Fraunhofer Institut Autonome Intelligente Systeme

Coordination of Decisions in Spatial Multi-Agent Systems

Frank Schweitzer[1,2]

[1] Institut für Physik, Humboldt-Universität zu Berlin

[2] FhG Institut für Autonome Intelligente Systeme, Sankt Augustin

schweitzer@ais.fhg.de http://www.ais.fhg.de/~frank/

in collaboration with:

L. Behera, R. Höns, H. Mühlenbein, J. Zimmermann

Schedule

1. What is the problem?
2. Non-linear voter models (Cellular Automata)
3. Decisions based on information dissemination (Brownian agents)
4. Conclusions

Complex Systems

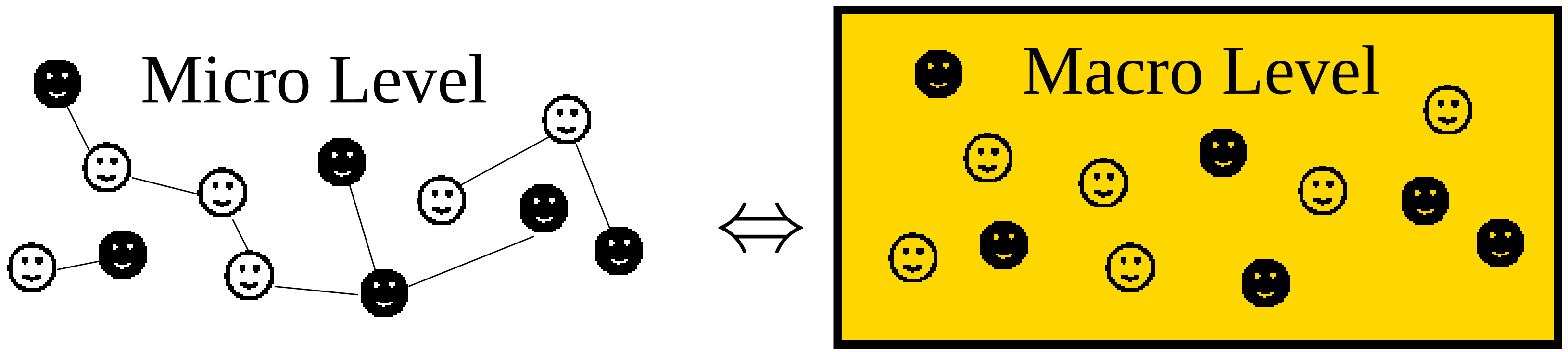
"Complex systems are systems with multiple interacting components whose behavior cannot be simply inferred from the behavior of the components. ...

New England Complex Systems Institute

"By complex system, it is meant a system comprised of a (usually large) number of (usually strongly) interacting entities, processes, or agents, the understanding of which requires the development, or the use of, new scientific tools, nonlinear models, out-of equilibrium descriptions and computer simulations."

Journal "Advances in Complex Systems"

Change of View



The micro-macro link:

How are the properties of the elements and their interactions ("microscopic" level) related to the dynamics and the properties of the whole system ("macroscopic" level)?

Lessons from Physics ?

- statistical theory of *many-particle systems*
  - microscopic properties: velocity, short-range interactions
  - macroscopic properties: conductivity, hardness ...
- *Synergetics*
  - subsystems commonly generate *order parameters*
    $\Rightarrow$ "enslaving principle" (circular causality)
  - collective effects, emergence of new qualities

Questions:

- Can we transfer methods/tools of statistical physics to *non-physical many-particle* systems?
- Can we derive a statistical theory of *multi-agent* systems?

Answer: YES ... but at what price?

Reasonable Reductions?

- 1^{st} (rigorous) solution:
  - reduce "humans" to "atoms": identical, "rational", ...
  - reduce complexity of interactions: time independent, identical, symmetrical, ...
  - successful examples in game theory, neoclassical economy, ...
- 2^{nd} (weaker) solution:
  - compromise between "realistic" and "solvable"
    $\Rightarrow$ focus on particular features, purposeful neglection
  - explore: how far can we get with "minimalistic" models?

Coordination of Decisions

- decisions: *basic* process (micro-economics, social system)
- based on *information*:
  economy: prices, quality, ...
  social system: harms and benefits, ...
  decision of other agents, ...
- classical approach: *rational agent*
  calculation of utility function
  common knowledge assumption
  dissemination of information: fast, loss-free, error-free
- *bounded rationality:*
  decisions based on incomplete (limited) information

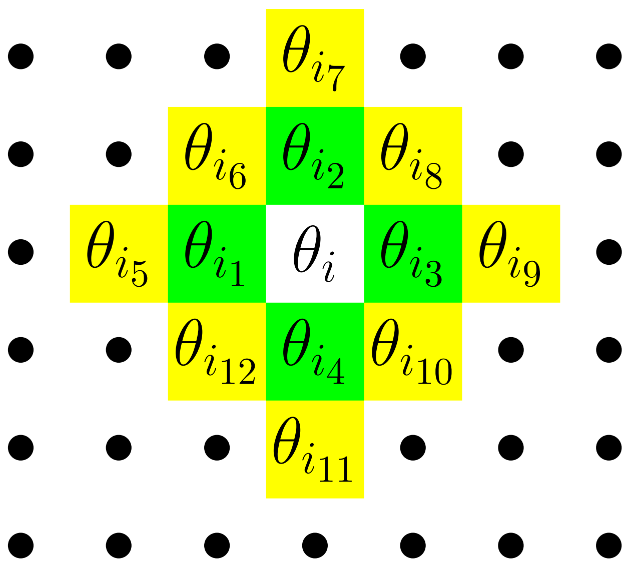
How to reduce the risk?

- *imitation strategies*
  biology, cultural evolution: adapt to the community
  economy: copy successful strategies
- *information contagion*: agents can observe payoffs
  transmission of two information: decision, payoff
  social percolation: hits and flops
- *herding behavior:* agents just imitate decisions without complete information about consequences
- our assumption: agent i more likely does what others do
  neighbourhood: spatial effects
  communication: exchange/lifetime of information

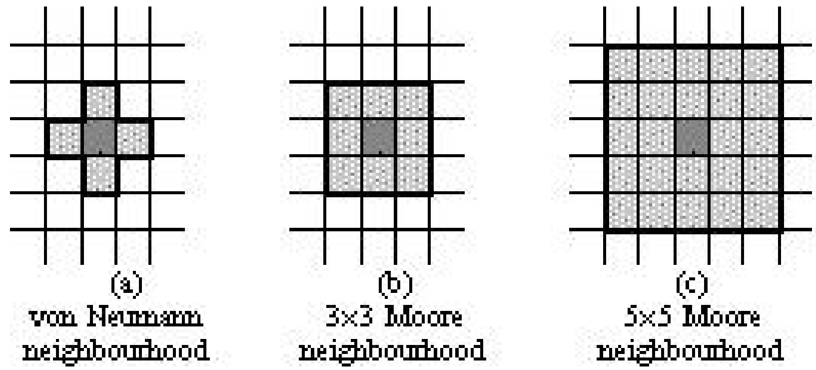
Non-linear Voter Models

- *assumtions:* voter's decision depends on neighborhood, only short-term memory
- *toy model* to investigate survival/extinction/coexistence of opinions
- interdisciplinary enterprise:
  linear voter model: domain of mathematical investigations
  relation to population biology, ecology
- our interest: investigation of spatial effects
  derivation of macro-dynamics from microscopic interactions

Cellular Automaton



- cell i with different states θ_i
- interaction with neighbors j



History: v. Neumann, Ulam (1940s), Conway (1970), Wolfram (1984), ...

Socio/Economy: Sakoda (1949/1971), Schelling (1969), Albin (1975), ...

Local Interaction Rules

- “frequency dependent process”: $\underline{\theta}_i \Rightarrow$ local frequency:

$$z_i^{\sigma} = \frac{1}{n}\sum_{j=0}^{n-1} \delta_{\sigma\theta_{i_j}} \; ; \qquad z_i^{(1-\sigma)} = 1 - z_i^{\sigma} \; ; \qquad \sigma \in \{0,1\}$$

- symmetric rules: same for $\theta_i \in \{0,1\}$

z_i^{σ}	$z_i^{(1-\sigma)}$	$w(1-\theta_i \vert \theta_i=\sigma, z_i^{\sigma})$
1	0	ϵ
4/5	1/5	α_1
3/5	2/5	α_2
2/5	3/5	$\alpha_3 = 1-\alpha_2$
1/5	4/5	$\alpha_4 = 1-\alpha_1$

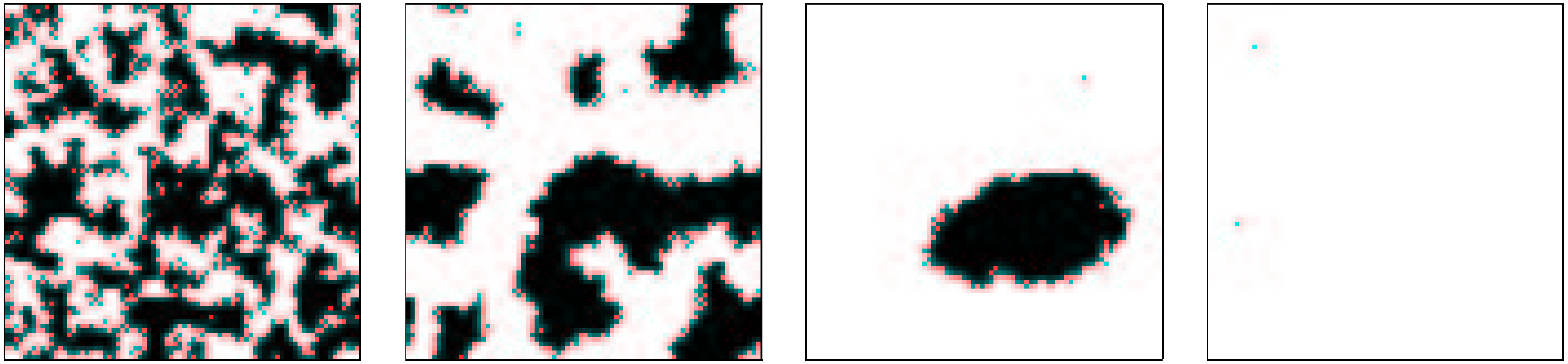
Results of Computer Simulations

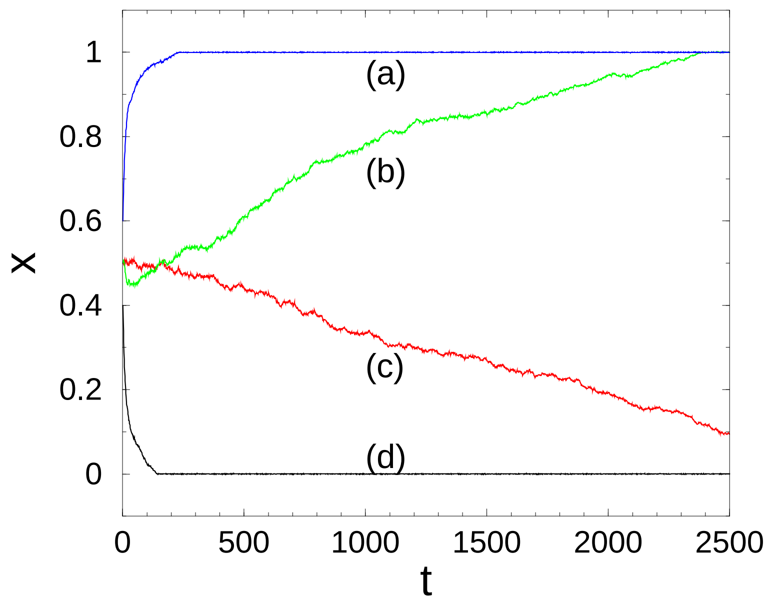
- initially $x = 0.5$, random distribution
- **Stochastic CA**

  Start Online-Simulation $\epsilon = 10^{-4}$, $\alpha_1 = 0.1$, $\alpha_2 = 0.3$

  - Result: coordination of decisions on medium time scales
    asymptotically: "no opposition"

$\epsilon = 10^{-4}$, $\alpha_1 = 0.1$, $\alpha_2 = 0.3$ $t = 10^1, 10^2, 10^3, 10^4$





Coexistence?

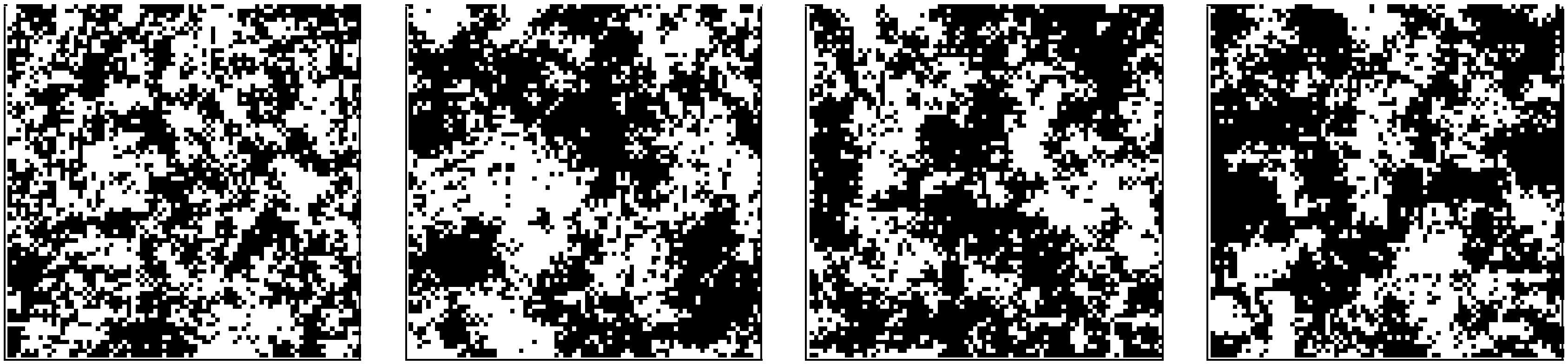
➤ Start Online-Simulation $\epsilon = 10^{-4}$, $\alpha_1 = 0.3$, $\alpha_2 = 0.4$

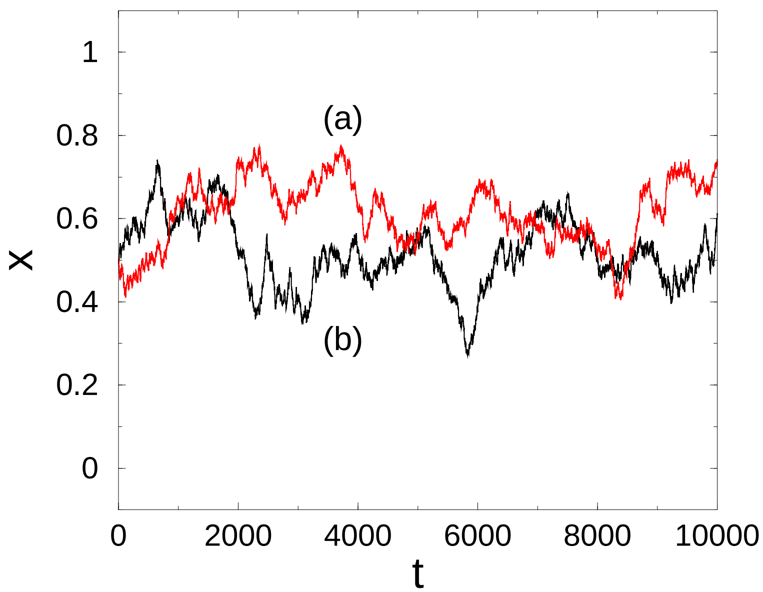
- Result: coexistence, but no spatial coordination

➤ Start Online-Simulation $\epsilon = 10^{-4}$, $\alpha_1 = 0.22$, $\alpha_2 = 0.3$

- Result: coordination of decisions on long time scales
  asymptotically: coexistence, but non-equilibrium

$\epsilon = 10^{-4}$, $\alpha_1 = 0.25$, $\alpha_2 = 0.25$ $t = 10^1, 10^2, 10^3, 10^4$

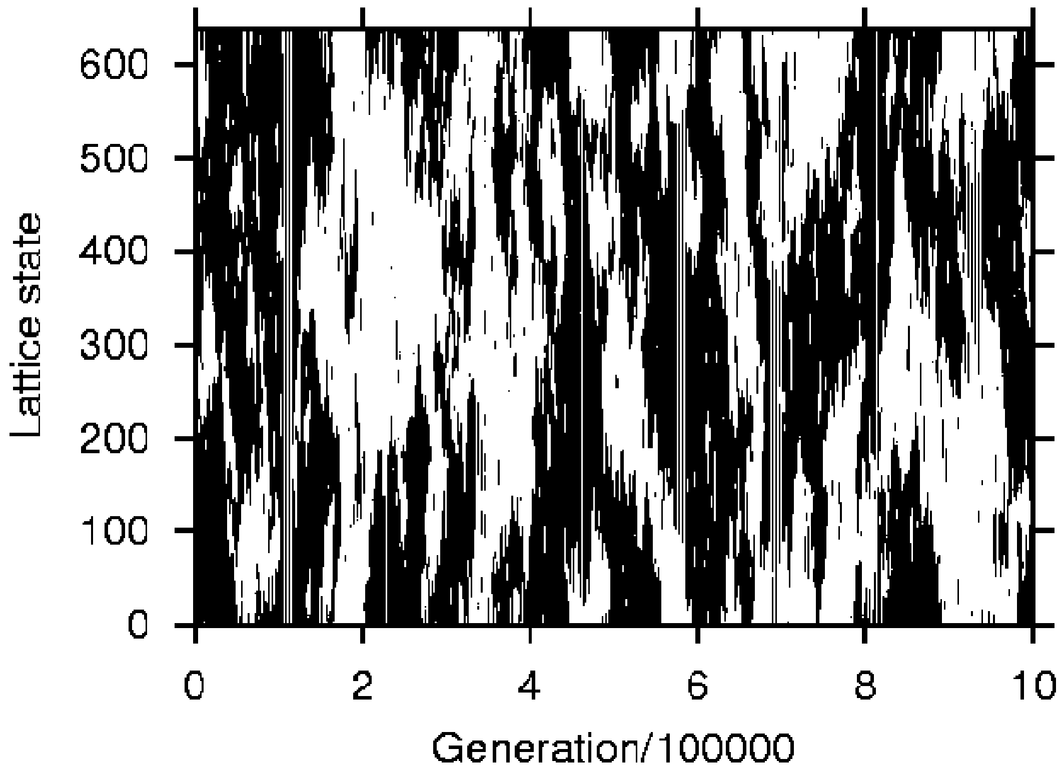




(a) $a_1 = 0.2$, $\alpha_2 = 0.4$
(voter model)

(b) $\alpha_1 = 0.25$, $\alpha_2 = 0.25$

1d CA:



long-term nonstationarity; temporal domination of one opinion

Two tasks:

- 1. define range of parameters for coexistence
- 2. describe spatial correlations between decisions

Microscopic Description

- individual i with "opinion" $\theta_i \in \{0, 1\}$
- "socio-configuration" $\mathbf{\Theta} = \{\theta_1, \theta_2, ..., \theta_N\}$
  state space $\Omega \sim 2^N$
- stochastic description: $p_i(\theta_i, t) = \sum_{\underline{\theta}'_i} p(\theta_i, \underline{\theta}'_i, t)$,
  local neighborhood: $\underline{\theta}_i = \{\theta_{i_1}, \theta_{i_2}, ..., \theta_{i_{n-1}}\}$
- Dynamics: one-step memory (Markov Process)
  transition rates: $w(1-\theta_i|\theta_i, \underline{\theta}_i)$; $w(\theta_i|(1-\theta_i), \underline{\theta}_i)$

➤ *Master equation:*

$$\frac{d}{dt}p_i(\theta_i,t) = \sum_{\underline{\theta}_i'} \Big[w(\theta_i|(1-\theta_i),\underline{\theta}_i')\; p(1-\theta_i,\underline{\theta}_i',t) - w(1-\theta_i|\theta_i,\underline{\theta}_i')\; p(\theta_i,\underline{\theta}_i',t) \Big]$$

Macroscopic Equations

- macro variable: $x_\sigma = \frac{N_\sigma}{N}$; $N = \sum_\sigma N_\sigma = N_0 + N_1 = \text{const.}$
  spatial correlation $c_{1|1}$

- expectation value: $\langle x(t) \rangle = \frac{1}{N} \sum_{i=1}^{N} p_i(\theta_i = 1, t)$

$$\frac{d}{dt} \langle x(t) \rangle = \sum_{\underline{\sigma}'} \Big[w(1|0, \underline{\sigma}') \, \big\langle x_{0,\underline{\sigma}'}(t) \big\rangle - w(0|1, \underline{\sigma}') \, \big\langle x_{1,\underline{\sigma}'}(t) \big\rangle \Big]$$

calculation of $\big\langle x_{\sigma,\underline{\sigma}'}(t) \big\rangle$: consideration of *all* possible $\underline{\sigma}'$ (!)

➤ **1st Approximation:** Mean-field limit
no spatial correlations

$$\langle x_{\underline{\sigma}^0} \rangle = \langle x_\sigma \rangle \prod_{j=1}^{m} \langle x_{\sigma_j} \rangle$$

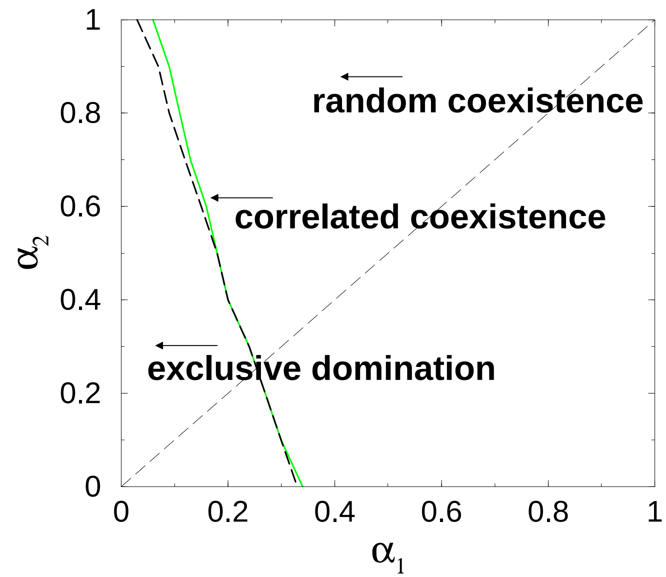
- tractable, but gives wrong results for the spatial case

- **2nd Approximation:** pair approximation
  estimation of spatial effects by considering pairs of nearest neighbor cells σ, σ'
- closed macroscopic dynamics: doublet frequency: $\langle x_{\sigma,\sigma'} \rangle$
  spatial correlation: $c_{\sigma|\sigma'} := \langle x_{\sigma,\sigma'} \rangle \,/\, \langle x_{\sigma'} \rangle$

$$\frac{d}{dt}\langle x(t) \rangle = \sum_{\underline{\sigma}'} \Big[\quad w(1|0,\underline{\sigma}')\,(1-\langle x \rangle) \prod_{j=1}^{m} c_{\sigma_j|\sigma} \\ - w(0|1,\underline{\sigma}')\,\langle x \rangle \prod_{j=1}^{m} c_{\sigma_j|(1-\sigma)} \Big]$$

$$\frac{dc_{1|1}}{dt} = -\frac{c_{1|1}}{\langle x \rangle}\frac{d}{dt}\langle x \rangle + \frac{1}{\langle x \rangle}\frac{d}{dt}\langle x_{1,1} \rangle \;; \qquad \frac{d\,\langle x_{1,1} \rangle}{dt} = \ldots$$

Phase Diagram for Coexistence

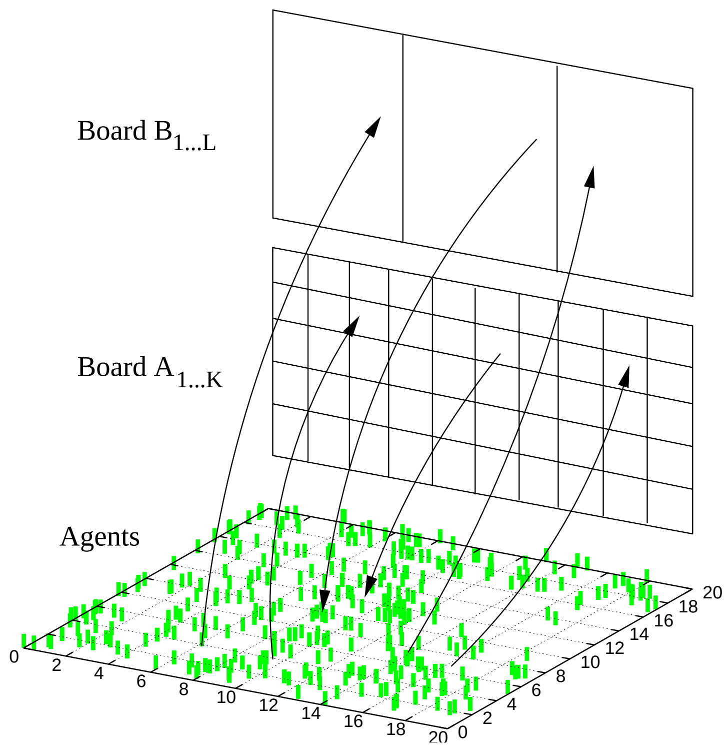


Results:

- simple local interaction model: coordination of decisions on medium/long time scales
- pair approximation predicts $\langle x(t) \rangle$ and $c_{1|1}(t)$
- missing: memory effects, dissemination of information

Spatial Communication via Tagboards

- arrays of "blackboards" with restricted access
  agents post/retrieve *locally* different information (tags)
- limited *lifetime* and *dissemination* of information



Spatial MAS of Communicating Agents

- N agents: position $\boldsymbol{r}_i \in \mathbb{R}^2$, “opinion” $\theta_i \in \{-1, +1\}$
- *binary choice*: depends on information $h_\theta(\boldsymbol{r}_i, t)$
- our assumption: agent i more likely does what others do

$$w(-\theta_i|\theta_i) = \eta \exp\left\{-\frac{h_\theta(\boldsymbol{r}_i,t) - h_{-\theta}(\boldsymbol{r}_i,t)}{T}\right\}$$

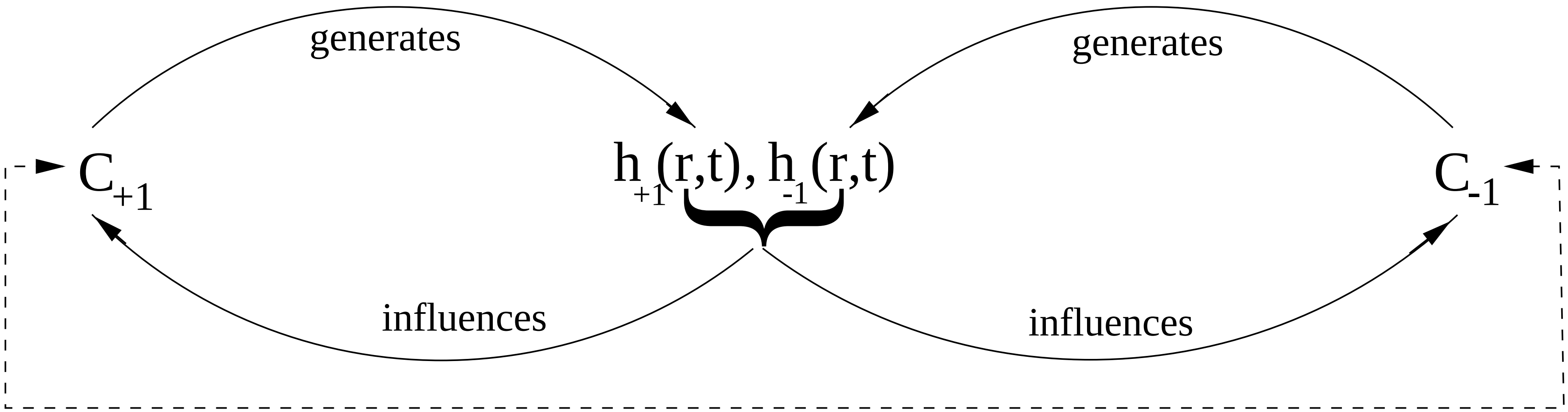
- tagboard $\Rightarrow$ spatio-temporal communication field

$$\frac{\partial}{\partial t} h_\theta(\boldsymbol{r},t) = \sum_{i=1}^{N} s_i\, \delta_{\theta,\theta_i}\, \delta(\boldsymbol{r}-\boldsymbol{r}_i) - k_\theta h_\theta(\boldsymbol{r},t) + D_\theta \Delta h_\theta(\boldsymbol{r},t)$$

neighbourhood: spatial effects

communication: exchange/lifetime of information

non-linear feedback:



Fast Information Exchange

- no spatial heterogeneity $\Rightarrow$ mean-field approach
- mean communication field: $(s_i \longrightarrow s_\theta)$

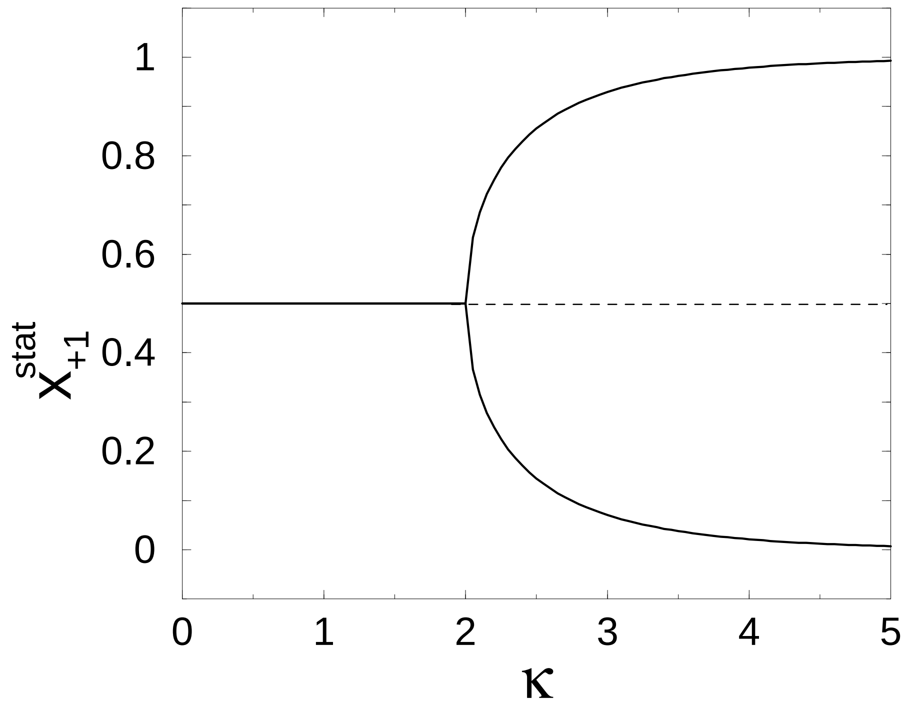
$$\frac{\partial \bar{h}_\theta(t)}{\partial t} = -k_\theta \bar{h}_\theta(t) + s_\theta \bar{n}_\theta$$

- subpopulations: $x_\theta(t) = N_\theta(t)/N$
- stationary states: $\dot{x}_\theta = 0$, $\dot{h}_\theta = 0$
  with $s_{+1} = s_{-1} \equiv s$, $k_{+1} = k_{-1} \equiv k$

$$(1 - x_{+1})\ \exp\left[\kappa\, x_{+1}\right] = x_{+1}\ \exp\left[\kappa\,(1 - x_{+1})\right]$$

  - bifurcation parameter: $\kappa = \dfrac{2s\,N}{A\,k\,T}$

Bifurcation diagram:

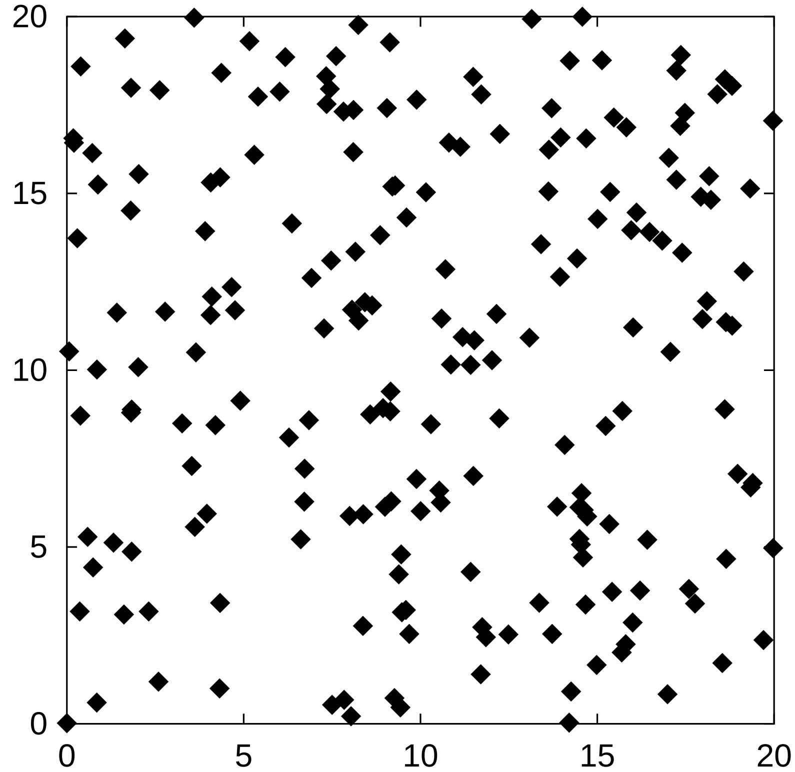


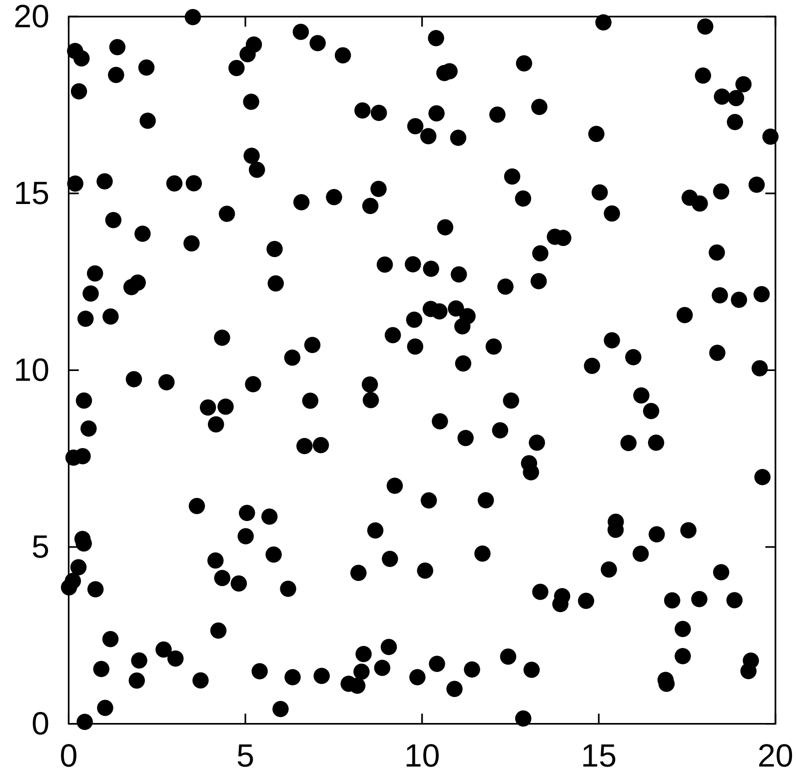
$$\kappa = \frac{2s\,N}{A\,k\,T} = 2 \Rightarrow \text{critical population size: } N^c = \frac{k\,A\,T}{s}$$

Emergence of minority and majority

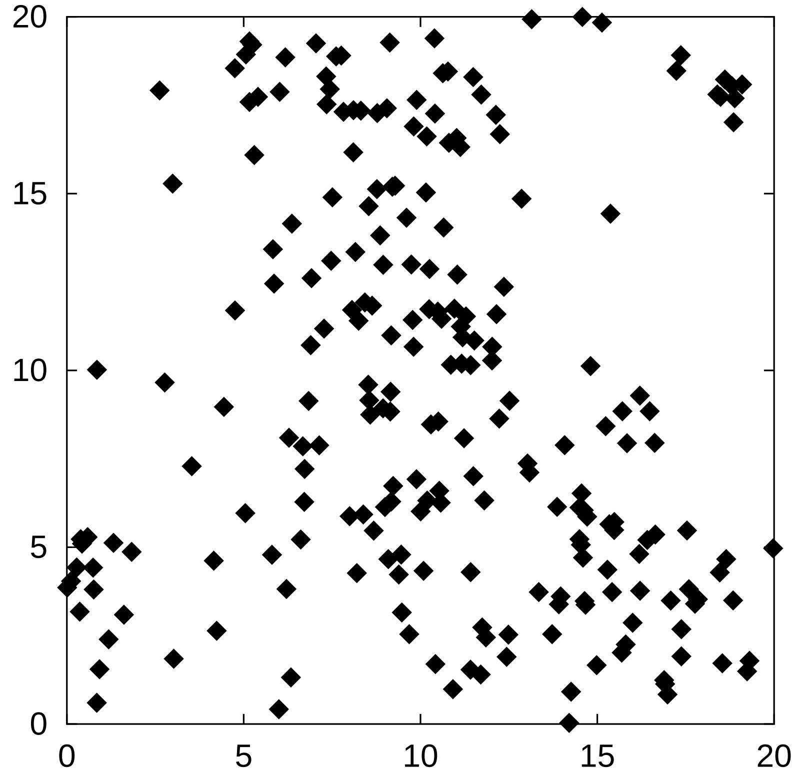
Spatial Influences on Decisions

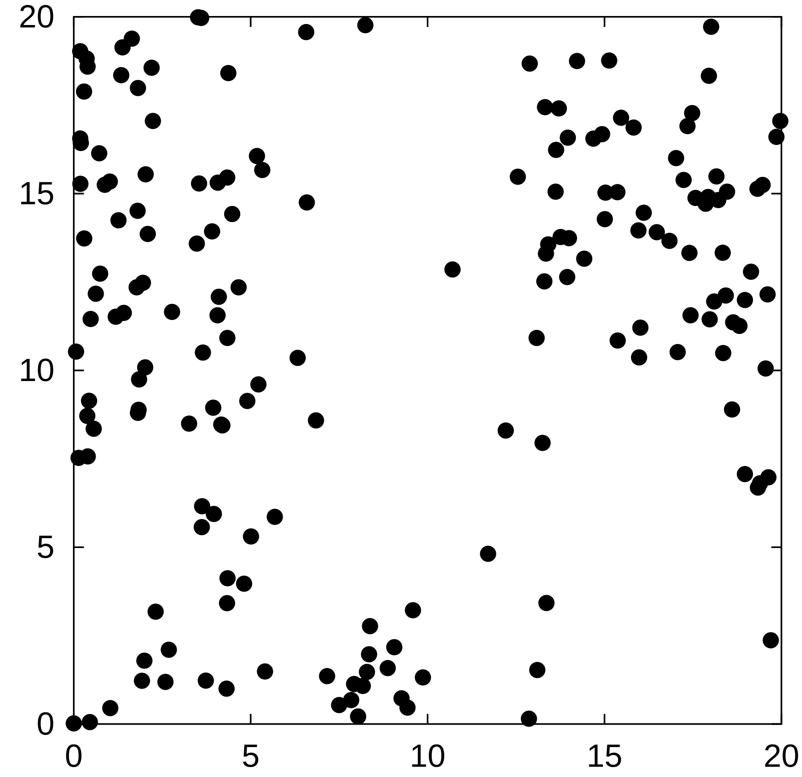
$$s_{+1} = s_{-1} \equiv s,\ k_{+1} = k_{-1} \equiv k,\ D_{+1} = D_{-1} \equiv D$$



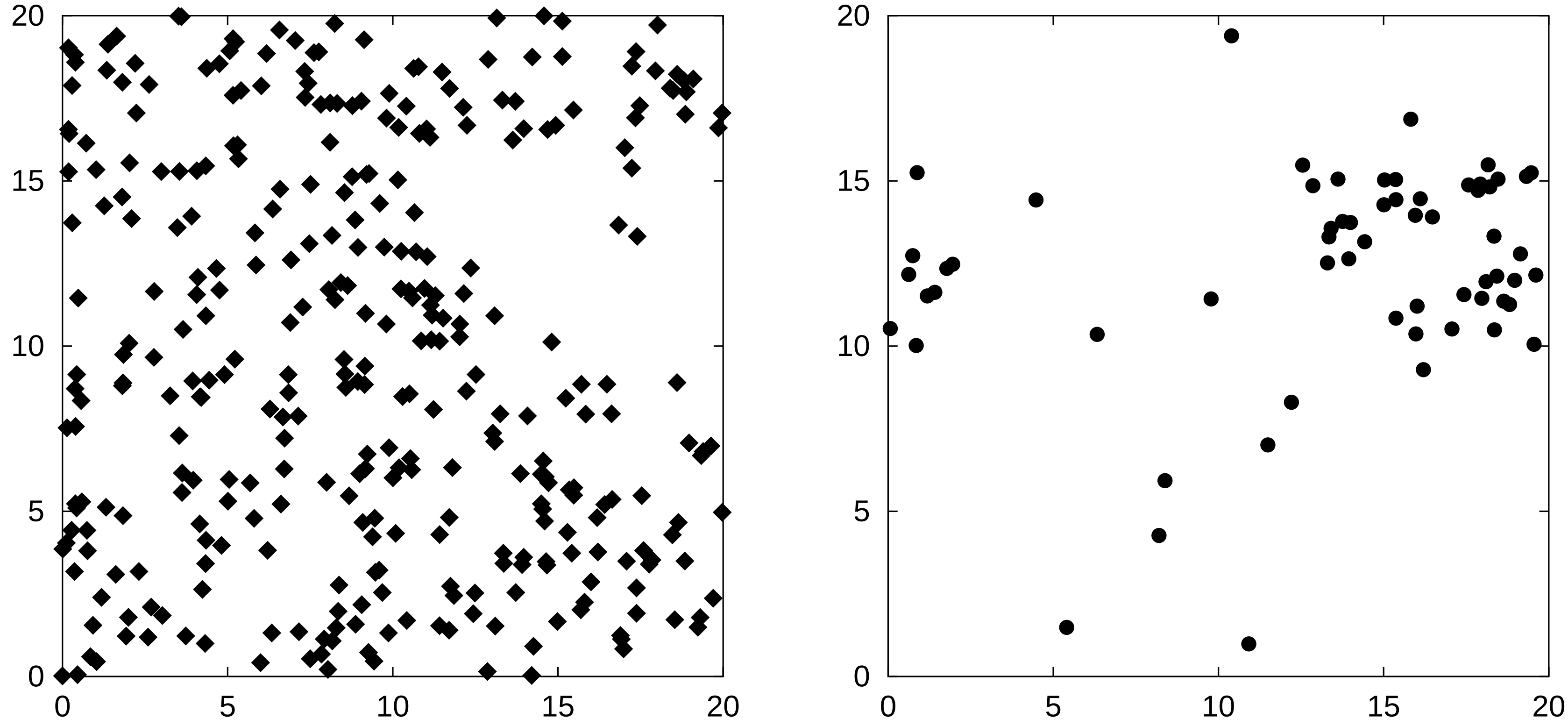


$$t = 10^0$$





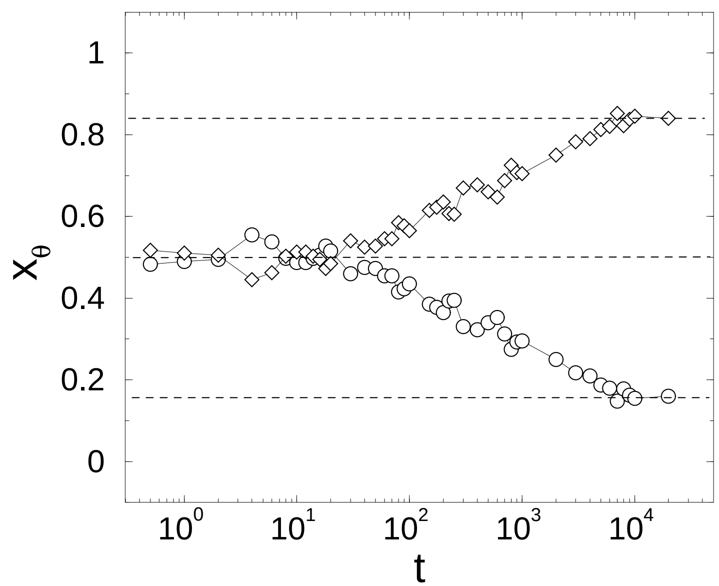
$$t = 10^2$$

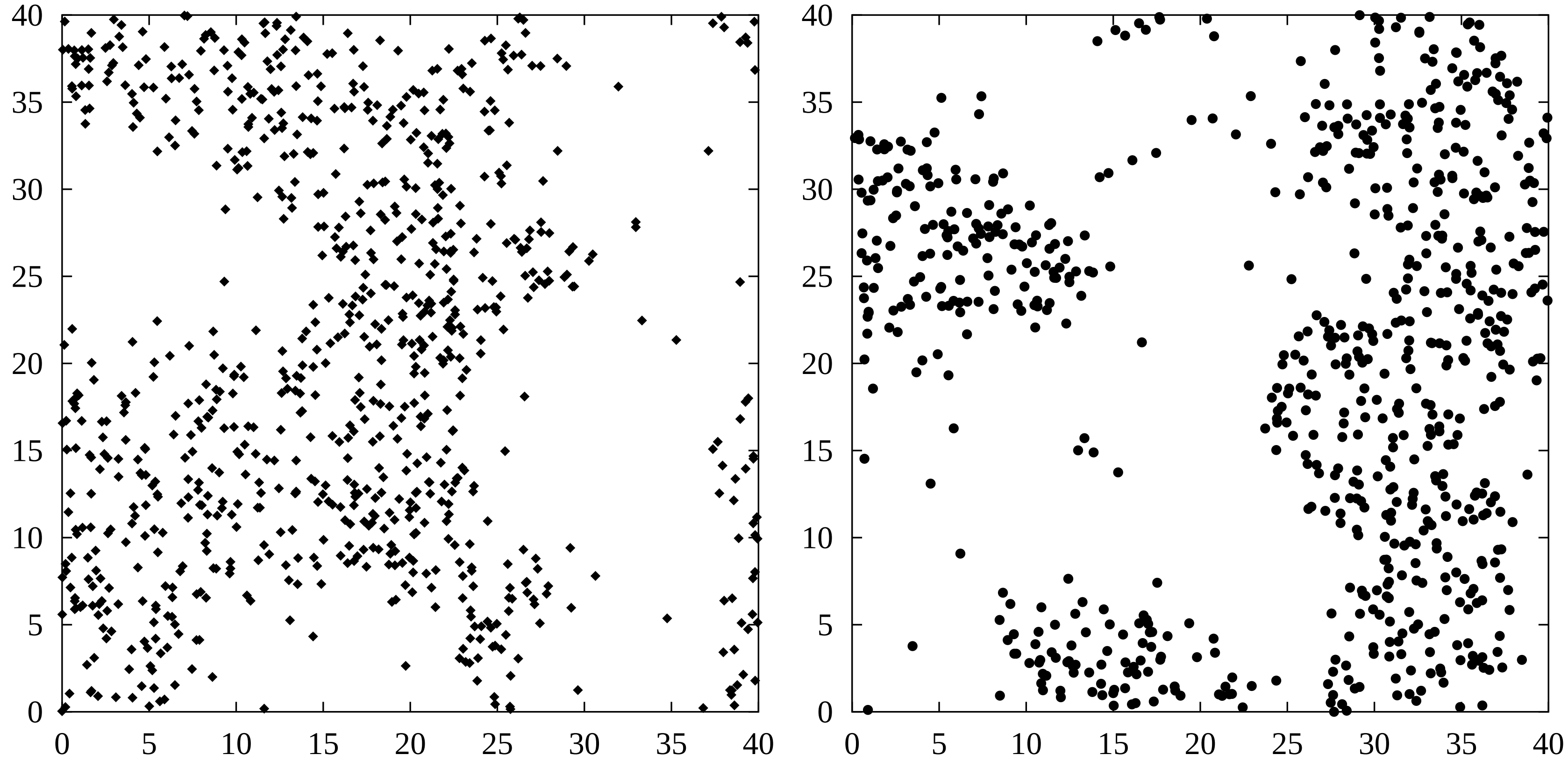


$$t = 10^4$$

Results: (first glimpse)

1. *spatial* coordination of decisions: concentration of agents with the same opinion in different spatial domains
2. emergence of minority and majority
3. random events decide about minority/majority status

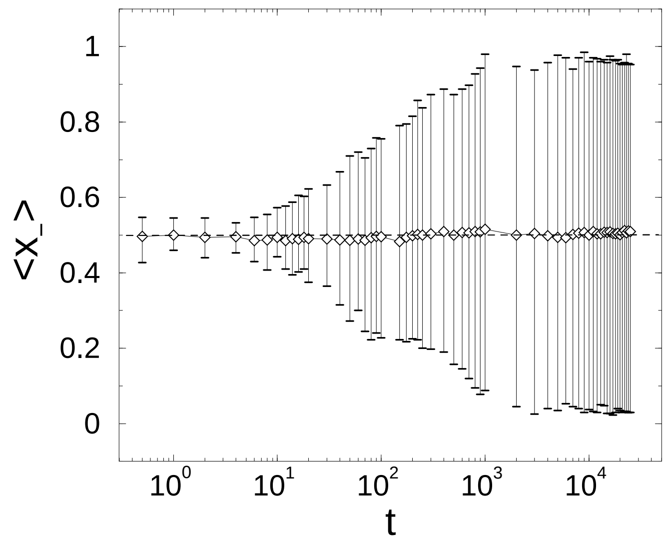




System size: $A = 1600$, total number of agents: $N = 1600$, time: $t = 5 \cdot 10^4$, frequency: $x_+ = 0.543$

Results: (closer inspection)

- *single-attractor regime:* fixed minority/majority relation
- *multi-attractor regime:* variety of spatial patterns
  almost every minority/majority relation may be established



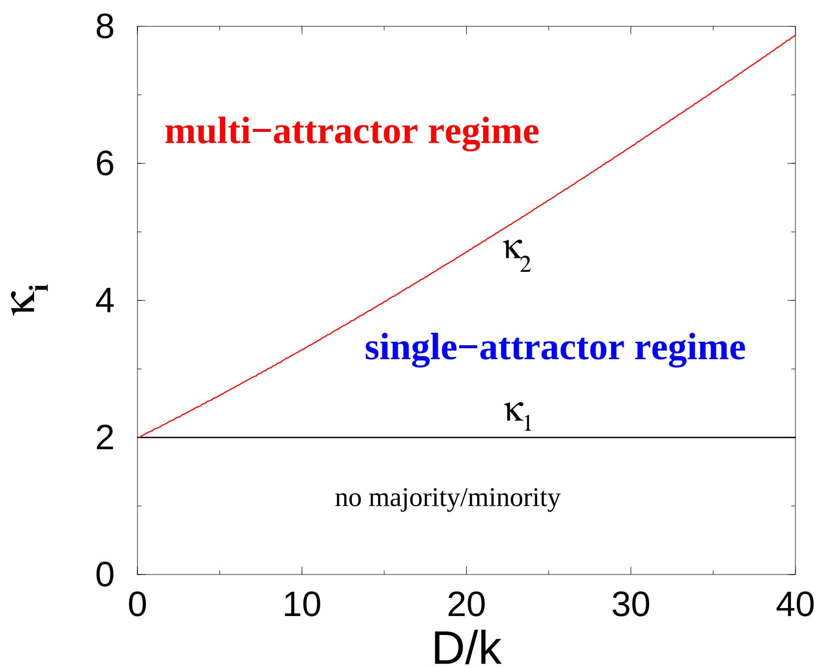
- dependence on information dissemination (D), memory (k), agent density (N/A) ??

Analytical Investigations

- $\kappa = 2\nu/T$, net information density: $\nu = \bar{n}\ s/k$
- existence of two bifurcations:

  $\kappa > \kappa_1 = 2$: minority/majority

  $\kappa > \kappa_2(D/k)$: multi-attractor regime



Result:

➤ to *avoid* multiple outcome (i.e. uncertainty in decision)

- speed up information dissemination (mass media, ...)
- reduce memory effects (distraction, ...) (k)
- increase randomness in social interaction (T)

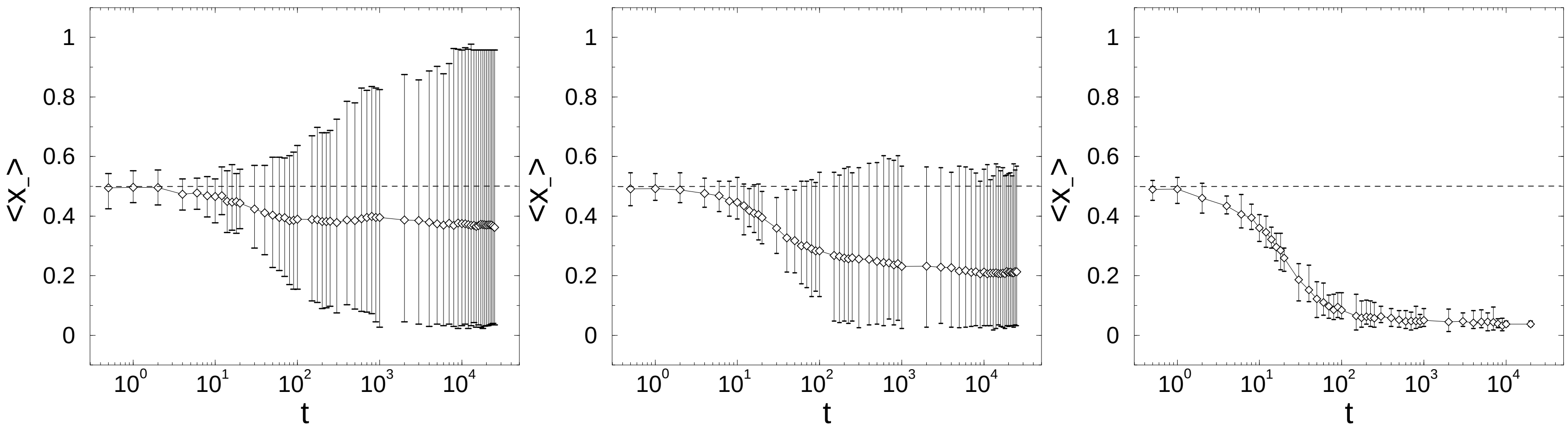
$\Rightarrow$ system “globalized” by ruling information

➤ to *enhance* multiple outcome (i.e. openess, diversity)

- increase self-confidence, local influences (s)
- prevent “globalization” via mass media (small D)

Influence of information dissemination

vary: $d = D_{+1}/D_{-1}$



d=1.1 d=1.2 d=1.5

- subpopulation with the more efficient communication becomes “always” the majority

Conclusions: Coordination of decisions

- based on local NN interaction (persuasion)
  $\Rightarrow$ non-linear voter models (CA)
- based on dissemination of information
  $\Rightarrow$ spatial model of communicating (Brownian) agents (BA)
- emergence of spatial domains of likeminded agents
- emergence of majority/minority
- non-stationary coexistence or extinction (CA)
- multi-attractor regime: multiple outcome (BA)
- “efficient” communication supports majority status

- advantage:
  - link agent-based (microscopic) model to analytical (macroscopic) model
  - allows prediction of collective behavior

Frank Schweitzer: *Brownian Agents and Active Particles. Collective Dynamics in the Natural and Social Sciences*
Springer Series in Synergetics, 2003 (422 pp, 192 figs)