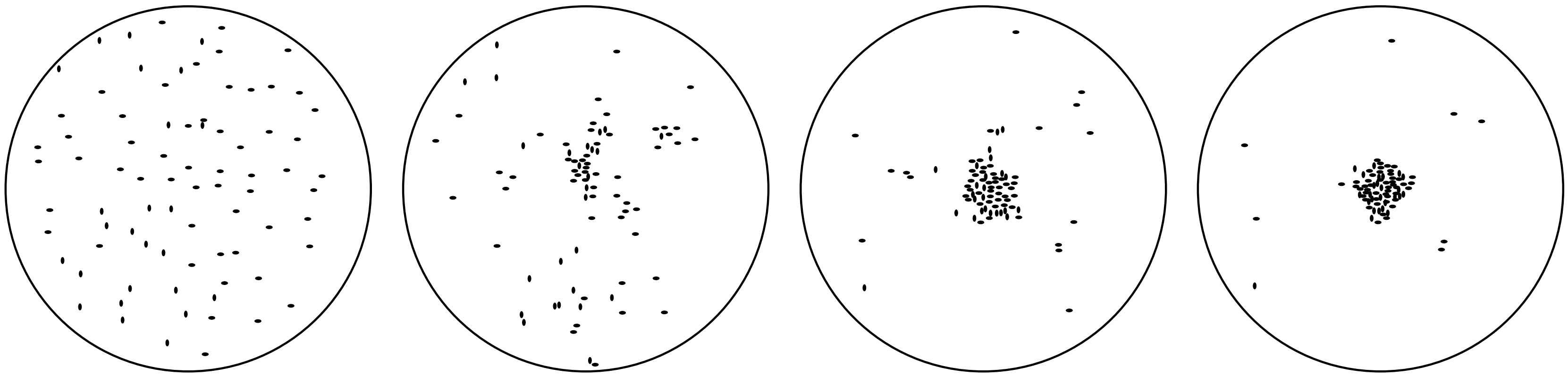
cells, slime mold amoebae, myxobacteria generate a *chemical field* to communicate



Deneubourg, J. L.; Gregoire, J. C.; Le Fort, E. (1990):
Kinetics of Larval Gregarious Behavior in the Bark Beetle Insect Behavior **3/2**, 169-182 (1990)

Application: biological aggregation

cells, slime mold amoebae, myxobacteria generate a *chemical field* to communicate



Deneubourg, J. L.; Gregoire, J. C.; Le Fort, E. (1990):
Kinetics of Larval Gregarious Behavior in the Bark Beetle Insect Behavior **3/2**, 169-182 (1990)

skipped here ...

Coherent Motion via Chemical Communication

- active particles with two different states $\theta \in \{-1, +1\}$, transitions possible

Coherent Motion via Chemical Communication

- active particles with two different states $\theta \in \{-1, +1\}$, transitions possible
- state dependent production rate

$$s_i(\theta_i, t) = \frac{\theta_i}{2} \Big[(1+\theta_i)\, s^0_{+1}\ \exp\{-\beta_{+1}\,(t - t^i_{n+})\} - (1-\theta_i)\, s^0_{-1}\ \exp\{-\beta_{-1}\,(t - t^i_{n-})\}\Big]$$

Coherent Motion via Chemical Communication

- active particles with two different states $\theta \in \{-1, +1\}$, transitions possible
- state dependent production rate

$$s_i(\theta_i, t) = \frac{\theta_i}{2} \Big[(1+\theta_i)\, s^0_{+1} \, \exp\{-\beta_{+1}\,(t - t^i_{n+})\} - (1-\theta_i)\, s^0_{-1} \, \exp\{-\beta_{-1}\,(t - t^i_{n-})\} \Big]$$

- two-component field

$$\frac{dh_\theta(\boldsymbol{r}, t)}{dt} = -k_\theta\, h_\theta(\boldsymbol{r}, t) + \sum_{i=1}^{N} q_i(\theta_i, t)\, \delta(\theta - \theta_i(t))\, \delta(\boldsymbol{r} - \boldsymbol{r_i}(t))$$

- overdamped Langevin equation:

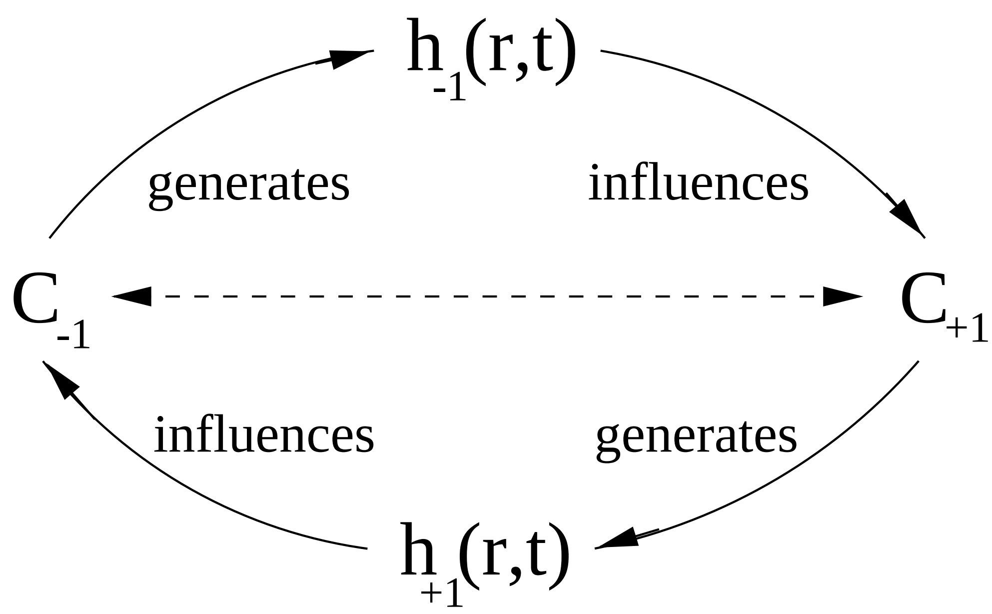
$$\frac{d\boldsymbol{r}_i}{dt} = \frac{1}{\gamma_0} \left. \frac{\partial h^e(\boldsymbol{r},t)}{\partial \boldsymbol{r}} \right|_{r_i,\theta_i} + \sqrt{\frac{2k_BT}{\gamma_0}} \boldsymbol{\xi}_i(t)$$

$$\boldsymbol{\nabla}_{\boldsymbol{i}} h^e(\boldsymbol{r},t) = \frac{\theta_i}{2} \left[(1+\theta_i) \, \boldsymbol{\nabla}_{\boldsymbol{i}} h_{-1}(\boldsymbol{r},t) \; - \; (1-\theta_i) \, \boldsymbol{\nabla}_{\boldsymbol{i}} h_{+1}(\boldsymbol{r},t) \right]$$

➤ overdamped Langevin equation:

$$\frac{d\boldsymbol{r}_i}{dt} = \frac{1}{\gamma_0}\,\frac{\partial h^e(\boldsymbol{r},t)}{\partial \boldsymbol{r}}\bigg|_{r_i,\theta_i} + \sqrt{\frac{2k_BT}{\gamma_0}}\,\boldsymbol{\xi}_i(t)$$

$$\boldsymbol{\nabla}_i h^e(\boldsymbol{r},t) = \frac{\theta_i}{2}\left[(1+\theta_i)\,\boldsymbol{\nabla}_i h_{-1}(\boldsymbol{r},t) - (1-\theta_i)\,\boldsymbol{\nabla}_i h_{+1}(\boldsymbol{r},t)\right]$$



applications:

- exploitation of food sources:
  ants mark trails from food sources to the nest with additional chemicals to guide nestmates to resources

Film

applications:

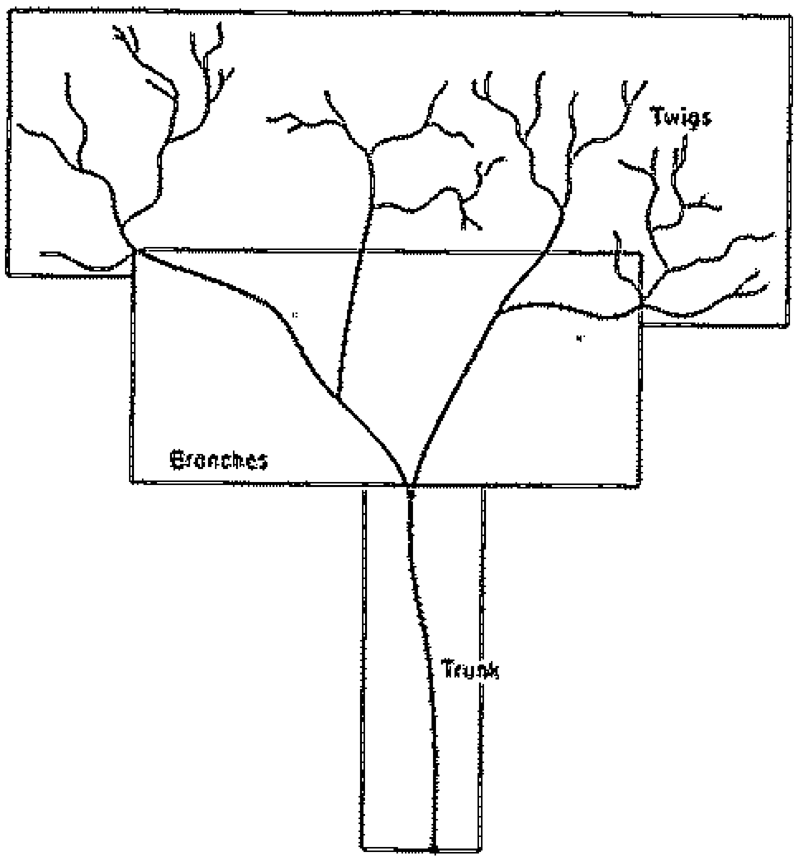
- exploitation of food sources:
  ants mark trails from food sources to the nest with additional chemicals to guide nestmates to resources

Film

- self-assembling of networks:
  particles responding to two different fields, link nodes with opposite potential

Film

Foraging Route of Ants (*Pheidole milicida*)



Schematic representation of the complete foraging route of *Pheidole milicida*, a harvesting ant of the southwestern U.S. deserts. Each day tens of thousands of workers move out to the dendritic trail system, disperse singly, and forage for food.

Hölldobler, B. and Möglich, M.: The foraging system of *Pheidole militicida (Hymenoptera: Formicidae), Insectes Sociaux* **27/3** (1980) 237-264

Conclusions

Model of Active Brownian Particles:

covers important features of biological motion

- take-up, storage and conversion of energy
- dissipation / metabolism
- active motion (acceleration)
- interaction with other particles / environment
- deterministic / stochastic influences

Examples:

- non-equilibrium velocity distribution
- uphill motion and directed transport
- intermittent (active/passive) “behavior” in the presence of localized energy sources
- coherent motion (“swarming”)
- directed motion (“foraging behavior”)

Advantages:

- particle-based stochastic approach to complex motion and interactive structure formation
- gradual transition from “physical” to “biological” phenomena
- can be extended towards biological Multi-Agent Systems: *example:* information exchange in biological “many-particle” systems