ETH zürich

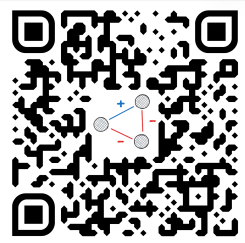
Structural balance in complex networks

Observation, Emergemence, and Implications

Dr. Giacomo Vaccario

Objectives of the review

- Clarify **consistency** of measures
  - Link to network structure
  - Catalogue of null models
- Theoretical **drivers**
  - Simultaneous Vs Alternative
  - Reinforcement Vs Weakening
- Practical **implications**
  - Ideological polarization
  - Performance, persistence, resilience, ...



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Introduction

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Emergemence

Implications

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Structural balance (SB)

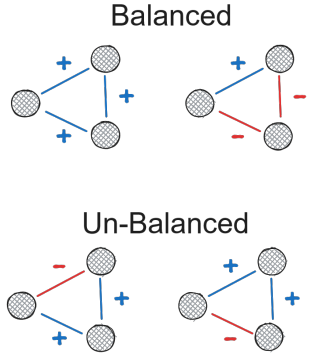
Original definition [Heider 1946]

- nodes → 2 individuals and 1 object
- edges → *signed* relations
- even number of negative links → balance

Extension to social networks [Cartwright & Harary 1956]

- nodes → *only* individuals
- edges → *signed* relations

micro-level → **triads**
macro-level → **Paradise** or **Polarizaion**



Why do we care about SB

- micro-macro link → **Emergence**
- very different balanced states → **Functionality** of networks
- **Applications** in social sciences, economics, ecology, biology, …

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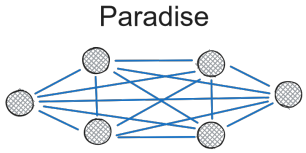

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What do we need to measure SB?

Data (N, E)

- **Nodes** $\rightarrow$ system elements $i \in N$
  - individuals
  - objects
  - groups
  - ...
- **Links** $\rightarrow$ signed relations, $(i, j, s) \in E$
  - $s \in \{0, 1\} \rightarrow$ binary $\rightarrow$ positive, negative
  - $s \in \{x, \ldots, 0, \cdots + x\} \rightarrow$ discrete
    $\rightarrow$ very negative to very positive [Harary 1959]
  - $s \in [a, b] \subset \mathbb{R}$continuous

Measure f

- **Node + links** $\rightarrow$ **signed networks** $G(N, E)$
- $f : G(N, E) \rightarrow \{0, 1\} \equiv$ **full** balance or unbalanced
  - $A \rightarrow$ adjacency matrix
  - $f : A \rightarrow \{0, 1\}$
- **Partial** balance $\rightarrow f : \cdot \rightarrow [a, b] \in \mathbb{R}$

Observation of structural balance: Data (N,E)

Direct signed relations

- Social networks
  - surveys → small scale
  - online → large scale
  - negative links are rare
- Ecology and chemistry
  - mutualistic VS predator-prey
  - activator VS inhibitor
- …

Inferred signed relations

- Social interaction data
  - text edits → wikipedia, github, …
  - co-location → face-to-face, …
  - communication → email, phone, …
- Others
  - co-activation → brain, …
  - correlation → stock market, …
- …

Data challange

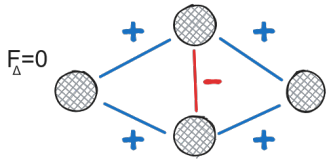
- **Signed relations** more scarce → **Interaction** data is largely available
- Methods are needed to infer signed relations, examples based on:
  - network ensemble [Andres et al., 2022], thresholds [Fraxnet et al., 2023; Wutchy and Uzzi., 2011]

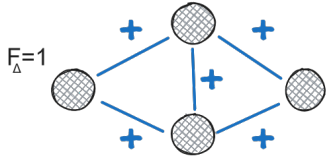
Measures of partial balance f

- **Structural measures** → look at the adj. A
  - Fraction of balanced triads F_Δ [Terzi and Winkler 2011]
    - simple and quick
  - Fraction of balanced cycles [Cartwright and Harary 1956]
    - longer paths → less important [Estrada and Benzi 2014]
- **Extremination** → How far away from **full balance**
  - min. number of sign changes → line index [Harary 1959]
  - min. number of nodes expelled → point index [Harary 1959]

SB depends on the measure

- **Details** of the network matter!
- Different measure → different results → trivial, but..
- Need for **reference** values to interpret results





Building reference cases: Null models

Null model → **random** object that **preserves** some properties:

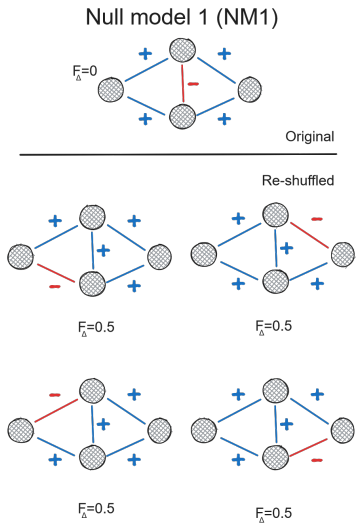
NM1 Number of positive link → preference to form positive links
- if $f(G(N,E)) \neq \langle f(G(N,E'))\rangle$
  ⇒ evidence for/against (partial) balance

NM2 Signed degree distribution → preference and heterogeneity
- if $f(G(N,E)) \sim \langle f(G(N,E'))\rangle$
  ⇒ partial balance due to the signed degree distribution

NMX …

Signed structure and SB

- Different null models → different results→ trivial, **but..**
- How to choose the null models? [Gallo et. al, 2024]
- Null models → hypothesis testing
- Generative mechanisms → emergence of SB



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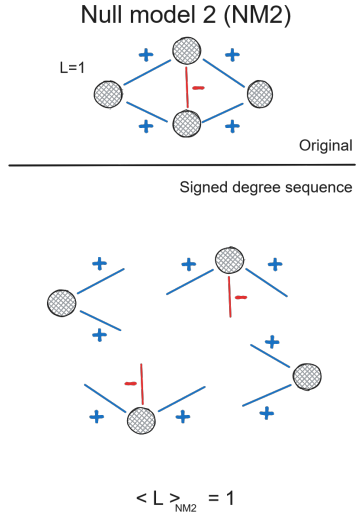

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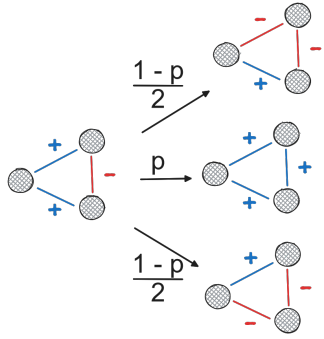
Emergence of structural balance

- **Generative mechanisms** → explain the emergence of SB
  - cognitive dissonance [Heider 1946]
  - homophily [Pham et al. 2022]
  - biotic interactions [Saiz et al. 2016]
  - in-group praise Vs out-group derogation [Wojcieszak et al. 2021]
- **Phase transition** → from disorder to oder [Macy et al. 2021]
  - Parameters → **Order** and **Control**
  - Continuous, first order, hysteresis, bifurcation
- **Example** of sign dynamic [Antal et al. 2005]
  - Control → probability to form positive ties p
  - Order → number of balanced triads F_Δ
  - Continuous phase transition at $p = 0.5$



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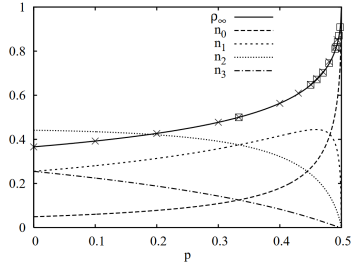

[Antal et al. 2005]

Emergence of structural balance

- **Perspective** → Who or what changes the signs
  - triad [Antal et al. 2005]
  - dyad [Yap & Arrigan 2015]
  - ego [Gorski et al. 2024]
- **Sign dynamics** → How signs change
  - stochastic [Antal et al. 2005]
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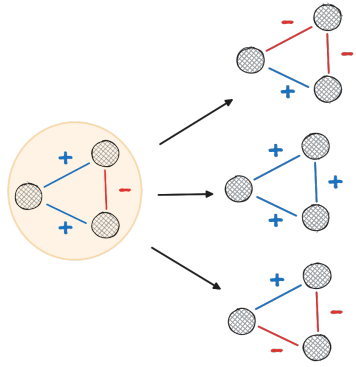
Energy landscape H

- Hamiltonian formalism [Marvel et al. 2009]
- Ragged vs smooth landscapes
- Accessibility of balanced states



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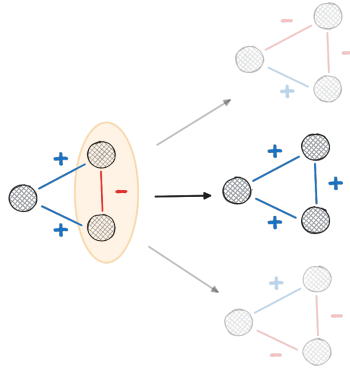

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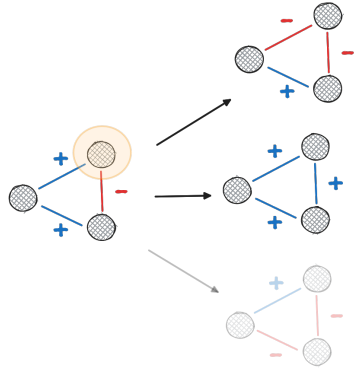
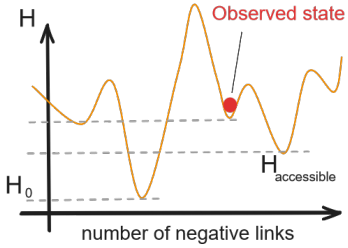


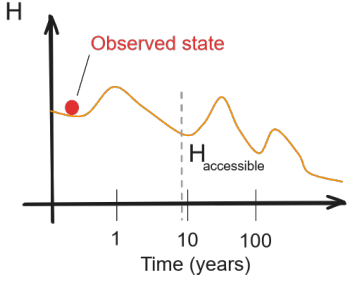
Models as new measures for SB

- **Usually** → models simulate sign dynamics
- **Interesting** → difference between real-world and simulations
  - How far are the **accessible and more balanced** states?
  - Partial balance as path dependence
- Define **measure** for this
  - **Energy** landscape → ΔH_{SB} [Facchetti et al. 2011]
  - **Time** to reach balance → e.g., first passage time

Possible research directions

- **Link** models to SB measures
- Use dynamics to **interpret** partial balance



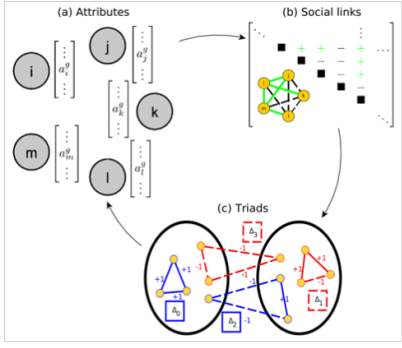


Richer dynamics: Coupling sign dynamics with node attributes

- **Node attributes** → opinions, ideology, demographics, …
- **Attributes → signed links**
  - homophily (selection), status, …
- **Sign links → attributes**
  - homophily (influence), evolutionary fitness, …
- Coupling the two → **feedback**: reinforcement or competition

Modeling SB and node attributes

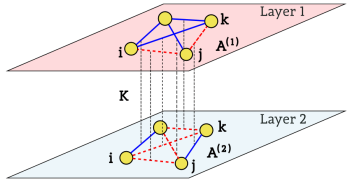
- Vast literature → **strong focus** on homophily
  - reproduce degree of balance [Pham et al. 2022]
  - ordered attributes → avoid polarization [Gorski et al. 2023]
  - …
- Real-world → **multiple simultaneous** mechanisms
- **Challenge**: connect mechanisms in a interpretable way



[Gorski et al. 2023]

Structural balance: beyond networks

- **Relations in different settings** → mutilayer networks
  - layer → environment, time [Mucha et al. 2010], etc.
  - each layer → different mechanisms
  - new paths to balance [Krishnadas et al. 2024]
- **Group relations** → higher-order models [Scholtes et al. 2019]
  - group interactions → group chats, emails [Battiston et al. 2021]
  - majority → beyond dyadic-rules [Papanikolaou et al. 2022]



What is there and what is missing

- Strucutal balance in multilayer networks → formal models exist [Krishnadas et al. 2024]
  - empirical validation is still missing
- Many formal higher-order models
  - none tailored to analyze structural balance
  - very little empirical analysis → missing data [Andres et al. 2024]

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Implications of structural balance: Politics

Trends in the literature

- increasing **ideological** polarization [Flamino et al. 2023]
- increasing **elite** polarization [Druckman et al. 2013]
- decreasing in international relations [Doreian et al. 1996]
- increasing in international relations [Aref ?]

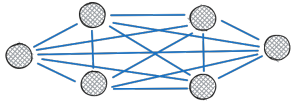
Possible **negative** effects

- "us versus them" [McCoy & Somer 2018]
- health of individuals [Fraser et al. 2022]
- increasing governments shutdowns [Jidalini & Smith 2019]

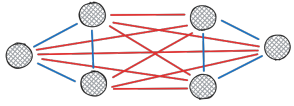
Possible **positive** effects

- "lifeblood of democratic politics" [Lipset 1959]
- quicker government formation [Fontan & Altafini 2021]
- slower diffusion of fake news [Flamino et al. 2023]

Paradise



Polarized



Ideological polarization

Duality: When it is good and when it is bad?

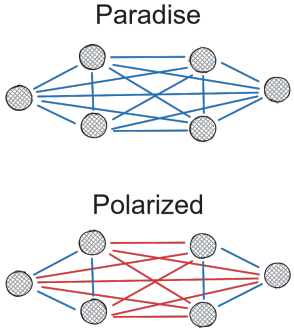
Implications of structural balance: Functionality

Possible **negative** effects

- focus on polarizing triads
- lower quality of articles [Lerner & Lomi 1019]
- shorter group persistence [Andres 2024]

Possible **positive** effects

- group effectiveness [Morrissette et al. 1967]
- performance in risky decision-making [Askarisichani et al. 2019]
- cohesion and longer group persistence [Andres 2024]
- robust opinion formation [Medo et al. 2021]
- legislative effectiveness [Aref & Neal 2020]
- link formation [Leskovec et al. 2010; Huang et al. 2013]



Mostly positive effects

Focus on **group** performance and resilience

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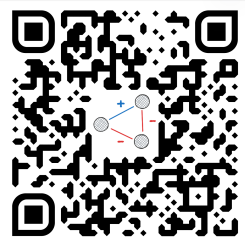

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