Faculty of Physics

WARSAW UNIVERSITY OF TECHNOLOGY

Under-representation of nonhierarchical triads in structural balance model

Author: Piotr Górski

in collaboration with Adam Sulik,
Giacomo Vaccario, Georges Andres, Janusz Hołyst

Zurich, 16.05.2024

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Social networks

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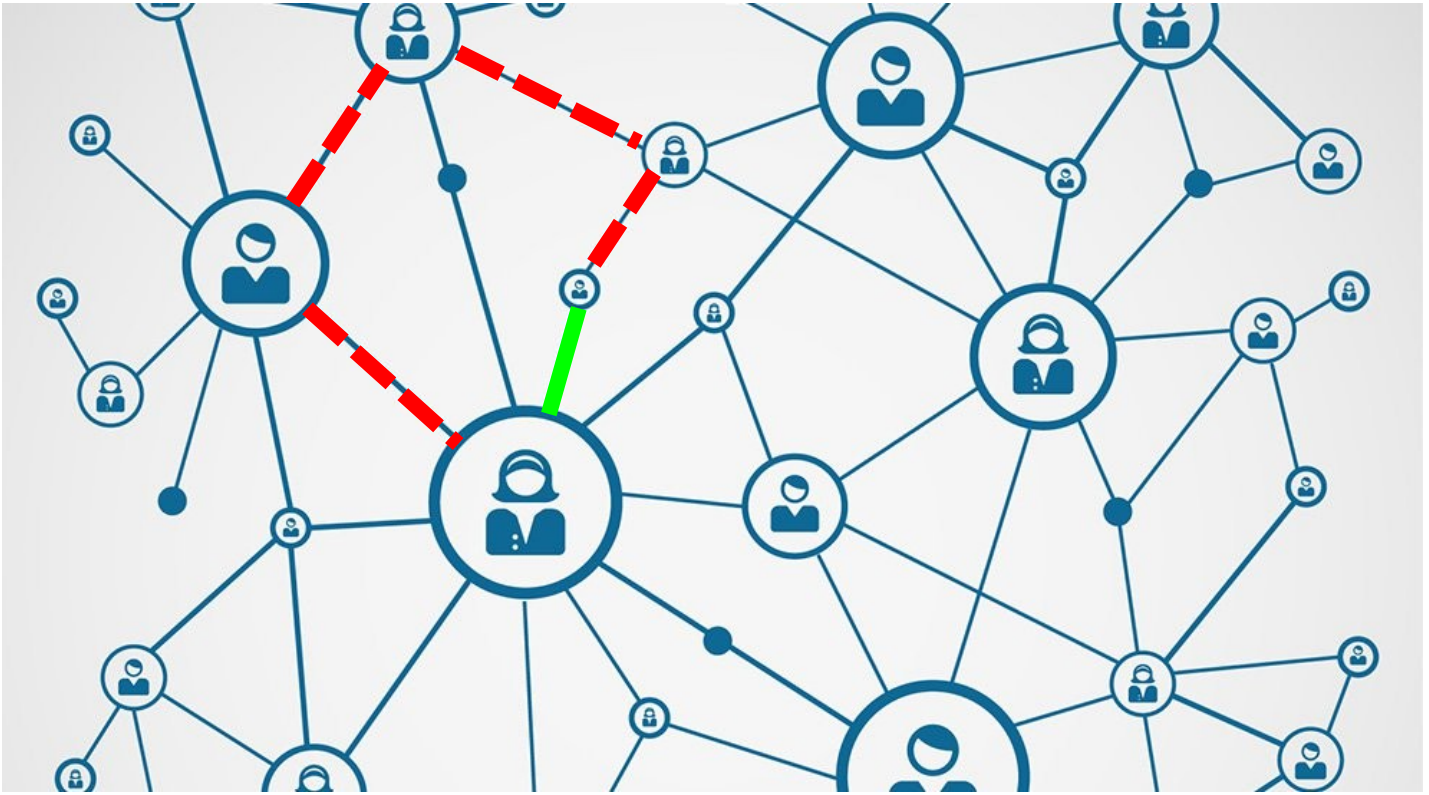
Credit: https://bitcoinwiki.co/wp-content/uploads/censorship-free-social-network-akasha-aims-to-tackle-internet-censorship-with-blockchain-technology.jpg

Social networks. Friends and enemies



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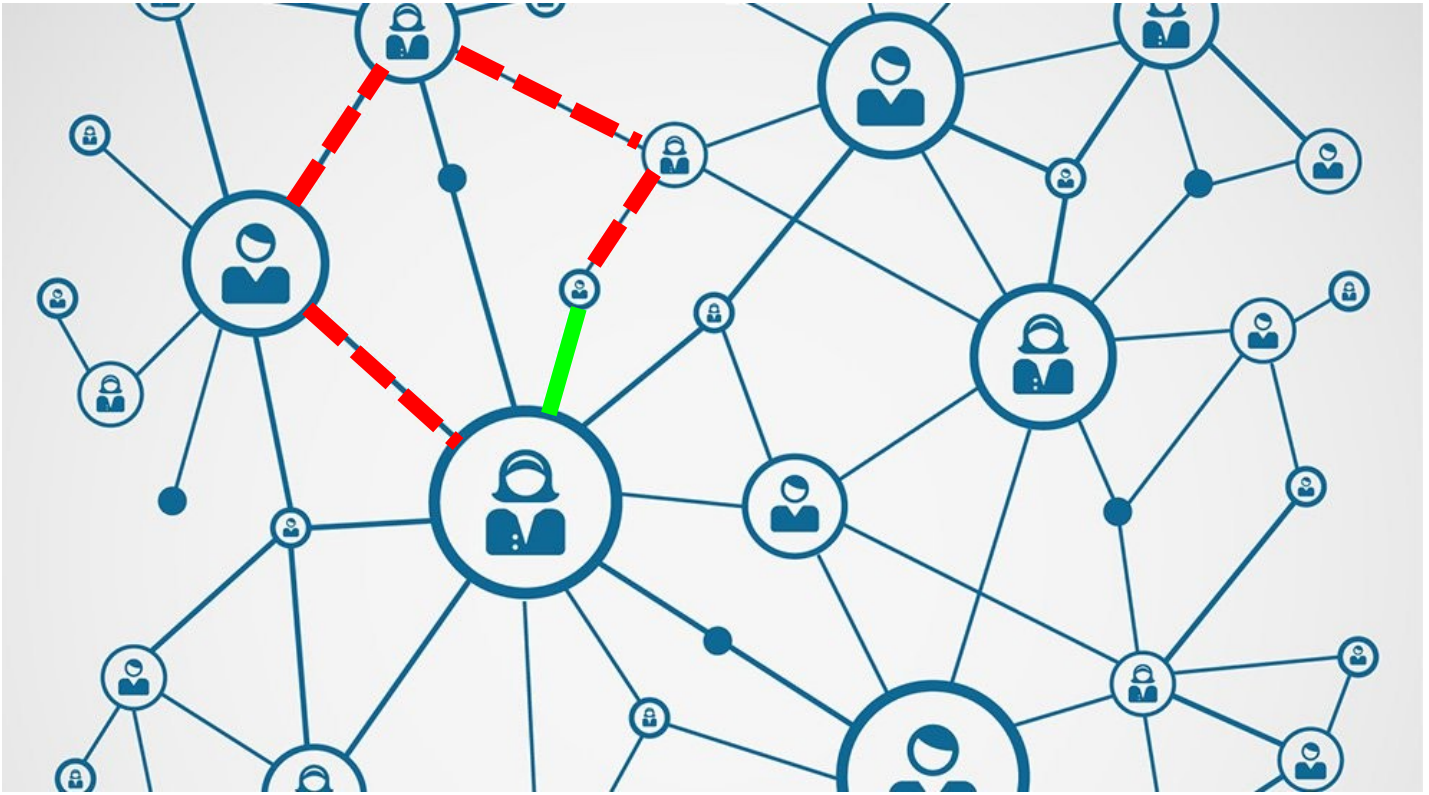
Why are people friends or enemies?



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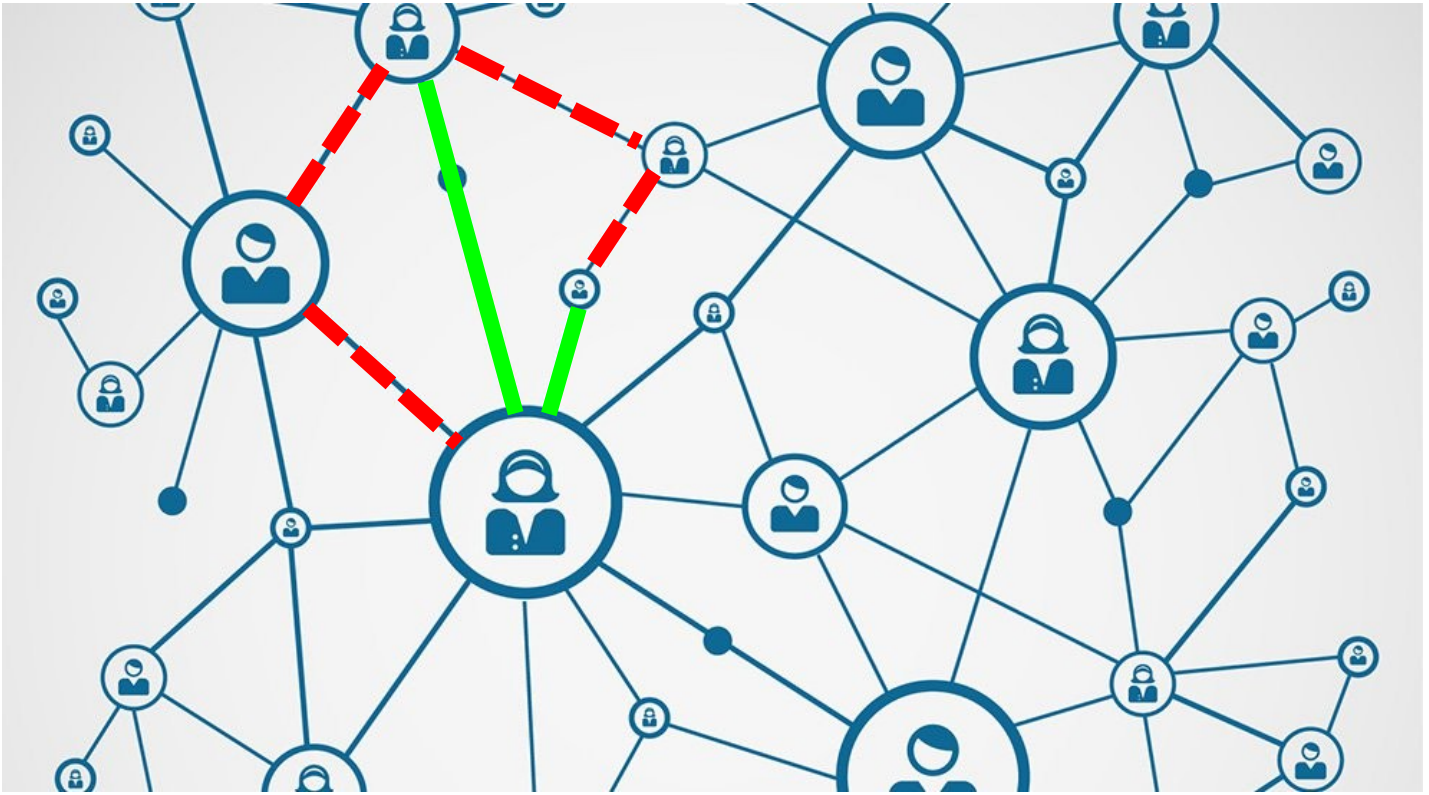

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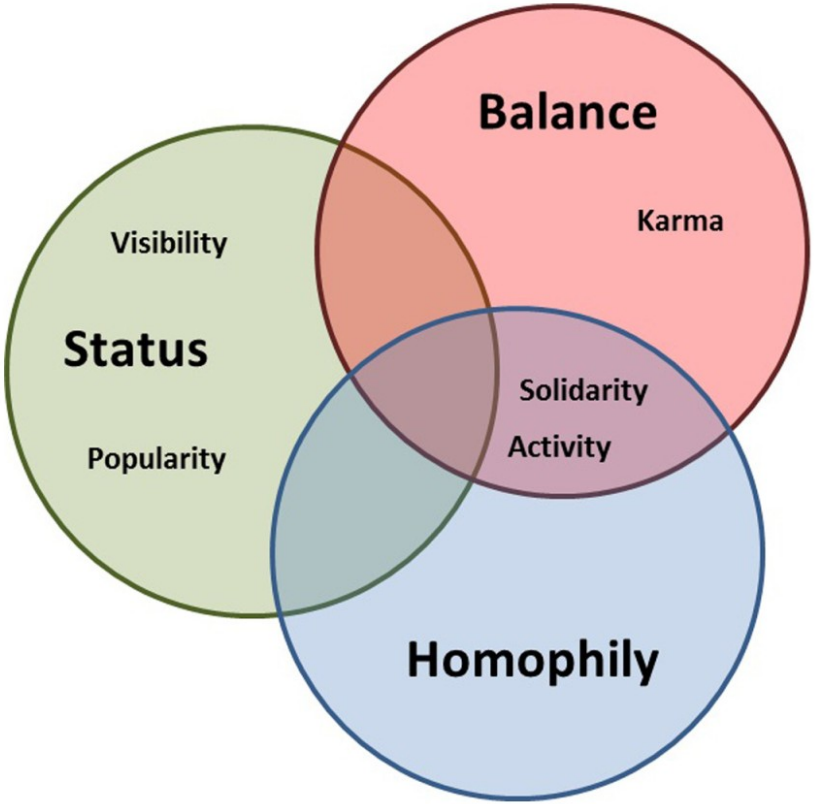
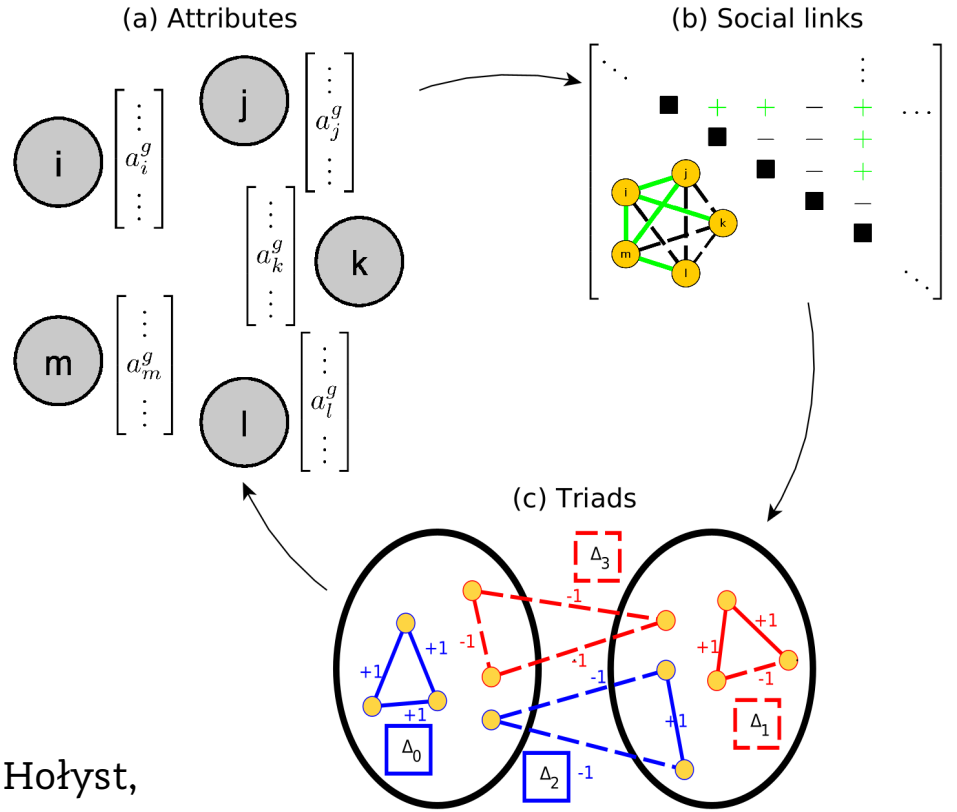
What influences formation of relations?



Source: Yap, J., & Harrigan, N. (2015). Why does everybody hate me? Balance, status, and homophily: The triumvirate of signed tie formation. Social Networks, 40, 103–122.

Previous work: connecting structural balance and homophily

Possible reason why structural balance is not observed are homophilic nature of relations between agents.



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P.G., Klavdiya Bochenina, Janusz Hołyst, Raissa D'Souza, PRL, 2019

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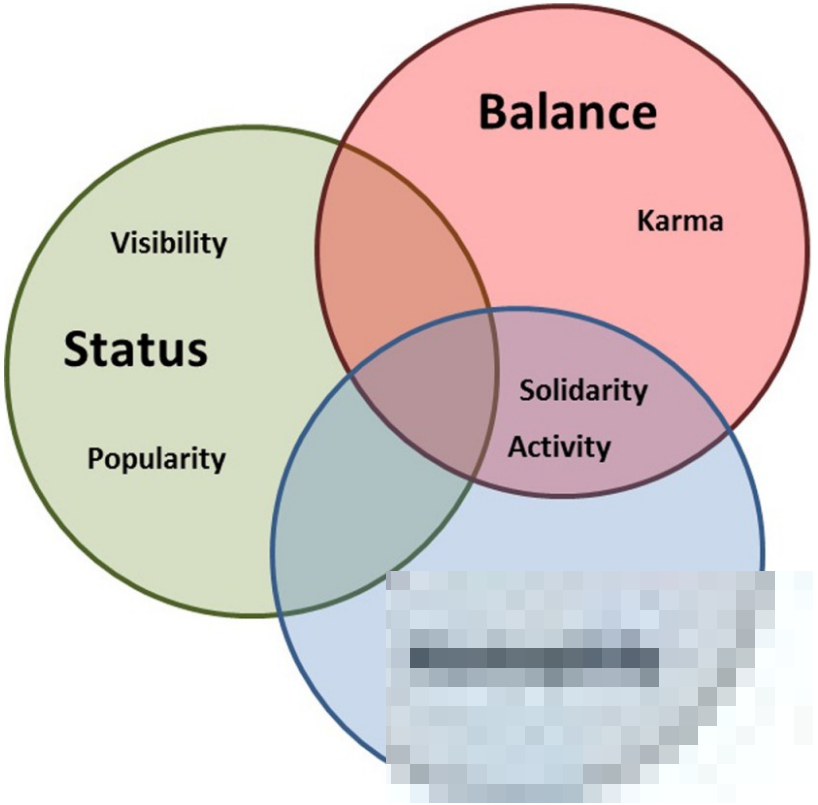

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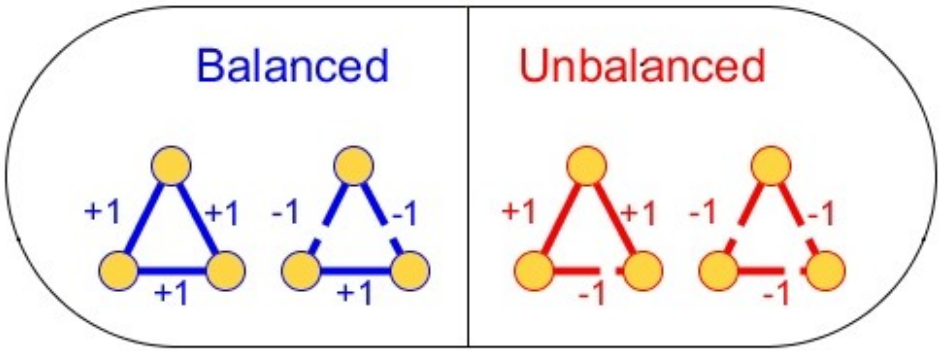
Structural balance theory

- Friend of my friend is my friend
- Friend of my enemy is my enemy
- Enemy of my friend is my enemy
- Enemy of my enemy is my friend



Status theory

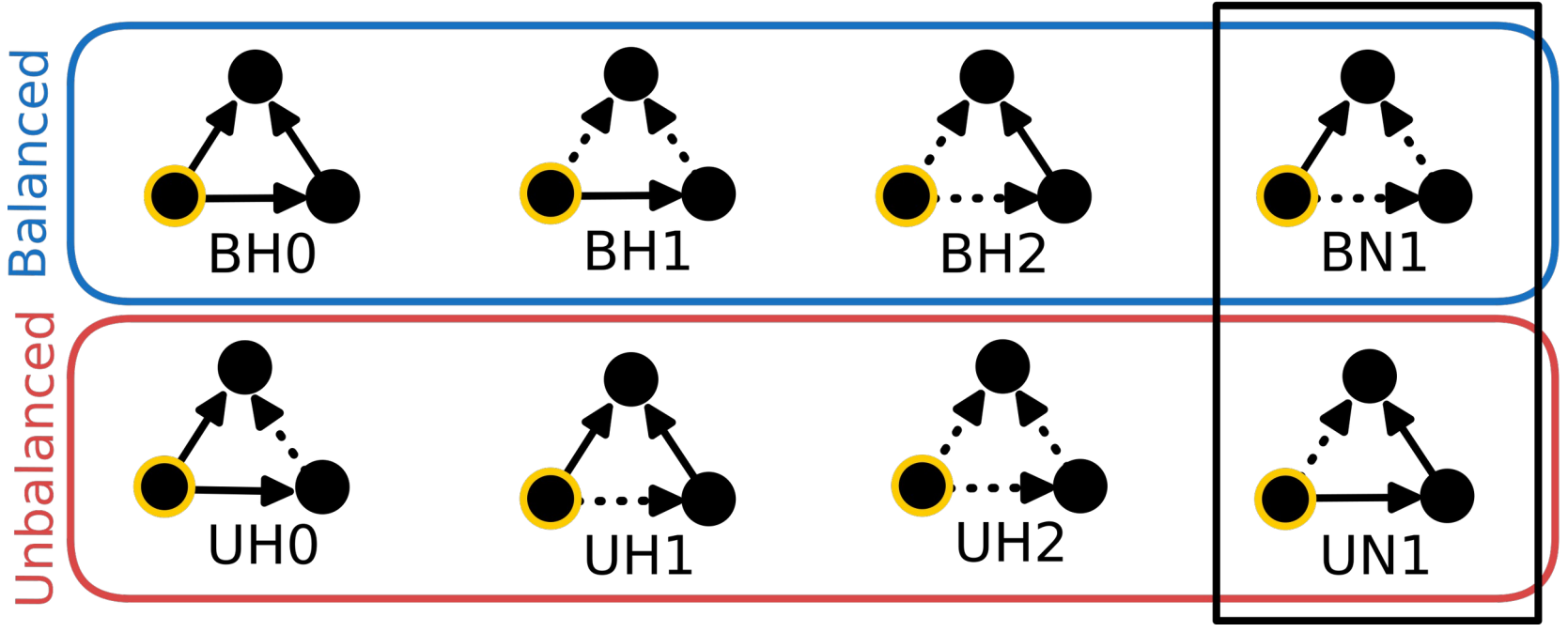
Introduced by Leskovec et al., 2010

- Agents with higher status tend to create negative links to those of lower status
- Agents with lower status tend to create positive links to those of higher status

Systemic result of such an approach is a hierarchical graph.

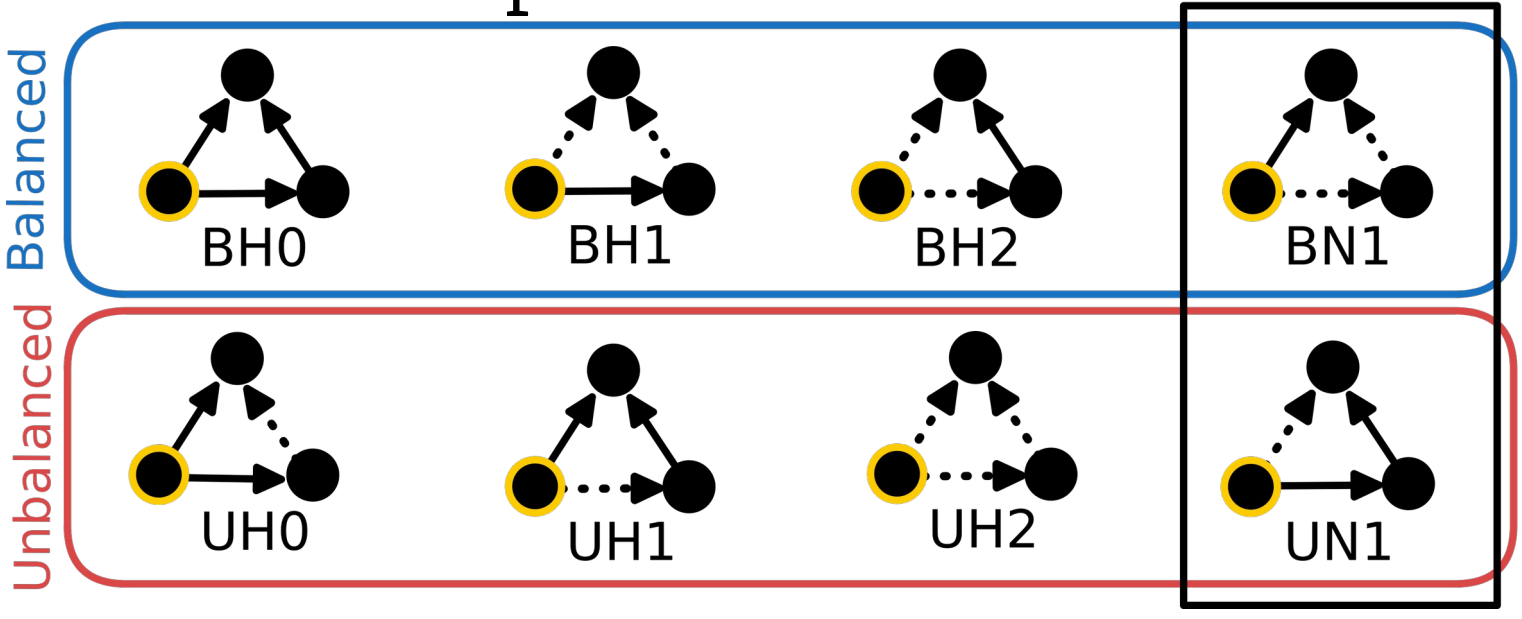
Bringing the theories together

- Status theory requires directed links
- More types of (un)balanced triads



Our approach

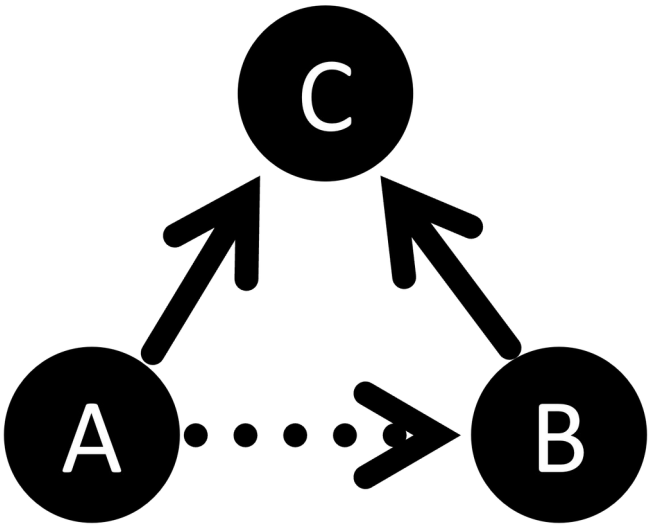
- Directed network
- Local information: egocentric agent perspective
  We do not consider loops.



What can we say about status in triads?

- From agent A's perspective:
- A considers B as of lower status
- A considers C as of higher status
  Thus, C > A > B
- A learns that B considers C
  as of higher status.

  Thus, C > B



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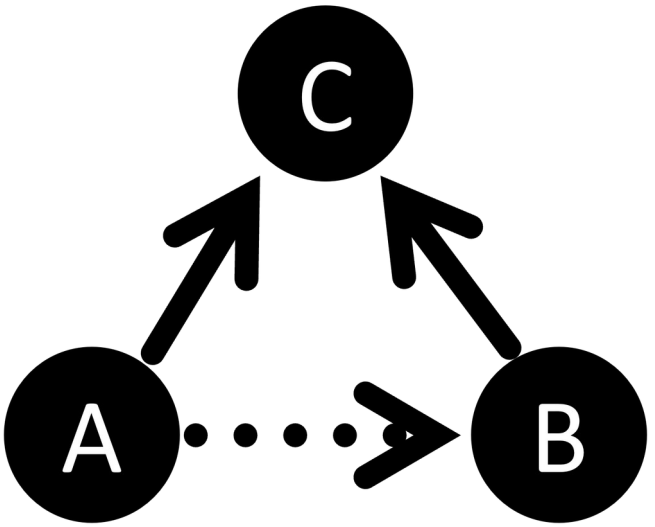

OK!

This triad may be stable
according to status theory.

What can we say about status in triads?

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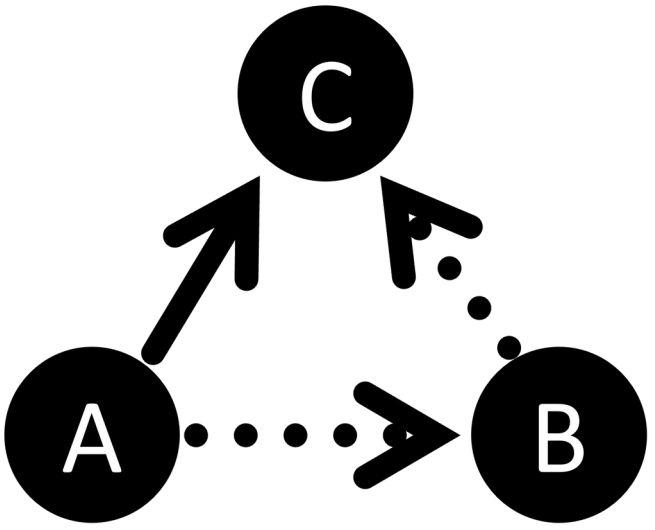

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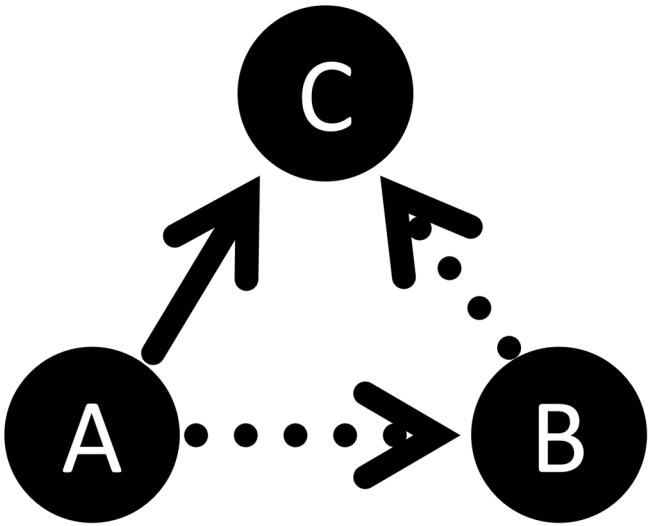
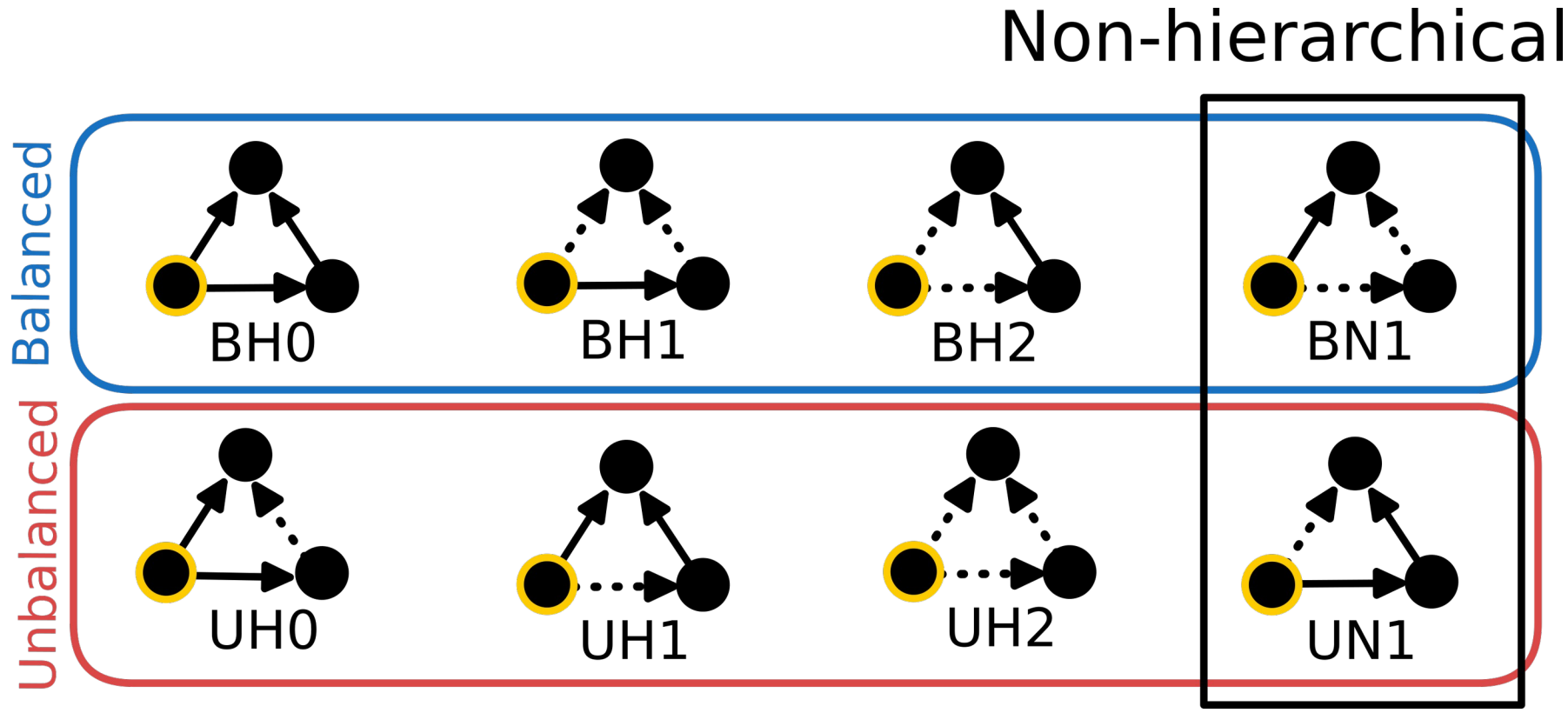


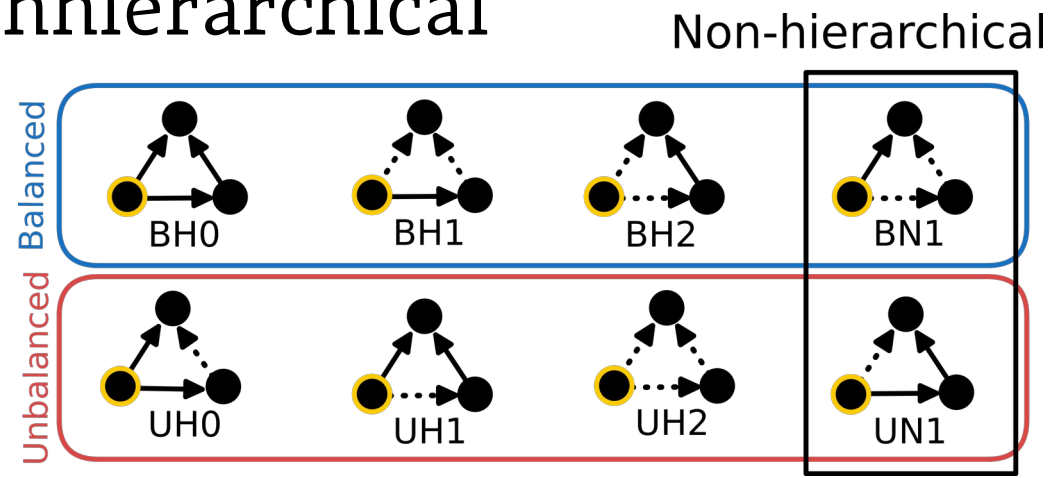
Unstable!
From A's perspective statuses are inconsistent with links.

Hierarchical and nonhierarchical triads



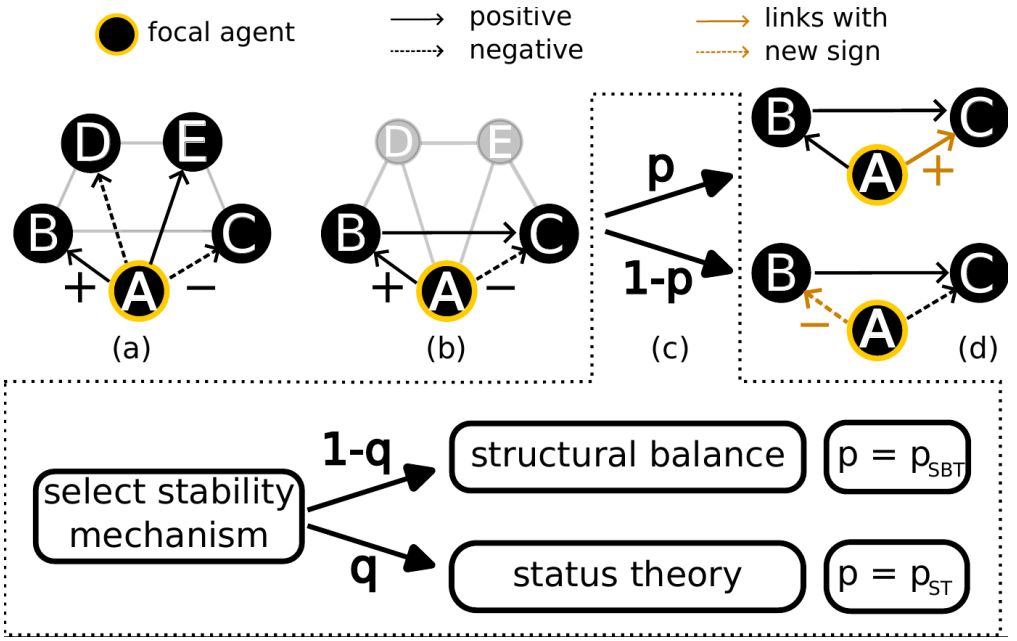
Classification of ego-triads and stability

- BH0 – **B**alanced **H**ierarchical triad with **0** negative outgoing links from the focal agent A.
- 4 balanced, 4 unbalanced
- 6 hierarchical, 2 nonhierarchical



Model dynamics and parameters

- At each step:
- (a) Choose a focal agent (agent A).
- (b) Construct an ego-based triad (triad ABC).
- (c) Select a theory to evaluate the triad's stability with probability q. Parameter p becomes p_{SBT} or p_{ST} depending on the chosen theory.
- (d) If the triad is unstable, then one of the links of the focal agent is flipped. If the links have differing polarities, then the negative becomes positive with probability p.



- q – hierarchy (status) importance
- p_{SBT} – probability of building friendly relations
- p_{ST} – probability of respecting others

Finding analytical solution

Possible simulation outcomes:

- Paradise state – all links positive (frozen state).
- Quasi-stationary, unbalanced state – density of positive links ρ fluctuates around a stationary level.

Analytical solution comes from detailed balance formula with π+ and π− describing rates of the evolution of positive and negative links, respectively:

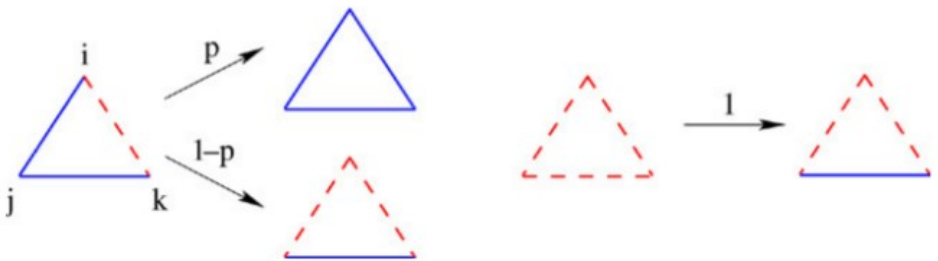
$$\pi^+(\rho; q, p_{SBT}, p_{ST}) = \pi^-(\rho; q, p_{SBT}, p_{ST})$$

Paradise is always a possible solution. Other possible solutions come from the quadratic equation in ρ:

$$\Big[2(1-q)(2p_{SBT}-1)\Big]\rho^2 - \Big[2(1-q)+(1-2p_{ST})q\Big]\rho + (1-q) = 0$$

Without status dynamics

- With the parameter *q=0*, ABM contains structural balance dynamics only.
- Such ABM is modified Local Triad Dynamics.
- Directed network
- Agent-based vs triad-based



Sources: T. Antal, P.L. Krapivsky, S.Redner Dynamics of social balance on networks, PRE, 2005
Social balance on networks: The dynamics of friendship and enmity, Physica D, 2006

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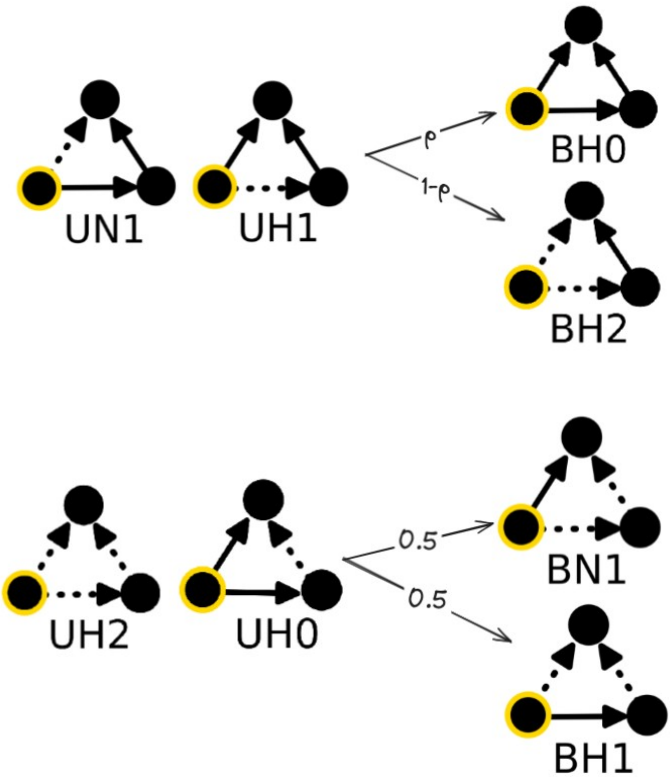
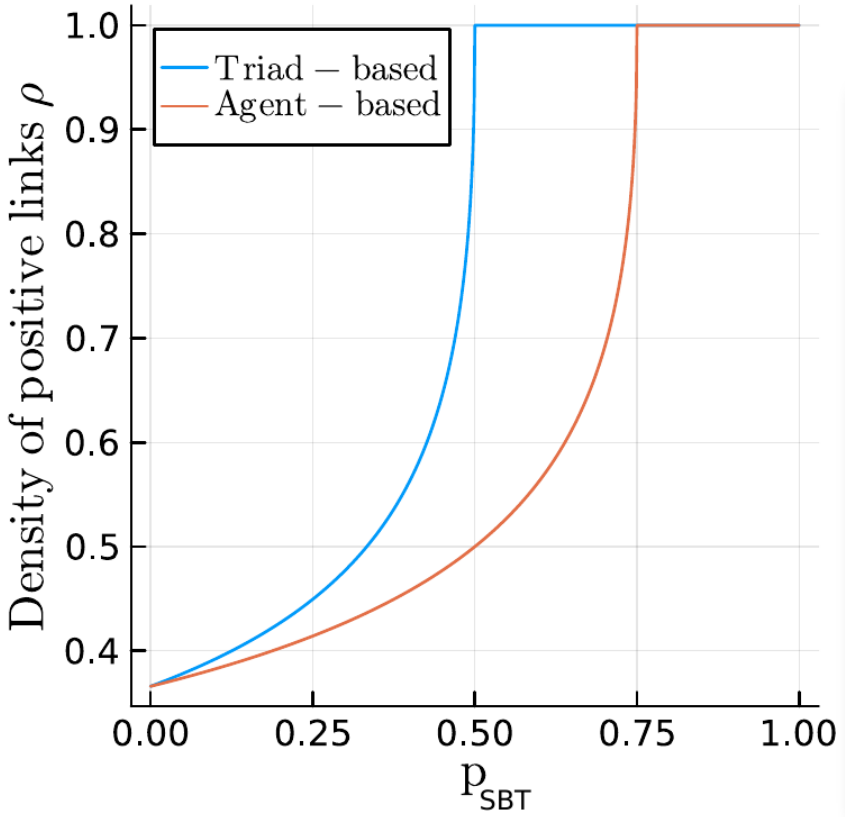
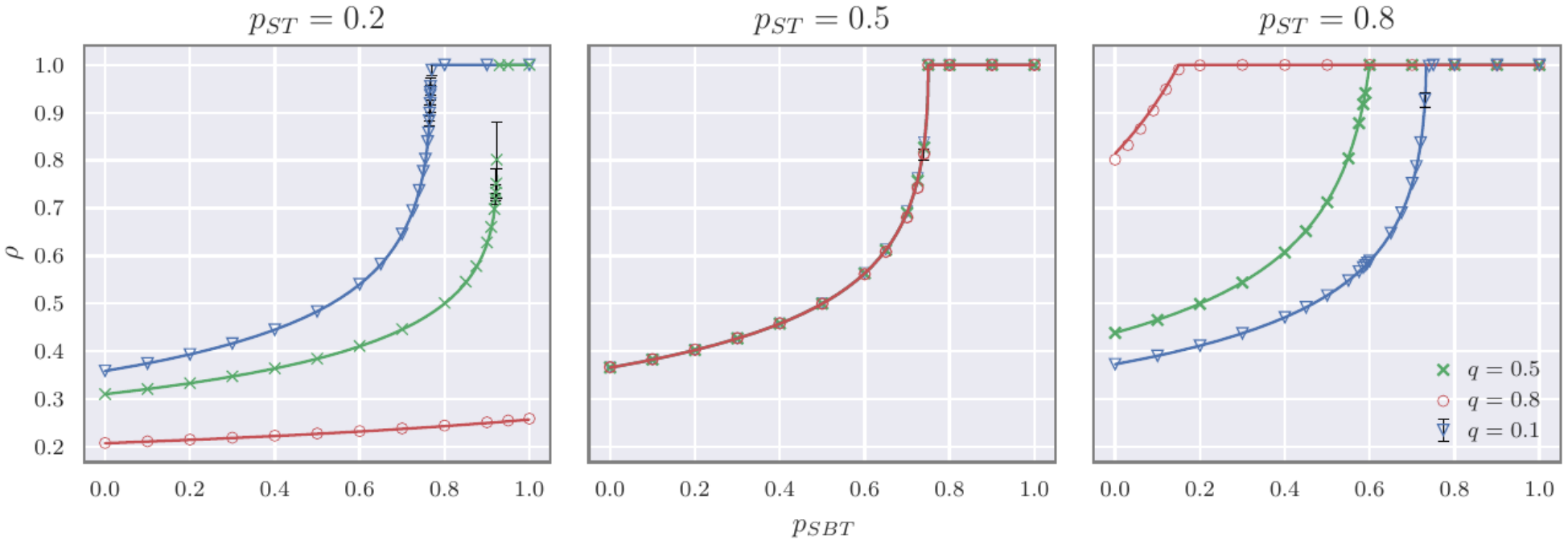


Without status dynamics

- With the parameter *q=0*, ABM contains structural balance dynamics only.
- Complete graph topology
- Continuous phase transition
- Agent-perspective makes paradise more difficult



Status introduces discontinuous phase transition

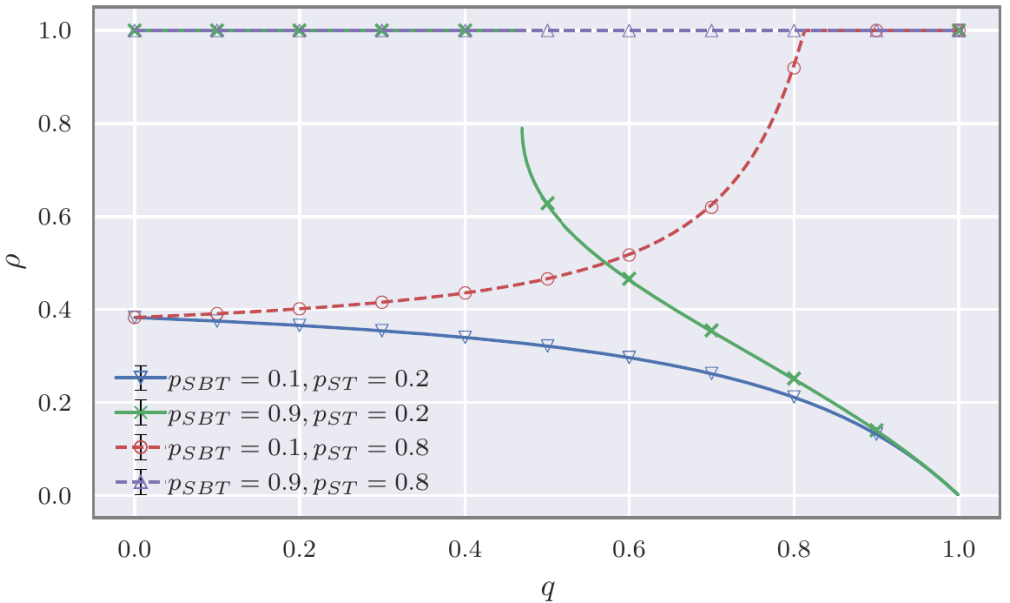


- Only with status (q > 0) and
- When $p_{ST} < 0.5$ (i.e., agents put themselves on top of their local hierarchies).

Any parameter can be a control parameter

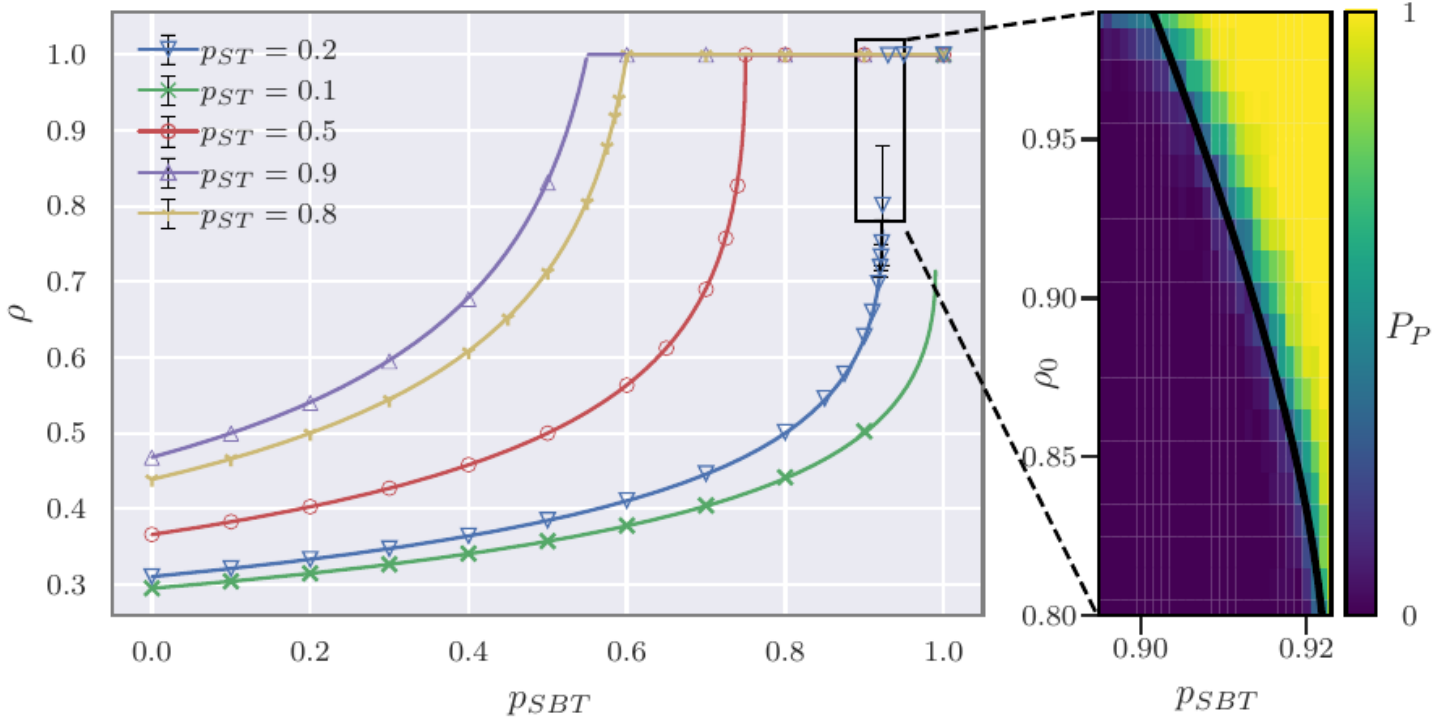
Discontinuous phase transition for q:

- When p_{SBT} is high (i.e., agents prefer friendly relations) and
- When p_{ST} is low (i.e., agents put themselves on top of their local hierarchies).



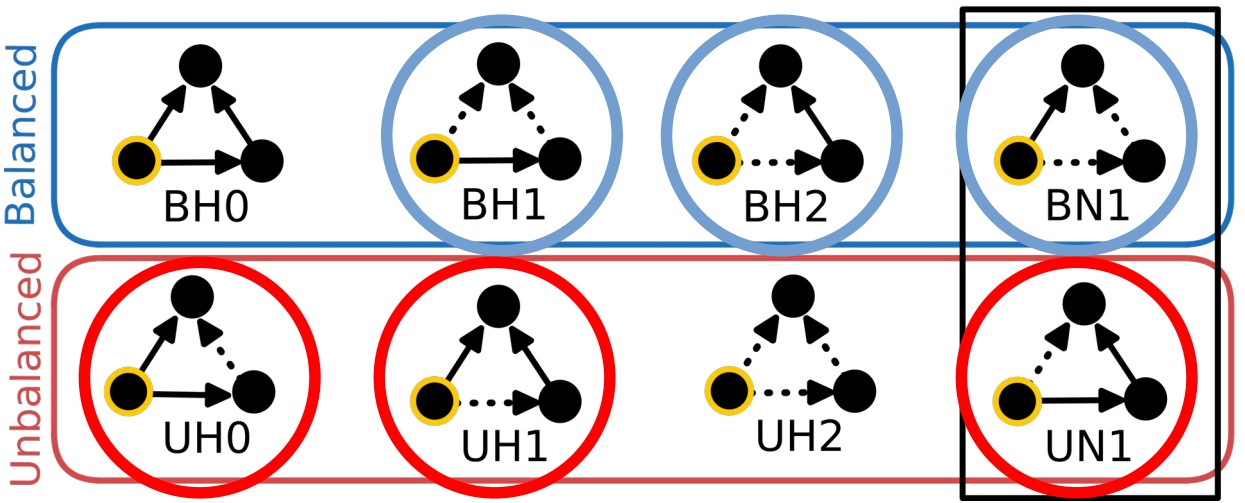
Discontinuous transition goes with separatrix

- Quadratic equation gives two solutions.
- Initial positive link density ρ_0 decides of the outcome



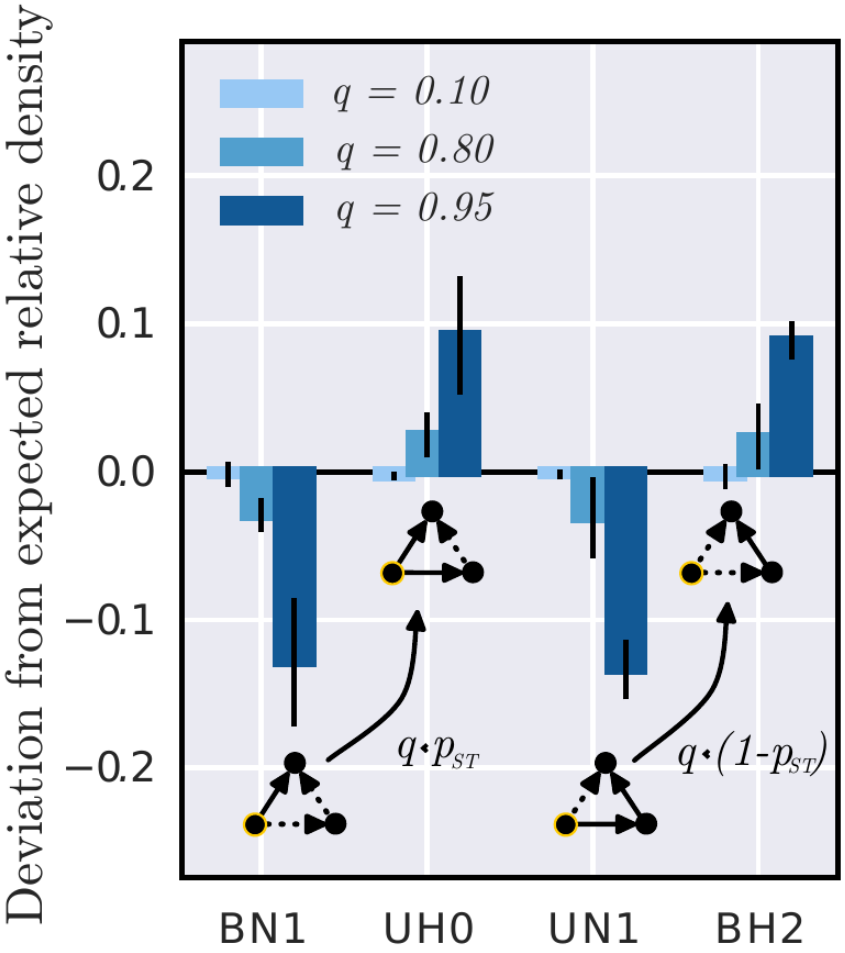
Under- and over-representation of triads

- Without status: same abundances of triads with same number of negative links.



Under- and over-representation of triads

- Increasing status and agent perspective: nonhierarchical triads – under-represented; hierarchical triad(s) – over-represented.
- Third pair of triads: not affected by status dynamics
- Expected relative density for each triad type is 1/3.



Digital traces of balance and status competition – chosen datasets

Large online social networks:

- Epinions (44k nodes, 11M edges)
- Slashdot (27k, 1.25M)
- WikiElections (4k, 745k)

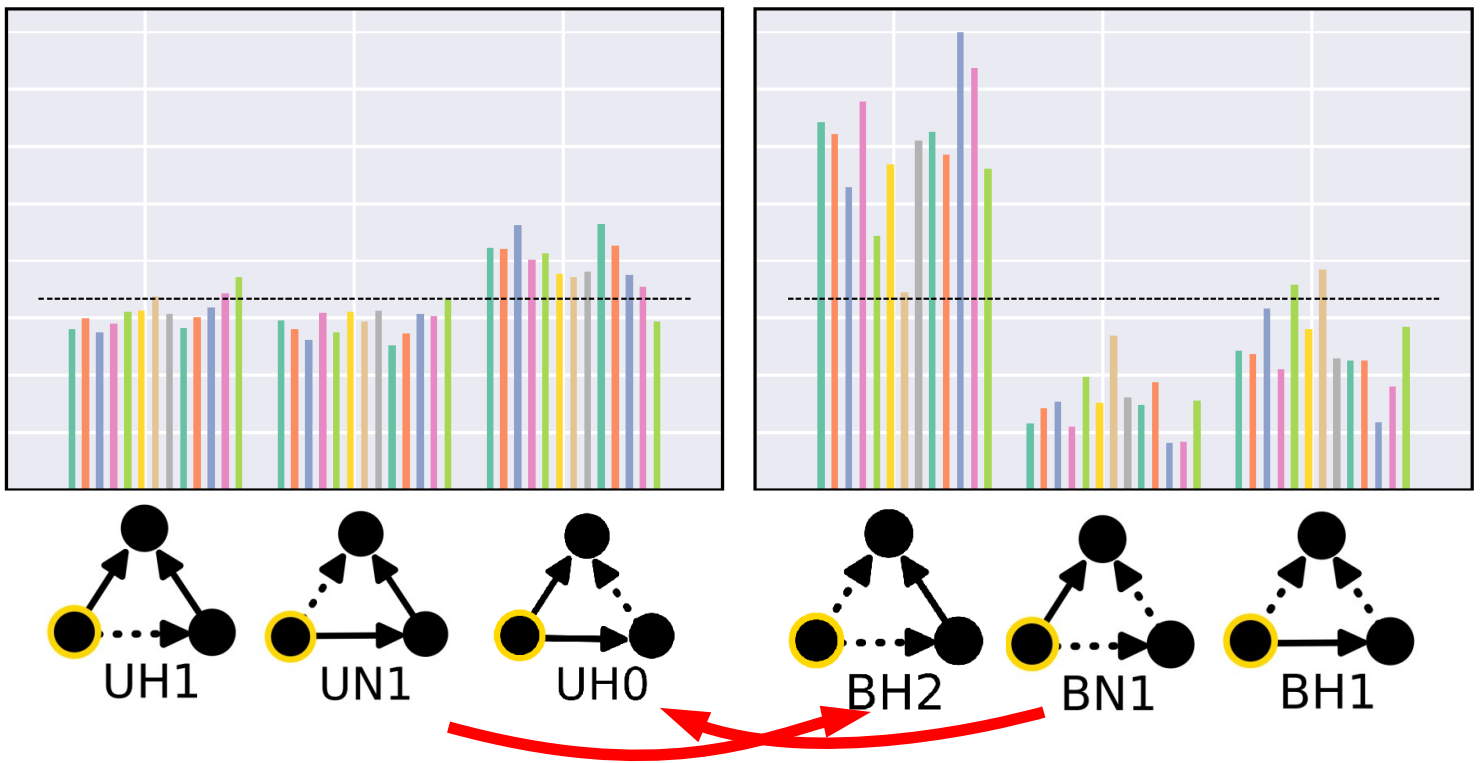
School networks*:

- 33 networks
- Agents: 3392
- Signed, weighted, directed edges: 57,568
- Agents' characteristics (e.g. prosociality, CRT scores)

*Source: M Ruiz-García, et al., Triadic influence as a proxy for compatibility in social relationships. PNAS, 2023

Digital traces of balance and status competition

School networks



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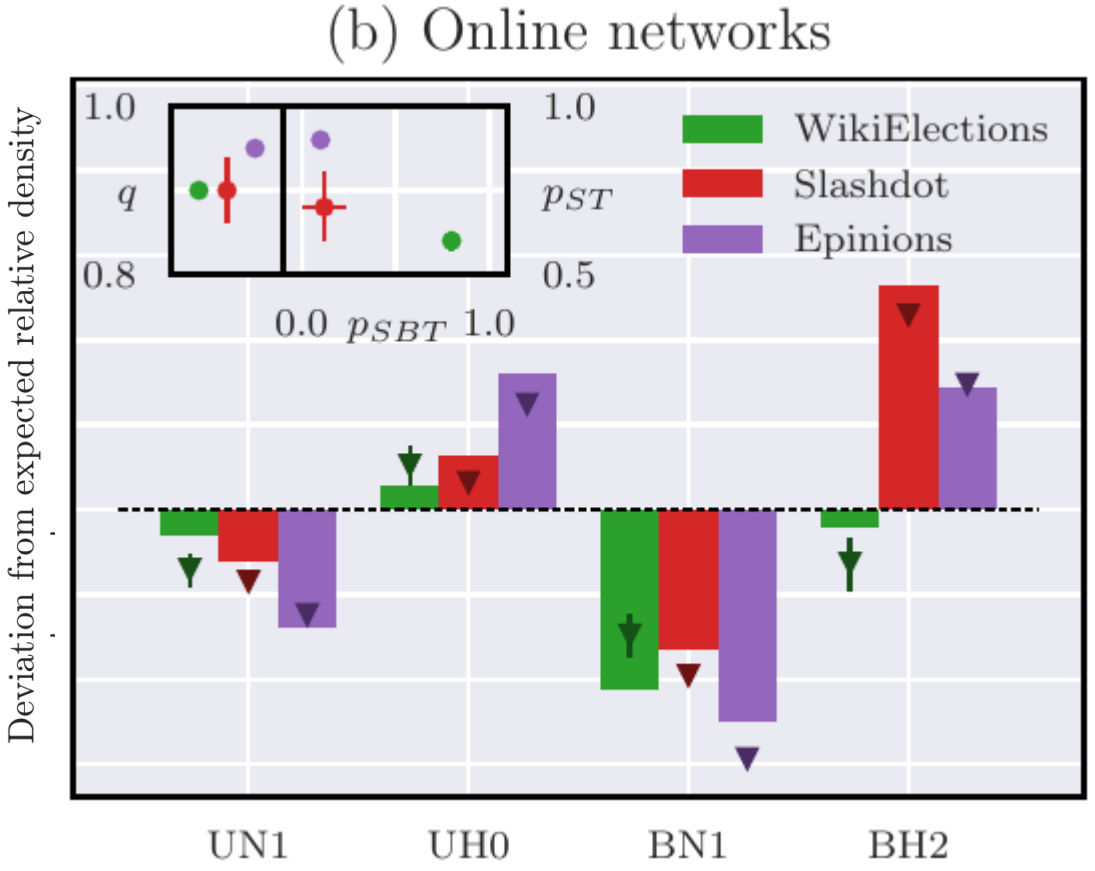
Fitting the model parameters to real systems

- For each dataset (WikiElections, Slashdot, Epinions, 33 school networks)
  - Compute:
    - Density of positive links
    - Ego-based triad density deviations
  - perform a grid search in parameter space
  - Look for parameter sets reproducing observed triad density deviations
  - Error function: MSE for obtained deviations

- Keep densities of positive links close to observed ones

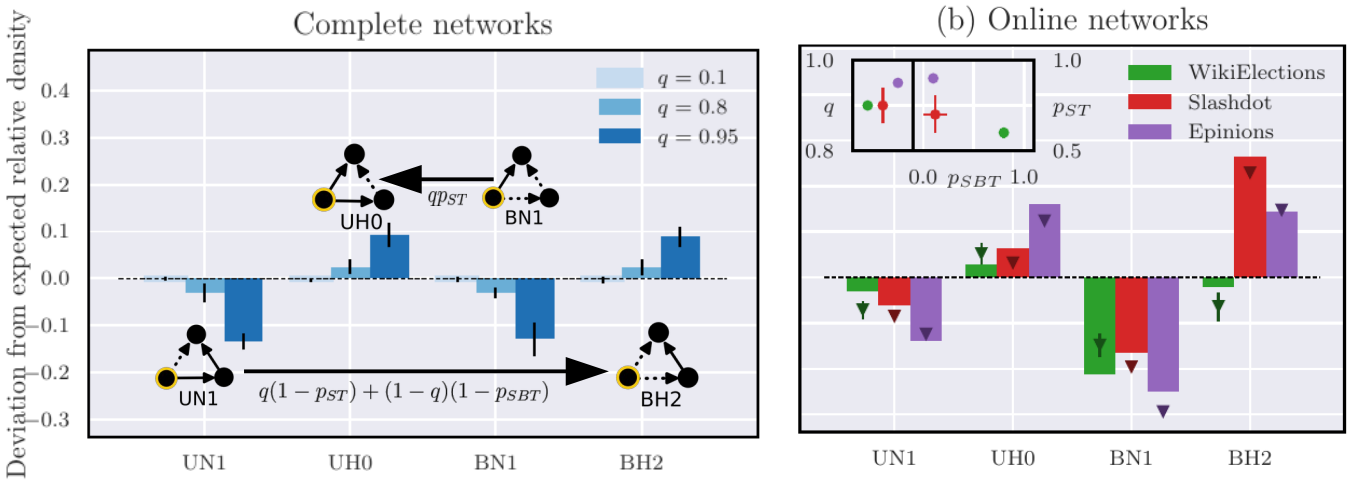
Fitting model parameters to large networks

- Status is more important in Epinions than in Slashdot and WikiElections
- Slashdot and Epinions are similar: (low p_{sbt}, high p_{st}).
  In conflict: favour negative relations, tend to respect others
- Question: how to connect obtained parameters to network/agent characteristics?



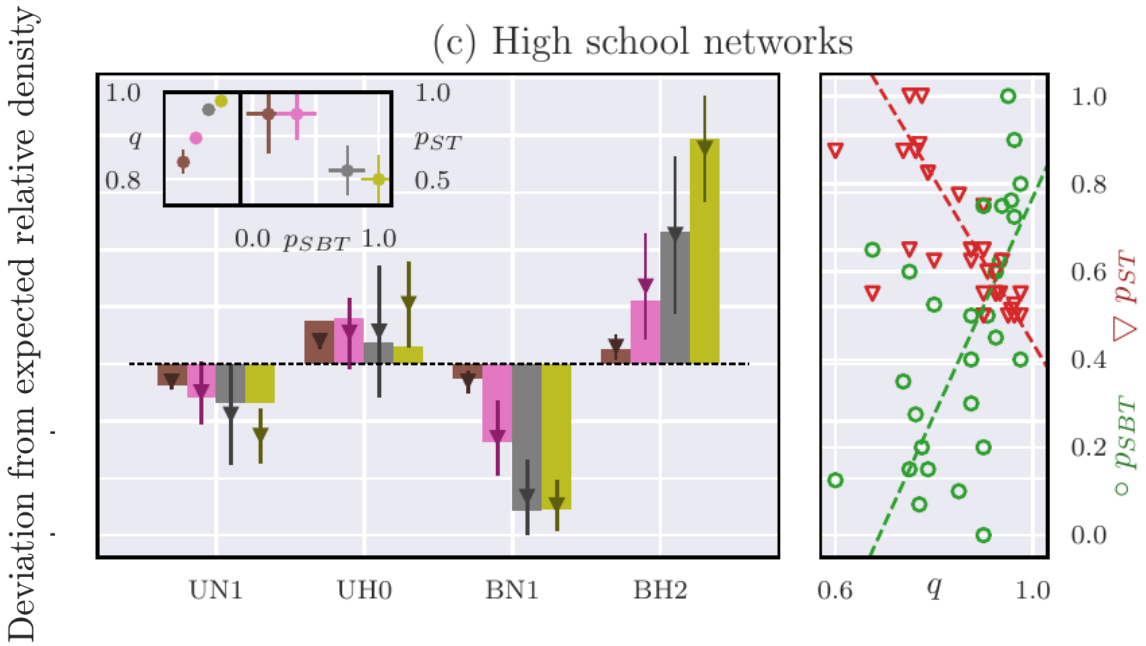
Fitting model parameters to large networks

- larger BH2 – low p_{SBT} (create enemy relations)
- larger UH0 – higher p_{ST} (respect others)



Fitting model parameters to school networks

- Varying level of status
- Correlation of parameters
- q is anti-correlated with p_{ST}: the more hierarchy is important, more frequently people place themselves on top of hierarchy.



Fitted parameters ~ System characteristics

- School dataset:
  - gender, *prosociality* score, CRT score of students.
  - weight of relations (strong/weak)
- Excluded 6 schools with high triangle interconnectivity
- Linear regression: What are the explanatory variables for status vs balance parameter q?

Fitted parameters ~ System characteristics

- Weak links: significant positive influence on q. Strong links: significant negative influence on q. => The stronger the relations are, the more often the individuals follow balance dynamics as compared to status.
- Weak but significant positive influence of mean prosociality on q => groups with higher antisociality tend to be less hierarchical. (~<=~ Halevy et al, 2011)
- Density of most antisocial people matter the most.

Conclusions

- Agent-based model combining structural balance and status theories with continuous and discontinuous phase transition analyzed numerically and analytically.
- ABM generates triad deviations which are also observed in real systems.
- We fit model parameters to real systems.
- We retrieve system factors that *take part* in balance vs status competition.