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# **Modelling Spatial Urban and Economic Aggregation Bottum Up**

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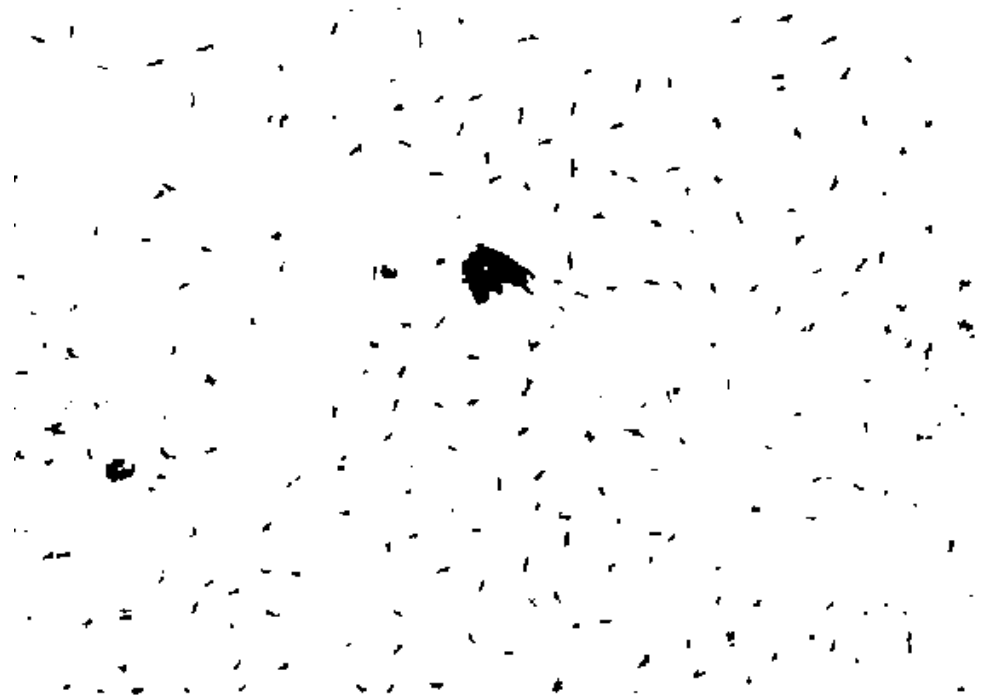
# **Schedule**

1. Modelling Urban Cluster Distribution
2. Spatial Information Field
3. Example: Aggregation by positive Feedback
4. Example: Urban Growth
5. Example: Economic Agglomeration
6. Conclusions

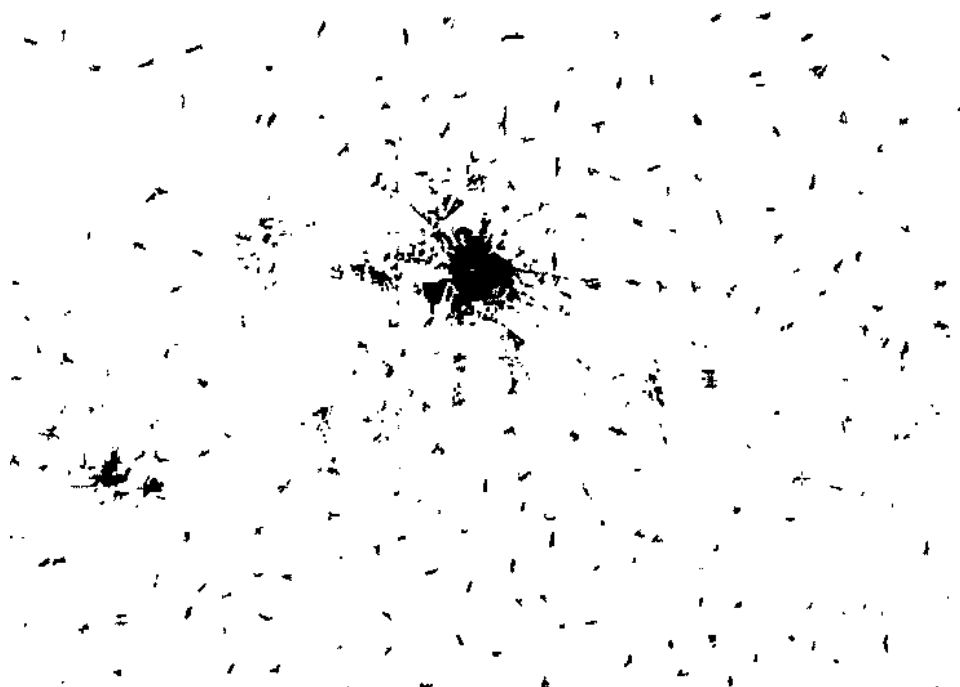
# Urban Growth

**Example: Berlin 1800 - 1945**

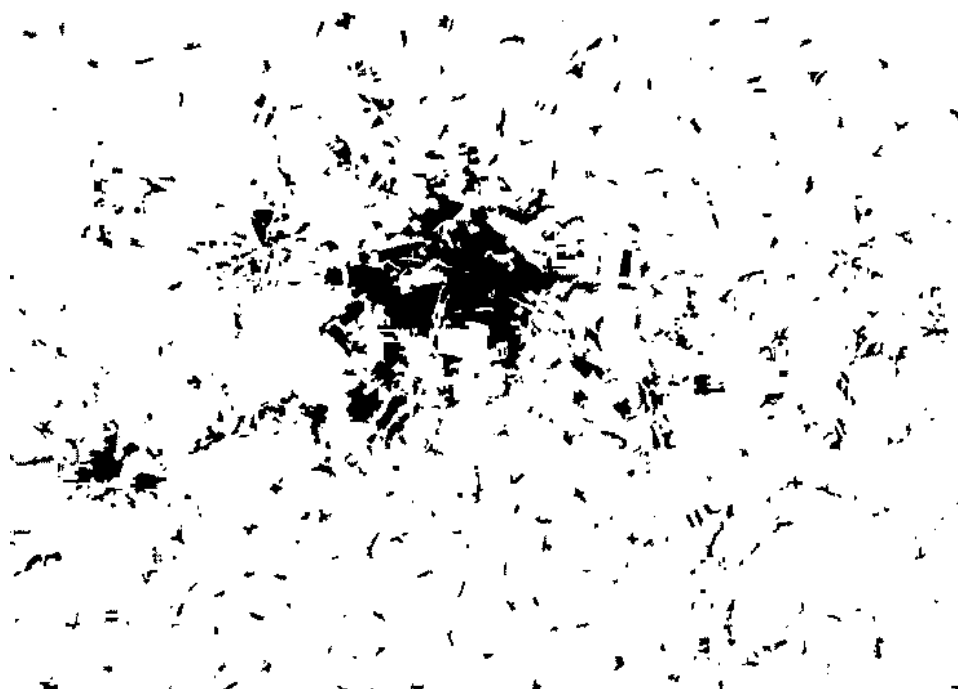
1800



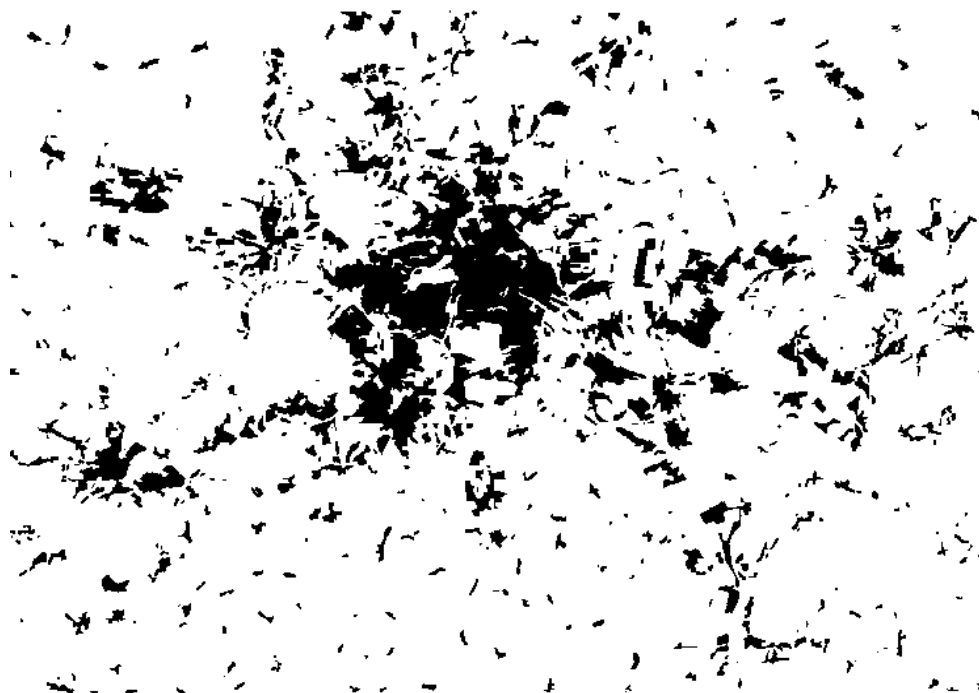
1875



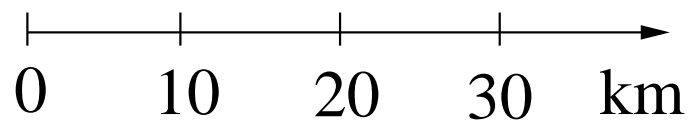
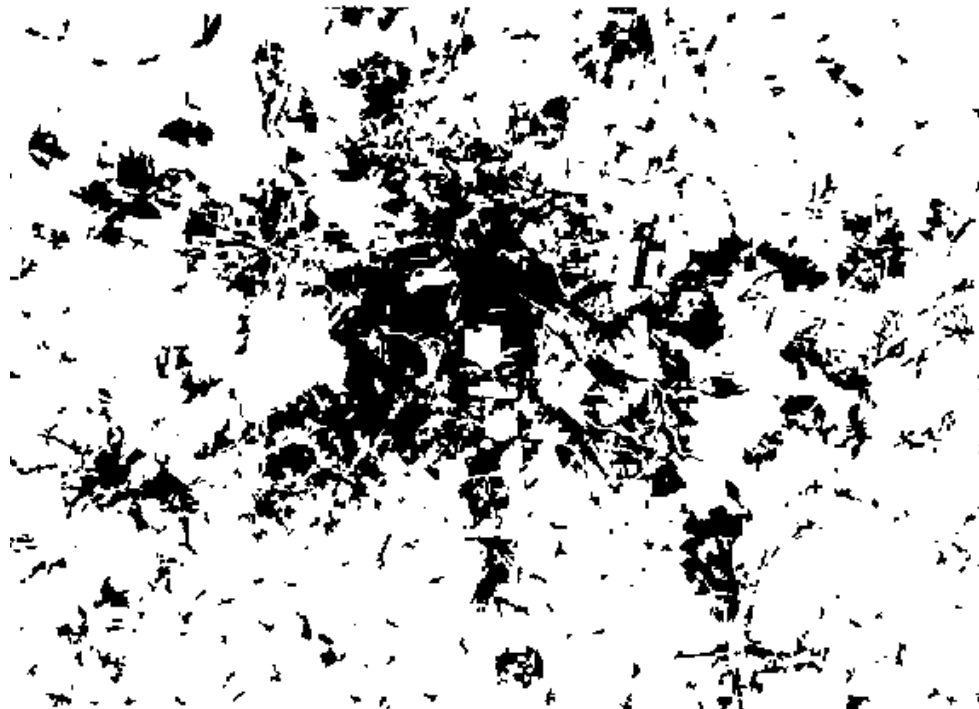
1910



1920



1945

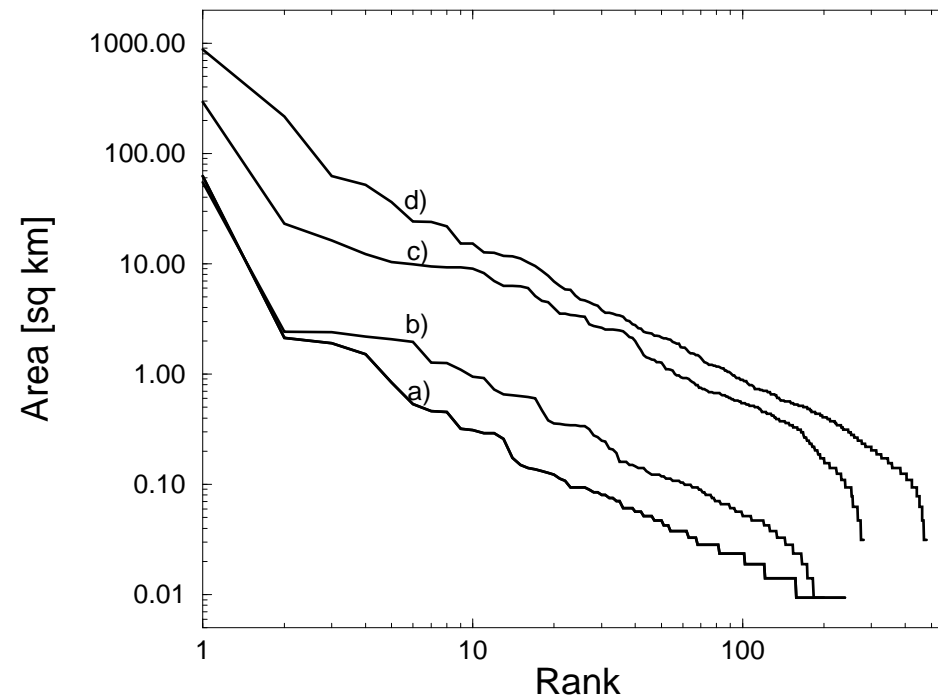




## Urban Growth: Common Features

- morphological transition: compact  $\Rightarrow$  fractal  
calculation of fractal dimension
- urban aggregate composed of many sub-clusters  
calculation of rank-size distribution

## Rank-Size Cluster Distribution



(a) Daegu (1988), (b) Munich (1965) (c) Moscow (1980), (d) Philadelphia (1980)

## Conclusion

- ▶ evolution of urban aggregates towards PARETO distribution  
⇒ aggregate hierarchically composed of clusters of all sizes
- ▶ deviations from PARETO distribution:  
⇒ *structural resources for potential development*

## Master Equation Approach to Urban Growth

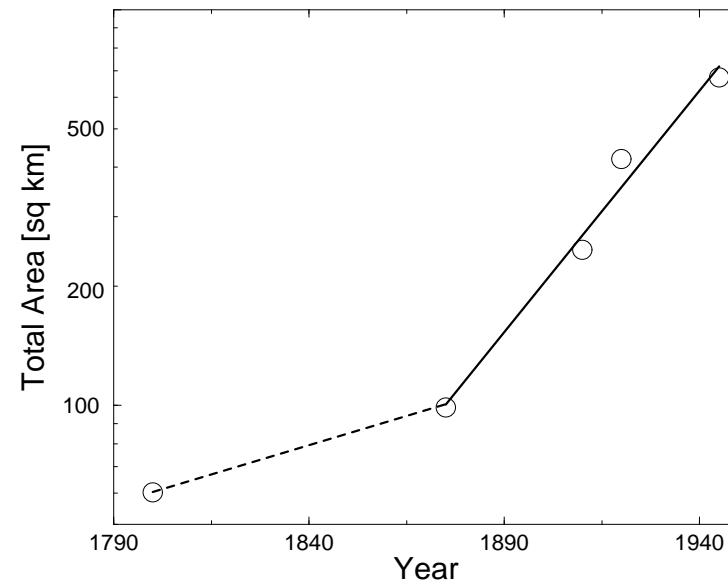
- ▶ distribution of clusters of different sizes:

$$n_1, n_2, \dots, n_k, \dots, n_A$$

$n_k$ : size (number of pixels) of cluster  $k$  ( $k=1, \dots, A$ )

- ▶ total mass:  $N_{tot}(t) = \sum_{k=1}^A n_k$

total growth: almost exponentially



## Two elementary processes:

### 1. *Formation of new clusters*

$$A \xrightarrow{w_1} A + 1$$

$$w_1 = w(A + 1, t + 1 | A, t) = c(N_{tot})$$

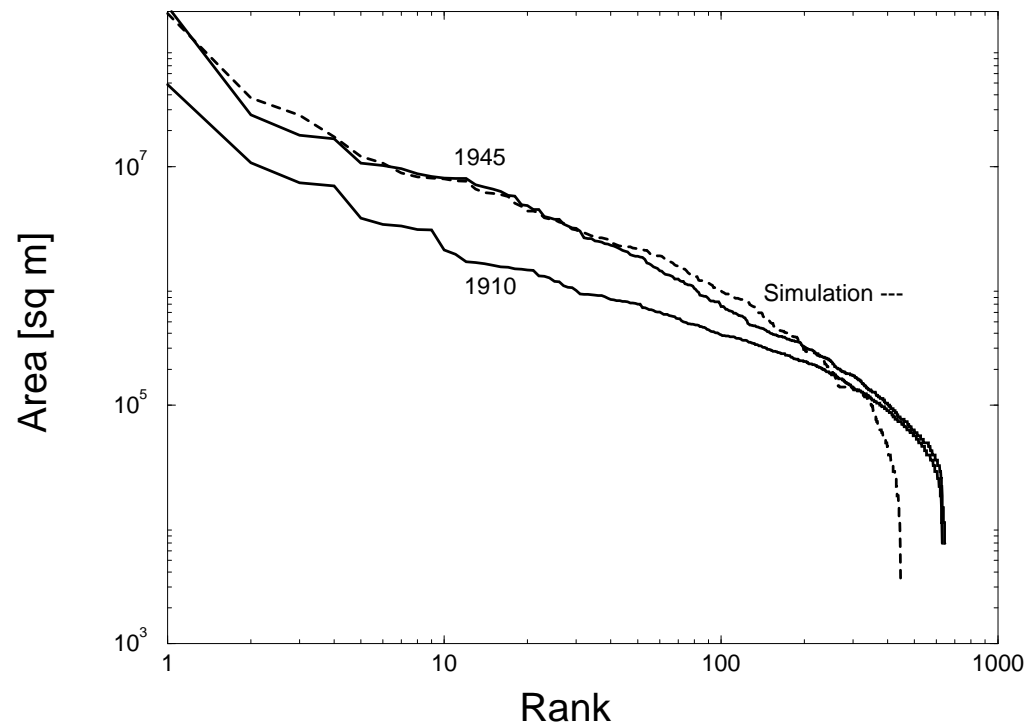
assumption:  $c = const.$

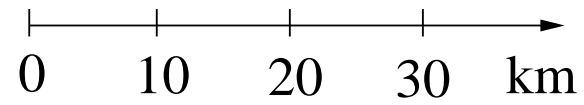
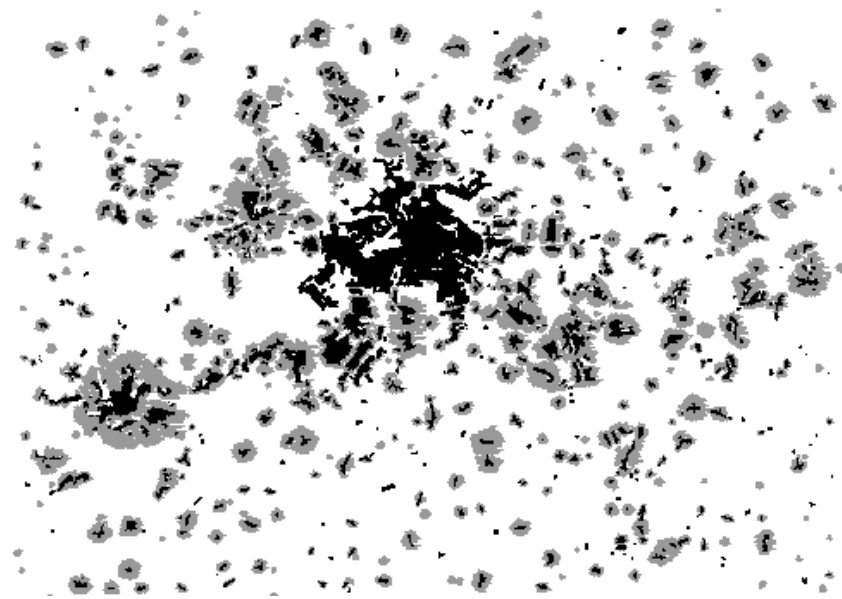
### 2. *Growth of existing clusters*

$$w_k = w(n_k + 1, t + 1 | n_k, t) = \gamma \frac{n_k}{N_{tot}}$$

$$\gamma = 1 - c(N_{tot})$$

## Results of Computer Simulations: Berlin 1910 - 1945







## Conclusion

- ▶ sub-cluster of rank 1 grows only by *coagulation*  
reason: *shift of growth zones* towards outer regions
- ▶ existence of *spatial correlations*:  
attraction (local growth)  $\iff$  repulsion (stop of growth)

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How to include spatial correlations ?

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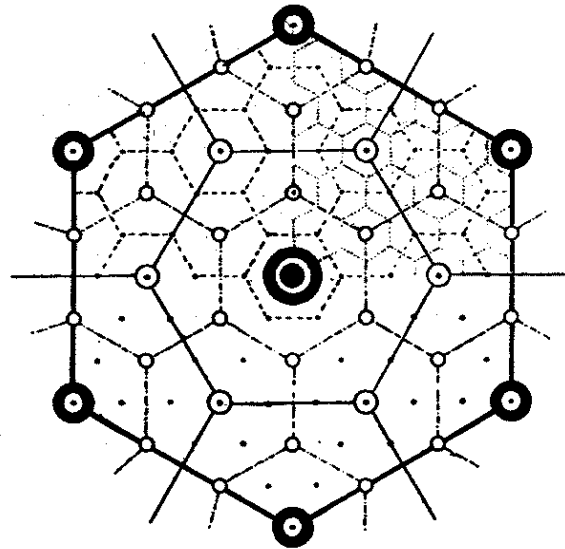
## Problem:

How to include spatial correlations ?

## Answer:

Distribution of spatial information !

# Central Place Theory



Walter Christaller: *Die zentralen Orte in Süddeutschland.*

Eine ökonomisch-geographische Untersuchung über die Gesetzmäßigkeit der Verbreitung und Entwicklung der Siedlungen mit städtischen Funktionen, Jena: Fischer, 1933  
(Reprint: Darmstadt: Wissenschaftliche Buchgesellschaft, 1980)

English translation by C.W. Baskin:

*Central Places in Southern Germany*, London: Prentice Hall, 1966

## Conclusion

- ▶ number and size of locations:
  - ⇒ hierarchical structure
  - ⇒ PARETO-like rank-size distribution
  
- ▶ spatial distribution of locations :
  - ⇒ depends on spatial correlations

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*Distribution of spatial information !*

## Spatial Information Field

$$\frac{\partial}{\partial t} h_{\theta}(r, t) = \sum_{i=1}^N q_i(\theta_i, t) \delta_{\theta, \theta_i} \delta(r - r_i) - k_{\theta} h_{\theta}(r, t) + D_{\theta} \Delta h_{\theta}(r, t)$$

### multi-component spatio-temporal field:

1. **production** of information:  
permanent *local* individual contribution:  $q_i(\theta_i, t)$
2. **distribution** of information: diffusion-like process,  $D_{\theta}$   
 $\Rightarrow$  determines how fast information is distributed
3. **memory effects**:  
information generated has a certain life time,  $1/k_{\theta}$
4. **different kind** of information  $\Rightarrow \theta$

## **Who is producing the information ?**

**Agents** (individuals, firms, existing urban aggregations, ...)



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## **What is the response ?**

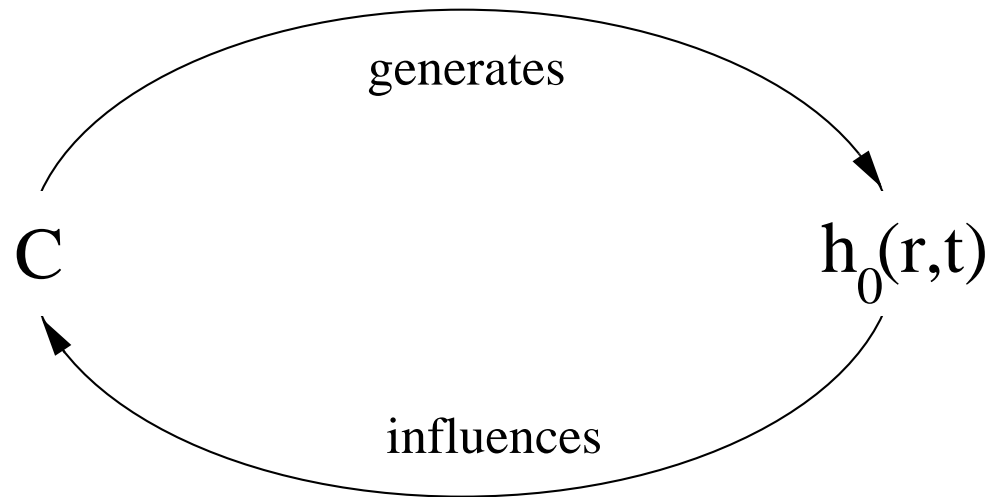
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## Example: Aggregation by Positive Feedback

migration of agents:

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{f}(\mathbf{r}_i) + \sqrt{2D} \boldsymbol{\xi}_i(t) ; \quad \mathbf{f}(\mathbf{r}_i) = \left. \frac{\partial h_0(\mathbf{r}, t)}{\partial \mathbf{r}} \right|_{\mathbf{r}_i}$$

$\mathbf{f}(\mathbf{r}_i)$ : guiding force, *local* influence

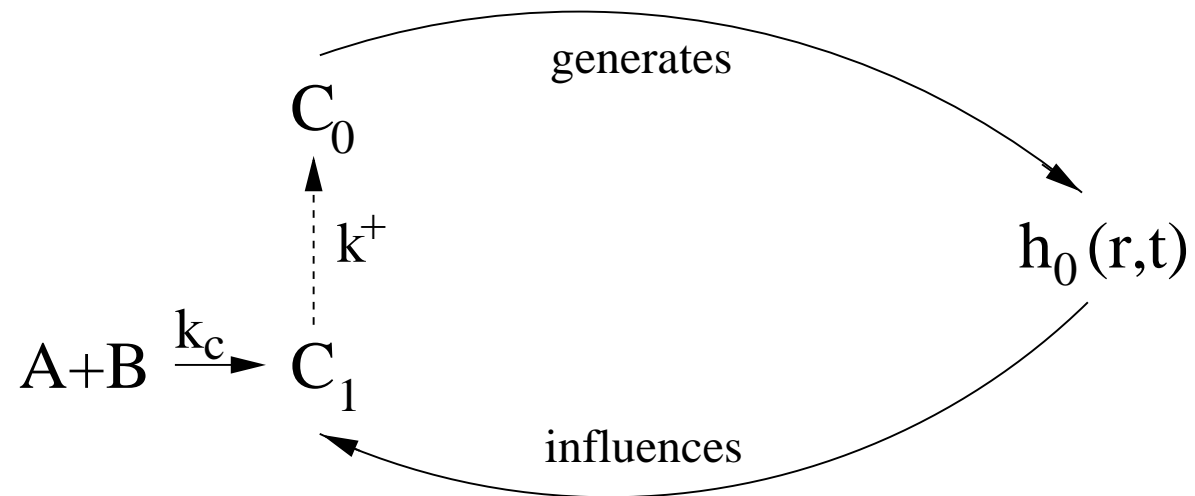


## Conclusion:

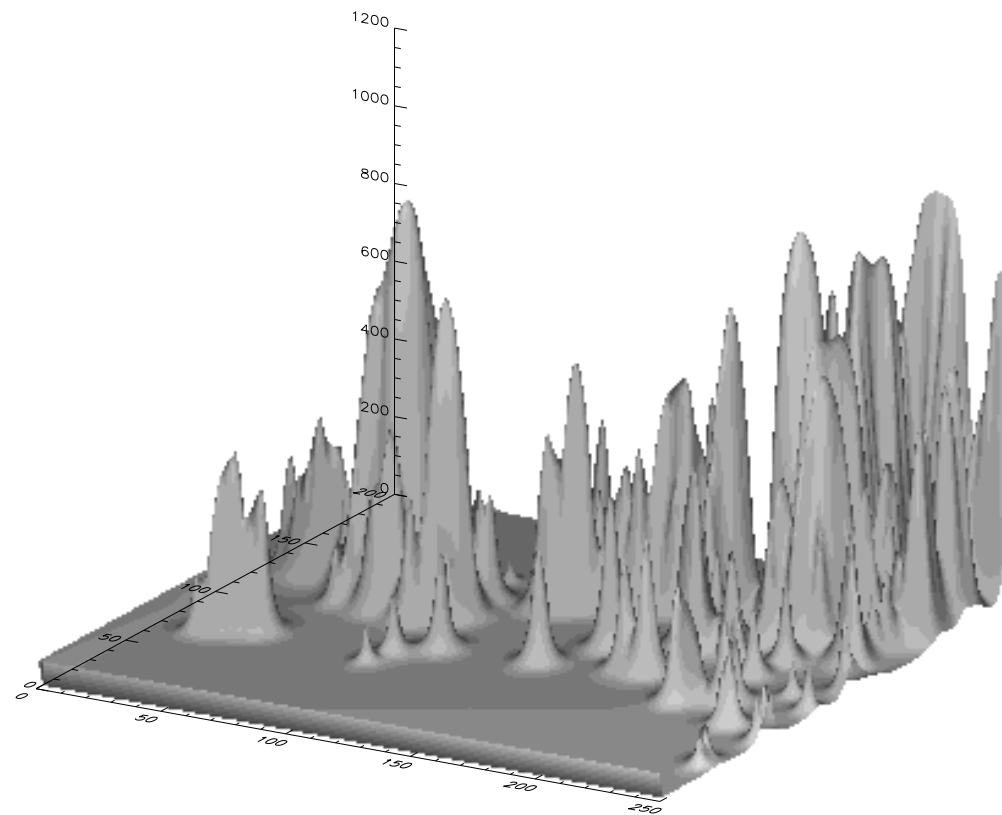
- ▶ positive feedback *only*  $\Rightarrow$  occurrence of multiple centers
- ▶ competition between centers: *ONE* big lump
- ▶ What about coexistence between aggregates ?  
Solution: **Inclusion of counteractive forces !**

## Example: Urban Growth

- ▶ non-linear feedback between existing aggregation ( $C_0$ ) and its further growth
- ▶  $C_0$ : generates spatial information  
“attraction potential”  $\Rightarrow h_0(\mathbf{r}, t)$
- ▶ “growth units” ( $C_1$ ) respond to attraction potential
- ▶ demand for further growth ( $B$ ) has to match existing free space ( $A$ )

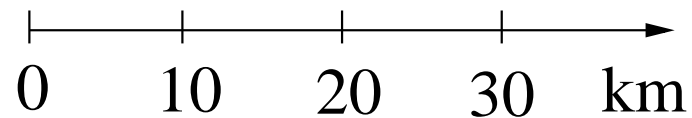
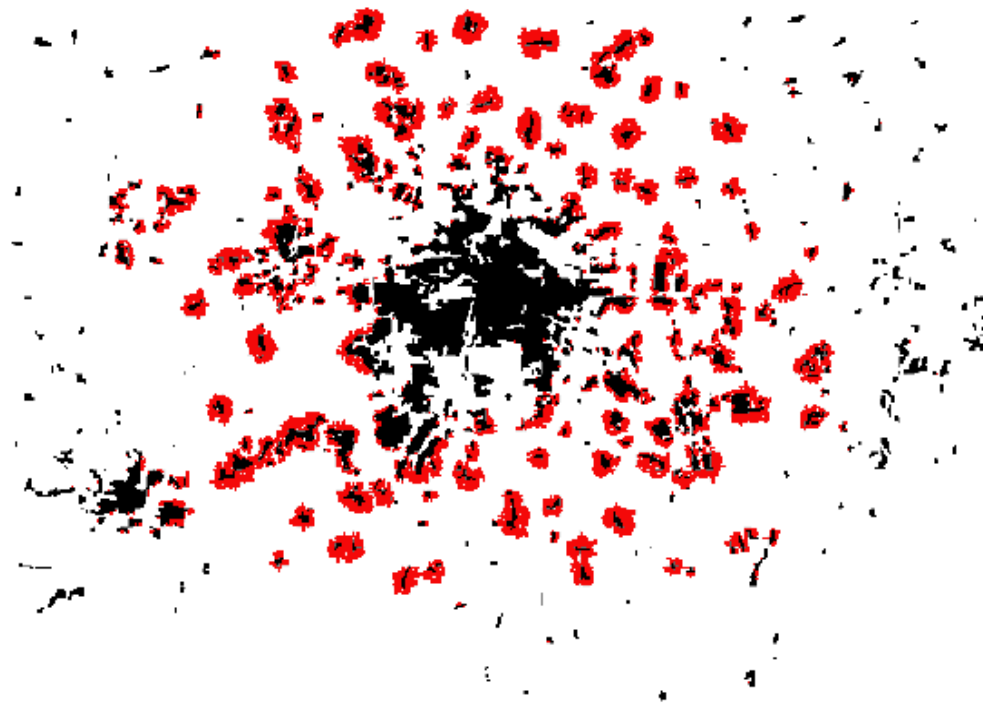


## Urban attraction field (Berlin/Potsdam area 1910)



## Result of Computer Simulations (Berlin 1910 - 1920)

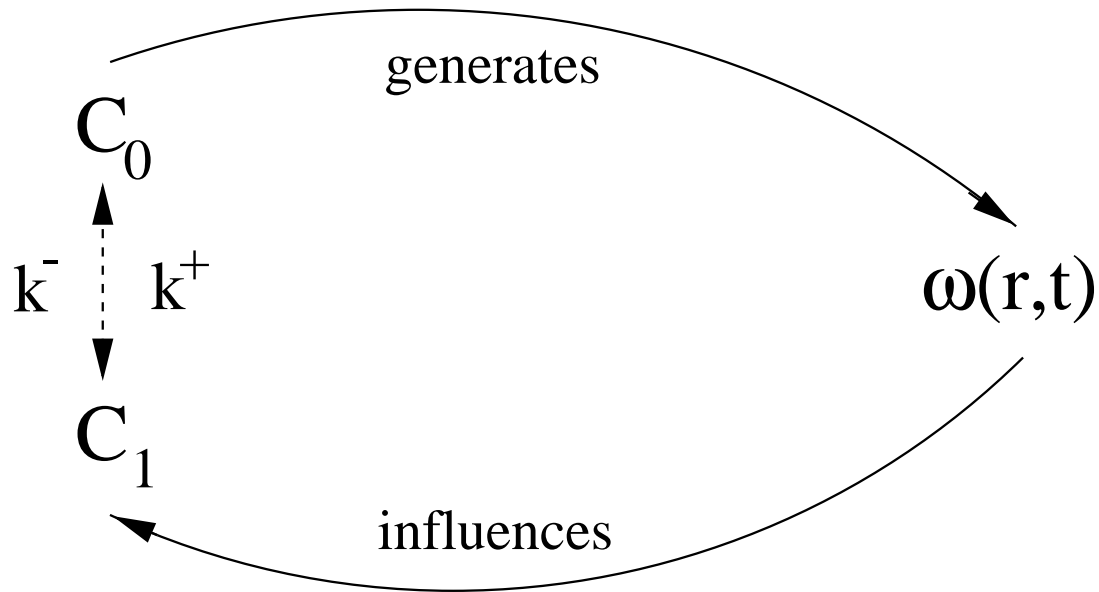
- ▶ shift of growth zones towards outer regions prevents urban collapse





# **Example: Economic Agglomeration**

## Example: Economic Agglomeration



- $\theta = 0$ : *employed agents*,  $C_0$  (immobile)  $\Rightarrow l(\mathbf{r}, t)$
- $\theta = 1$ : *unemployed agents*,  $C_1$  (mobile)  $\Rightarrow n(\mathbf{r}, t)$
- migration due to spatial wage differences:  $\omega(r)$
- “hiring” and “firing”:  $k^+$ ,  $k^-$

## Economic Assumptions

- ▶ wage: marginal product of labor:

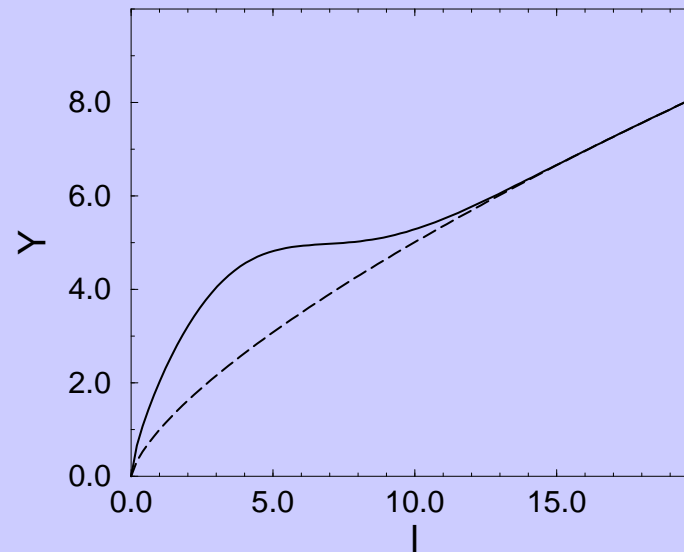
$$w\{l(\mathbf{r}, t)\} = \frac{\delta Y\{l(\mathbf{r}, t)\}}{\delta l}$$

- ▶ Cobb-Douglas production function

$$Y\{l(r, t)\} = A l^\beta(r, t), \quad \beta < 1$$

- ▶  $A$ : represents level of productivity  
considers *cooperative effects* resulting from interactions  
among the workers  $\Rightarrow$  non-linear function

$$Y(l) = \frac{\bar{A}}{2} [1 + \exp(a_1 l + a_2 l^2)] l^\beta$$



- ▶  $a_2 < 0$ : saturation effects  $\Rightarrow$  advantages of cooperative effects compensated by disadvantages of crowding

**“hiring” and “firing” rates:** $\omega^*$ : minimum wage

hiring rate  $k^+$ : firms hire workers as long as  $\frac{\delta Y}{\delta l} > \omega^*$   
(maximum profit condition)

$$k^+ = k^+ \{l(\mathbf{r}, t)\} = \eta \exp \left\{ \frac{\delta Y \{l(\mathbf{r}, t)\}}{\delta l} - \omega^* \right\}$$

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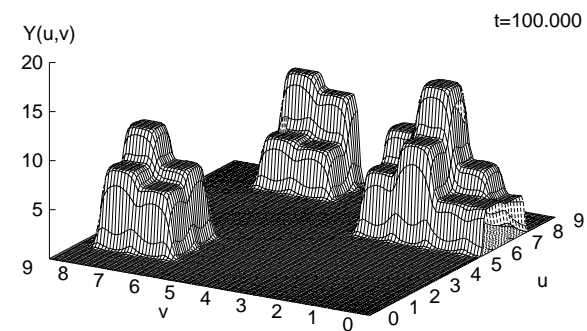
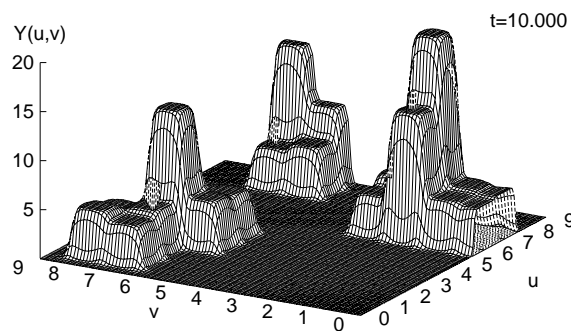
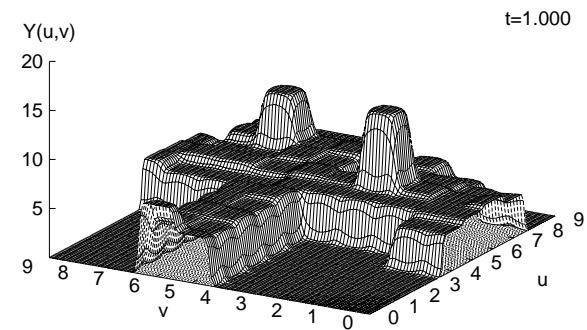
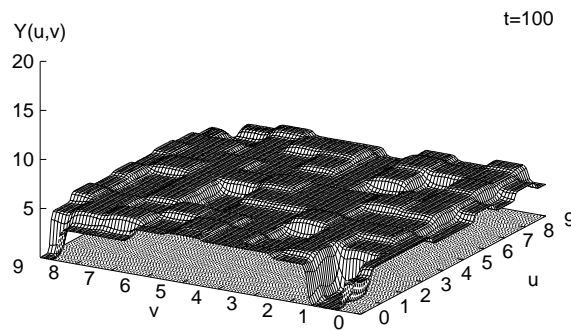
firing rate  $k^-$ : external and internal reasons

(i) workers are fired if  $\frac{\delta Y}{\delta l} < \omega^*$

(ii) workers can quit their job for better opportunities

$$k^- = k^- \{l(\mathbf{r}, t)\} = \eta \exp \left\{ - \left[ \frac{\delta Y \{l(\mathbf{r}, t)\}}{\delta l} - \omega^* \right] + c \frac{\partial \omega(\mathbf{r})}{\partial r} \right\}$$

# Spatial distribution of production



## Results of Computer Simulations

$t = 0$  : random initial distribution

$t = 100.000$  : distinct *extended major economic regions*

**(i)  $t < 1.000$  :**

- ▶ coexistence of numerous small economic centers  
basis: cooperative effects, mutual stimulations

**(ii)  $t > 1.000$  :**

- ▶ some small centers overcome economic bottleneck  
⇒ increase of marginal output
- ▶ competition: *local* attraction of labor force at the expense of the former small economic centers



## **Final stage: distinct extended economic regions**

(i) *stable coexistence* of the major economic regions

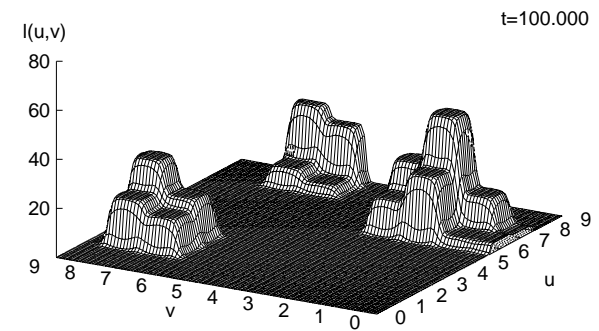
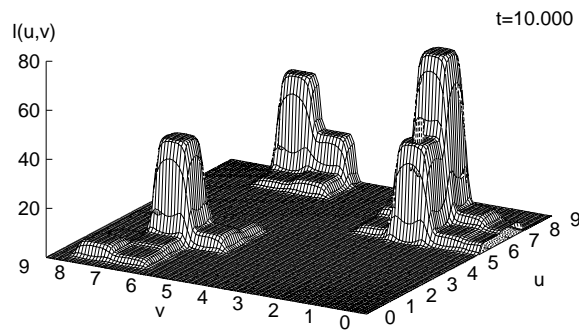
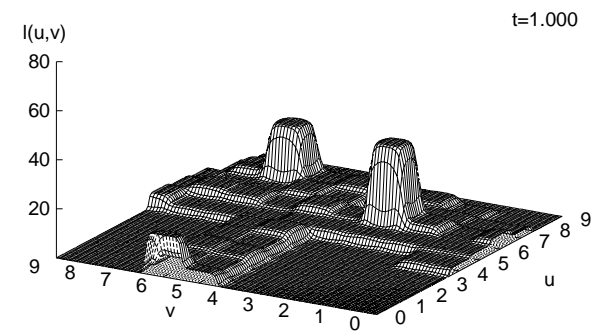
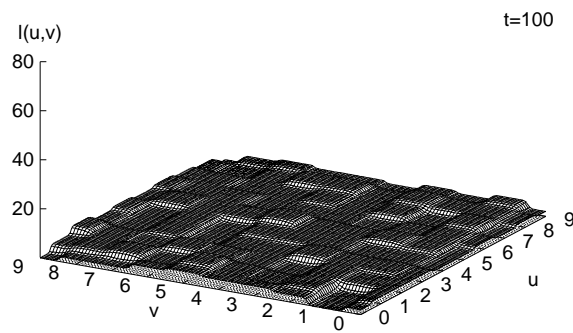
reason: *critical distance*

- ▶ each economic center has its own attraction/supply area  
⇒ prediction of the *central place theory*

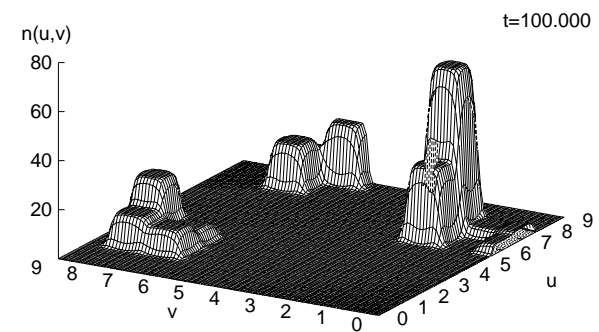
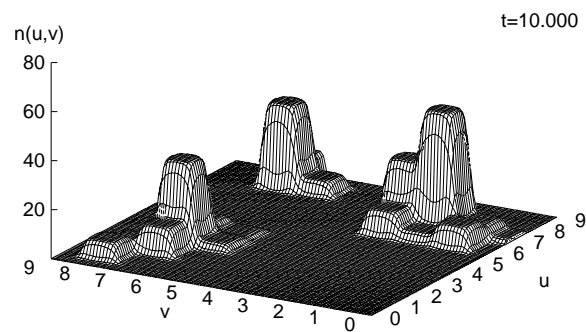
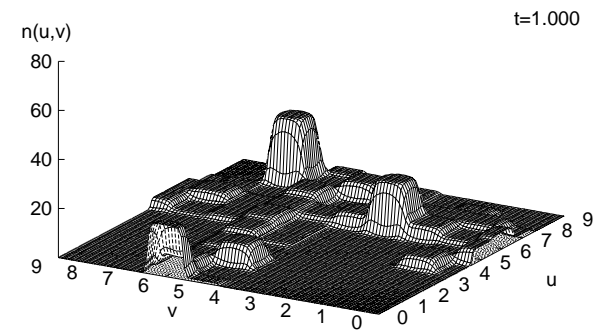
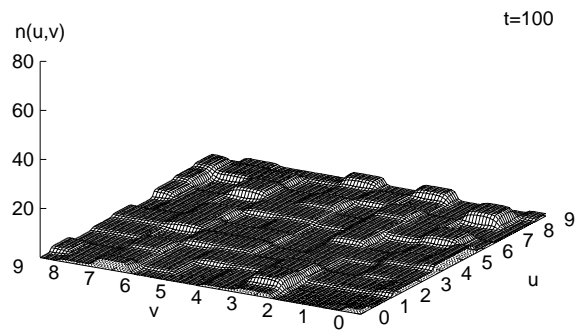
## Final stage: distinct extended economic regions

- (i) *stable coexistence* of the major economic regions  
reason: *critical distance*
  - ▶ each economic center has its own attraction/supply area  
⇒ prediction of the *central place theory*
  
- (ii) *quasi-stationary non-equilibrium* within the major economic regions
  - ▶ each economic region consists of some *subregions*  
centers do not have same number of employed agents  
⇒ still follow a *stochastic eigendynamics*

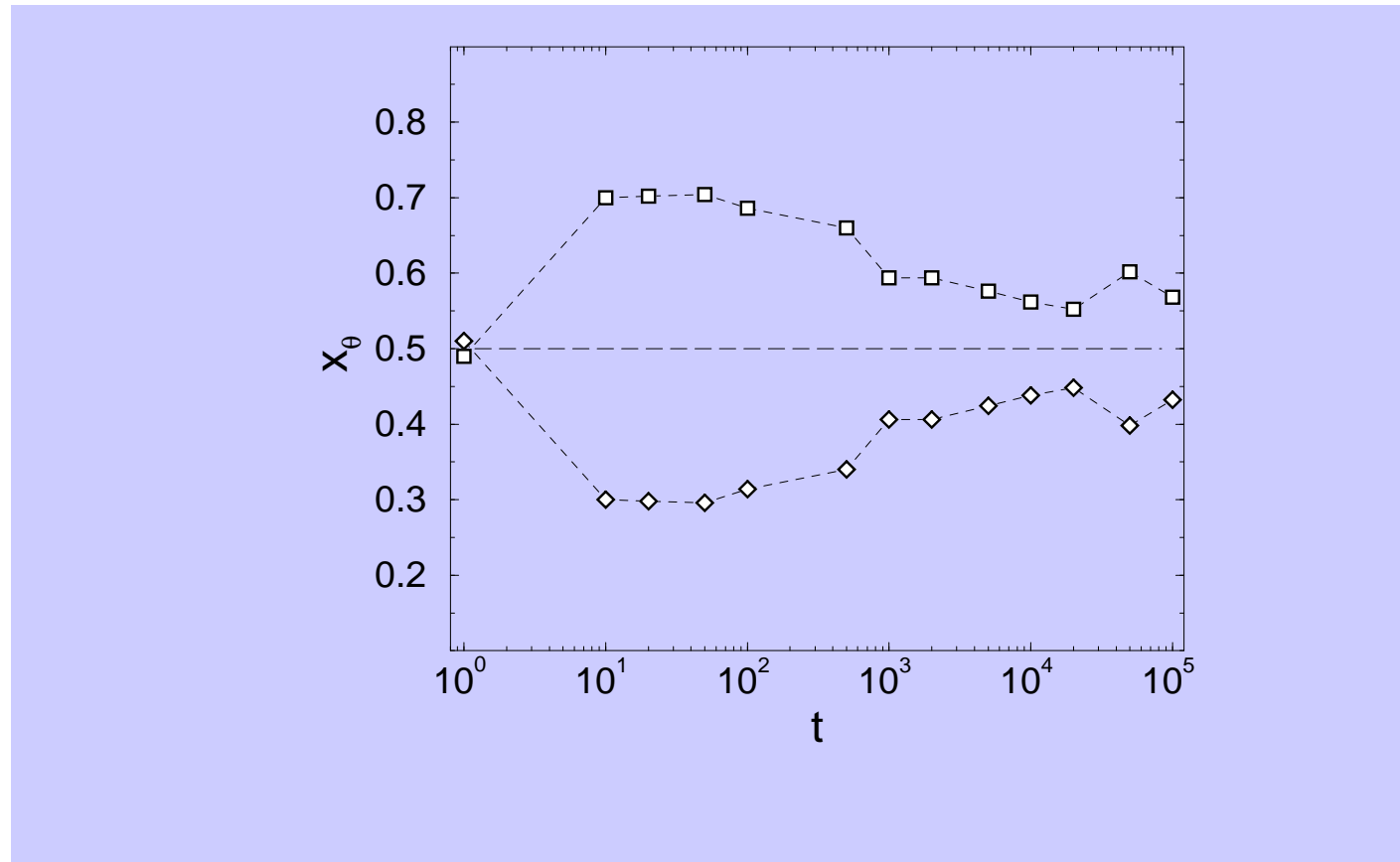
# Spatial density of employed agents



# Spatial density of unemployed agents



**Total share  $x_\theta = N_\theta/N$**



employed agents: ( $\square$ )

unemployed agents ( $\diamond$ )

# **Spatio-Temporal Evolution of Unemployment**

## **(i) small scale production**

- ▶ significant higher share of employed agents ( $\sim 70$  percent)  
broadly distributed

## **(ii) large scale production**

- ▶ increase of wage  $\Rightarrow$  affects migration of unemployed agents  
concentrate in the productive regions
- ▶ important for the further growth: agents to hire

# Spatio-Temporal Evolution of Unemployment

## (i) small scale production

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broadly distributed

## (ii) large scale production

- increase of wage  $\Rightarrow$  affects migration of unemployed agents  
concentrate in the productive regions
- important for the further growth: agents to hire
- concentration of employed *and* unemployed agents in the  
*same* regions
- new (larger) centers employ 60 percent of the agents  
 $\Rightarrow$  *increase of unemployment*

## Conclusions

- *Agents* (individuals, firms, urban “growth units”, ...)
- communicating via a *spatial information field*  
local production, distribution of information,  
memory effects



## Conclusions

- *Agents* (individuals, firms, urban “growth units”, ...)
- communicating via a *spatial information field*  
local production, distribution of information,  
memory effects
- *minimalistic* multi-agent system:
  - agents: simplex - not complex
  - producing / receiving “information”  $h(\mathbf{r}, t)$
  - action: local concentration / aggregation
  - path dependence  
further action is “enslaved” by previous history

## System's level:

- ▶ emergence of complexity
  - ⇒ complex spatial patterns
  - ⇒ local competition, global coexistence
- ▶ adaption to changes in the environment (resources)

Reason: *Self-Organization*

solutions result from the non-linear interaction

boundary conditions (semi-structured environment)

## Characteristics of the approach:

- ▶ *non-deterministic*  
random events (fluctuations) play a considerable role
- ▶ *non-finalistic*  
final (global) solutions cannot be predicted from local interactions  $\Rightarrow$  solutions *emerge*  $\Rightarrow$  *path dependent*
- ▶ *bottom-up approach*: create a solution  $\Rightarrow$  self-organization  
*top-down approach*: design a solution  $\Rightarrow$  planning

## **Self-Organization**

*Self-Organization is the process by which individual subunits achieve, through their cooperative interactions, states characterized by new, emergent properties transcending the properties of their constitutive parts.*

Biebricher, C. K.; Nicolis, G.; Schuster, P.: Self-Organization in the Physico-Chemical and Life Sciences, EU Report 16546 (1995)

*Self-organization is defined as spontaneous formation, evolution and differentiation of complex order structures forming in non-linear dynamic systems by way of feedback mechanisms involving the elements of the systems, when these systems have passed a critical distance from the statical equilibrium as a result of the influx of unspecific energy, matter or information.*

SFB 230 “Natural Constructions”, Stuttgart, 1984 - 1995