The fragility of opinion formation in a complex world

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International Workshop
“Signed Relations and Structural Balance in Complex Systems: From Data to Models”

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A motivating experience

In 2014, a series of protests and political demonstrations began in Venezuela...

Reuters

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Question #1:
Should we trust
Maduro's government?

A motivating experience

In 2014, a series of protests and political demonstrations began in Venezuela...

"Venezuela's government should address the people's legitimate grievances..."

"We must respect the right to peaceful protest..."

"We trust that the government of President Maduro will preserve the constitutional order..."
President of Syria Bashar al-Assad expressed his support in a letter to President Maduro,...

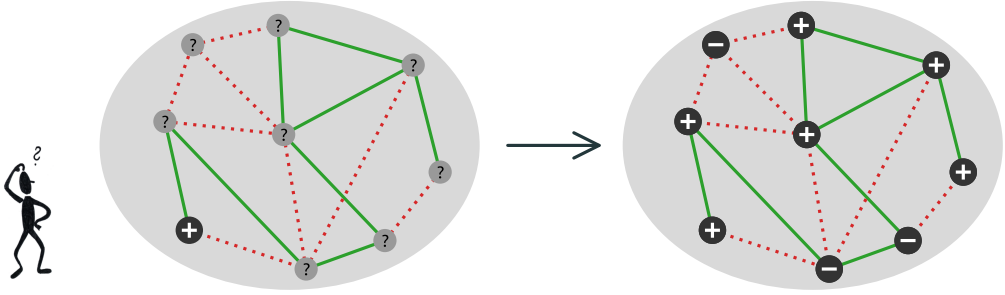
https://en.wikipedia.org/wiki/Reactions_to_the_2014-2017_Venezuelan_protests

A motivating experience

In 2014, a series of protests and political demonstrations began in Venezuela...

Question #2:
What happens if we generalize
from this single question
to a whole learning process?

Opinion formation on a signed network



- **Nodes:** Subjects on which opinions are to be made
  - Countries and other entities in world politics
- **Links:** Signed relations between the subjects
- **The observer:** Outside, no social network

Setting up a synthetic world

- N nodes of two types: $\theta_i \in \{-1, +1\}$; unknown to the observer
- N_S source nodes: The observer knows their types
- Link signs correlate with node types:
  - $\theta_i = \theta_j$: link is positive with probability $r \geq 0.5$
  - $\theta_i \neq \theta_j$: link is negative with probability $r \geq 0.5$
  - r is link reliability

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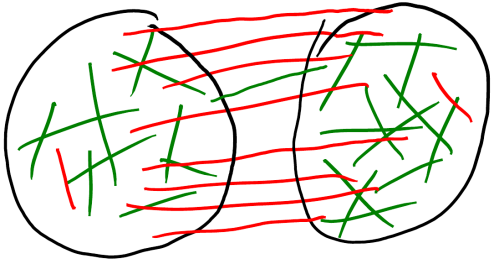

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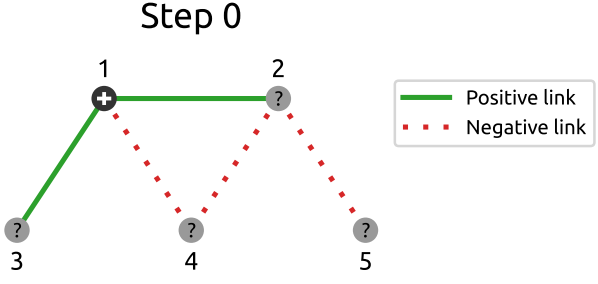
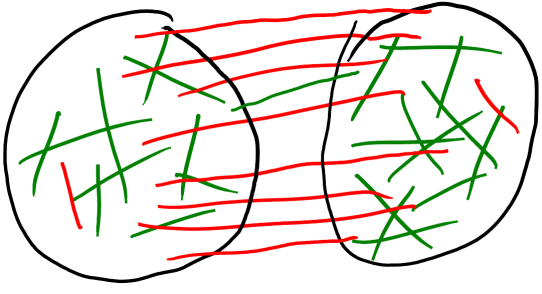


Possible applications of the model

1. Two opposing camps: Mainstream media and misinformation sources
   - You initially trust in some mainstream media
   - Do you end up trusting other mainstream media and distrusting misinformation sources?



Possible applications of the model

1. **Two opposing camps:** Mainstream media and misinformation sources
2. **Employee network:** Manager attempts to assess employee qualities
3. **Inter-firm network:** Which other firms to trust
4. **Social networks:** E.g., find a suitable roommate
5. ...

Random neighbor heuristic (Medo et al, 2021)

Local & easy to apply
(“average Joe”)

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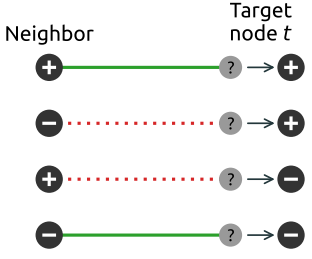
1. Choose target node *t* at random
2. Opinion on *t* is made using its random neighbor



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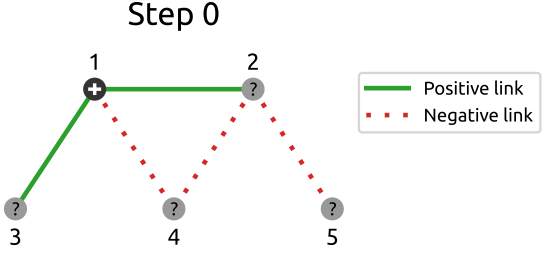

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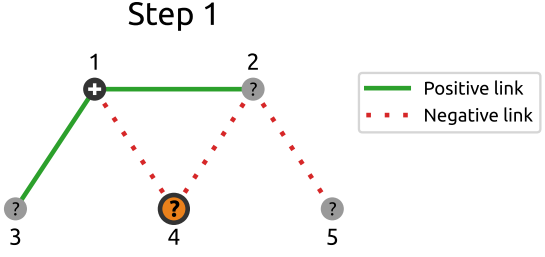

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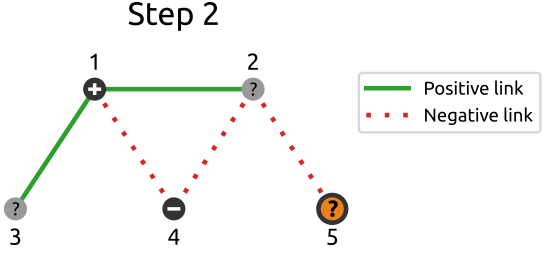

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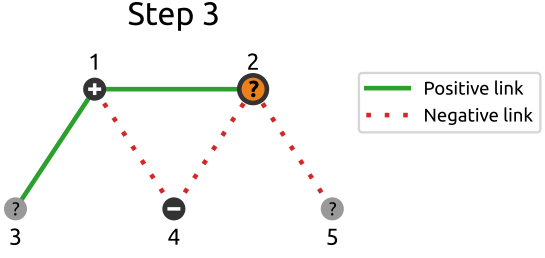

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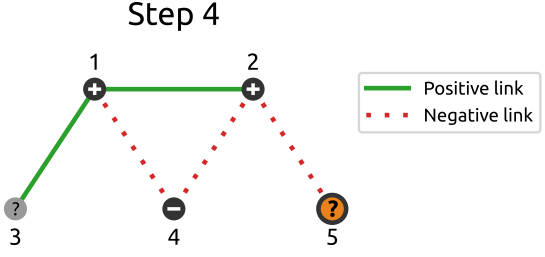
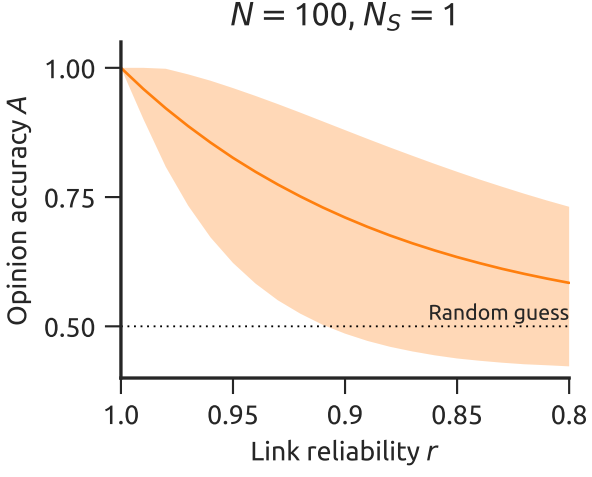


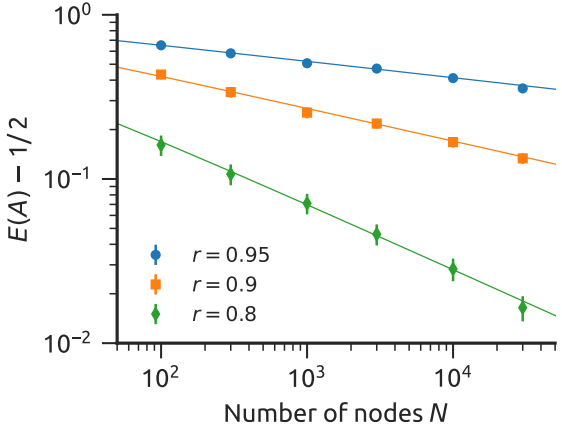
Random neighbor heuristic: The outcome



Shaded area: 10th–90th percentile range

Random neighbor heuristic: The outcome

$$E(A) - 1/2 \sim N^{-2(1-r)}$$



Lesson #1

Even at small noise,
resulting opinions show
low accuracy and high variability

Lesson #2

As the system size grows,
limit opinion accuracy is 1/2
regardless of how small is the noise

To make sense
of a complex world
is difficult

The majority rule

- Use all neighbors, not just a random one
- Choose the majority opinion signal

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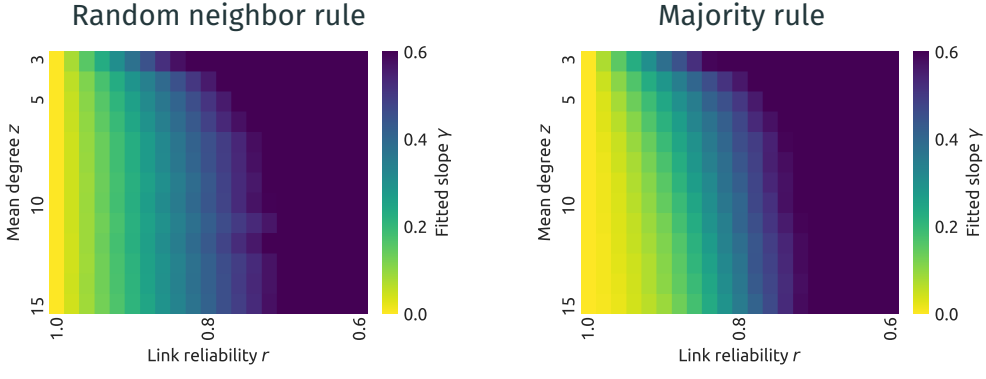
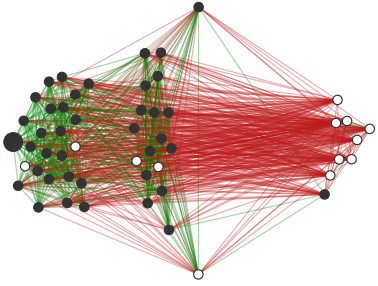
- Use all neighbors, not just a random one
- Choose the majority opinion signal
- Opinion accuracy still approaches 1/2 as $N^{-\gamma}$

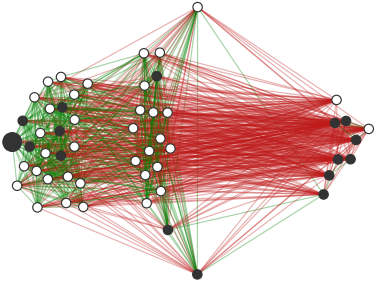


1st United Nations General Assembly network

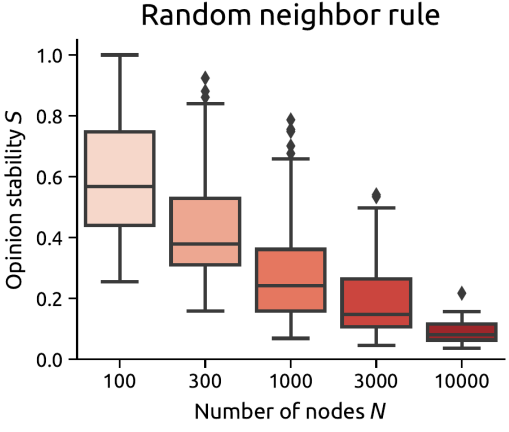
Run 1



Run 2



Opinion formation on real signed networks



Slashdot social network

Two new methods (Meng et al, 2022)

1. Bayesian solution:

$$P[\boldsymbol{\theta}|\boldsymbol{\sigma}, R] = \frac{P[\boldsymbol{\sigma}, R|\boldsymbol{\theta}] \cdot P[\boldsymbol{\theta}]}{P[\boldsymbol{\sigma}, R]} = \frac{q^{z_1(\boldsymbol{\theta})}(1-q)^{z_2(\boldsymbol{\theta})} r^{z_3(\boldsymbol{\theta})}(1-r)^{z_4(\boldsymbol{\theta})}}{\sum_{\boldsymbol{\theta}' \in \Theta} q^{z_1(\boldsymbol{\theta}')}(1-q)^{z_2(\boldsymbol{\theta}')} r^{z_3(\boldsymbol{\theta}')}(1-r)^{z_4(\boldsymbol{\theta}')}}$$

2. Shortest-path heuristic:
   - Based on shortest paths between each source node and a target node
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2. **Shortest-path heuristic:**
   - Based on shortest paths between each source node and a target node
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Shortest-path accuracy: $E(A) - 1/2 \sim N^{-\gamma}$, where

$$\gamma = -\ln(2r-1)/\ln z$$

on a random network.

How to avoid
ending up with
random opinions?

Option 1: Start with many source nodes

- More source nodes $\implies$ better accuracy
- Denoting $f_S := N_S/N$, the random neighbor rule gives

$$\lim_{N\to\infty} E(A) = \frac{1}{2} + f_S^{2(1-r)}/2$$

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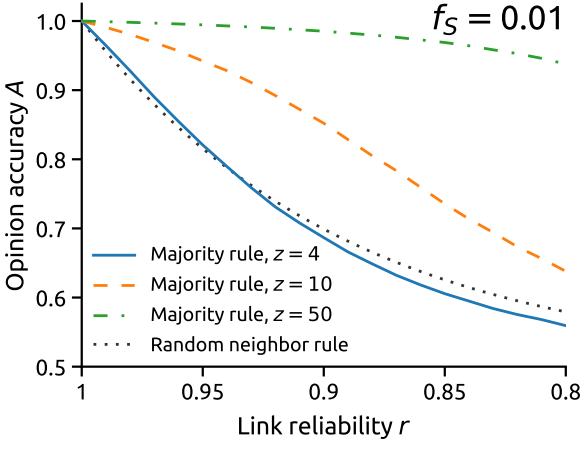

Option 2: Require consensus (Fenoaltea et al, 2022)

- From opinion formation to group growth:
  1. Individuals of two types: Fit or unfit for a group
  2. Group seed: N_0 fit members
  3. Each candidate is evaluated by m group members

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- From opinion formation to group growth:
  1. Individuals of two types: Fit or unfit for a group
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  3. Each candidate is evaluated by m group members

- Rules of the game:
  - A fit member positively evaluates a fit candidate with probability r
  - Admit a candidate only if all evaluations are positive
- Group cohesion, C: The fraction of fit nodes
  - The same as opinion accuracy, A

Group cohesion: Results

- When $m = 1$, cohesion vanishes (goes to $1/2$) as N grows

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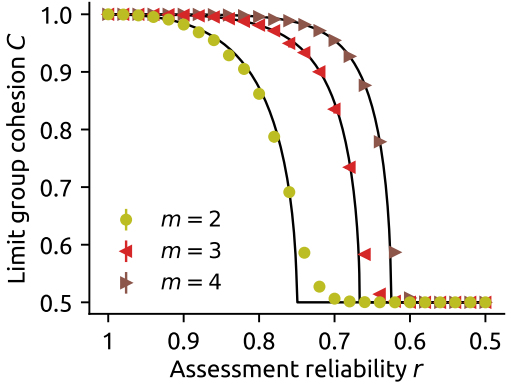
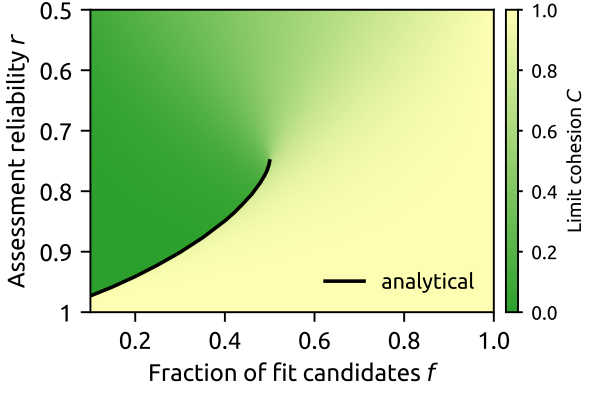


Group cohesion: Results

- When $m = 1$, cohesion vanishes (goes to $1/2$) as N grows
- When $m \geq 2$, a phase transition emerges at $r_c = 1/2 + 1/(2m)$
- The fraction of fit candidates matters too



Broader implications

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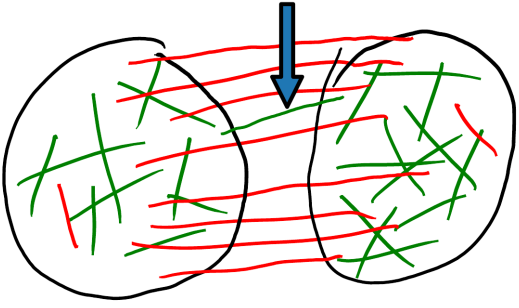
3. Do not be that misleading link



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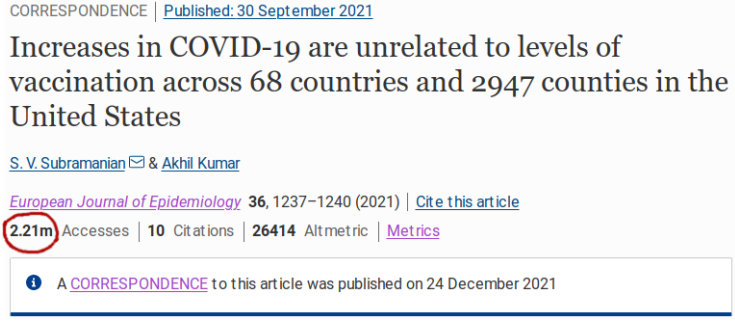

CORRESPONDENCE | Published: 30 September 2021

Increases in COVID-19 are unrelated to levels of vaccination across 68 countries and 2947 counties in the United States

S. V. Subramanian & Akhil Kumar

European Journal of Epidemiology **36**, 1237–1240 (2021) | Cite this article

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A CORRESPONDENCE to this article was published on 24 December 2021

Thank you for your attention!

M. Medo, M. S. Mariani, L. Lü, The fragility of opinion formation in a complex world, Communications Physics 4, 1 (2021)
F. Meng, M. Medo, B. Buechel, Whom to Trust in a Signed Network? Optimal Solution and two Heuristic Rules, Information Sciences 606, 742 (2022)
E. M. Fenoaltea, F. Meng, R.-R. Liu, M. Medo, Robustness of cohesion in a model of group formation, Physical Review Research 5, 013023 (2023)

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