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ORIGINAL ARTICLE

Inferring attitudinal spaces in social networks

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Abstract

Ideological scaling methods have shown that behavioral traces in social platforms can be used to mine opinions at a massive scale. Current methods exploit one-dimensional left–right opinion scales, best suited for two-party socio-political systems and binary social divides such as those observed in the US. In this article, we introduce a new method to overcome limitations of existing methods by producing multidimensional network embeddings and align them with referential attitudinal for a few nodes. This allows us to infer a larger set of opinion dimensions from social graphs, embedding users in spaces where dimensions stand for indicators of several social dimensions including (in addition to left–right cleavages) attitudes towards elites, or ecology among many other issues. Our method does not rely on text data and is thus language-independent. We illustrate this approach approach on a Twitter follower network. Finally, we show how our method allows us to analyze the opinions shared within various communities of social networks. Our analyses show that communities of users that have extreme political opinions are also more homogeneous ideologically.

Keywords Network scaling · Graph embedding · Ideology · Political attitude data · Party systems · Polarization

1 Introduction

Over the last 10 years, social media, with the wealth of granular behavioral data they produce, have been imagined as a privileged source to mine opinions (for a recent extensive review, see the survey by Messaoudi et al. 2022). Inferred opinions may help answer numerous questions about online social and political dynamics. One can use such data to investigate the effect of political preferences on algorithmic recommendation systems (Bakshy et al. 2015; Ramaciotti Morales and Cointet 2021), or to uncover political motivations driving certain groups of actors and social movements (Budak and Watts 2015; Cointet et al. 2021) among others. Current methods for opinion mining leverage a diverse set of principles, including the geometrical approaches. We adopt such spatial approaches and position users of a social network in a space where dimensions are informative of opinions, acting as indicators of positive and negative attitudes towards relevant issues. These methods can be traced back to *ideology scaling* methods in political sciences (Poole and Rosenthal 1985) in recently in online social networks (Barberá 2015; Bond and Messing 2015). More recently, we proposed an extension to these methodologies, the *ideological embedding* method (Ramaciotti Morales et al. 2021), with which we were able embed social graphs in multi-dimensional *ideological spaces* in which each dimension acts as indicators of positive or negative attitudes towards several grouped issues of public debate, such as issues related to globalization or immigration, going beyond traditional left–right opinion scales. Building on *ideological space* methods, we further tackle the problem of the interpretability of dimensions and distances in space, and the bounds and reference points of the dimensions used as opinion scales. Mapping attitudinal data from political surveys to inferred *ideological embeddings*,

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we show that we can interpret the latent dimensions which emerge from social graphs. Put differently, we can determine which dimensions are related with negative and positive attitudes towards which issues. Additionally, we propose a method to map the *ideological space* onto the scales of the external attitudinal data, meaning that all the social media users following representatives can be mapped onto the scales of any external attitudinal data, allowing for comparison between countries, or between opinions at different moments in time.

Opinions may have different forms and functions, which makes them difficult to conceptualize and operationalize. An important type of opinion is that of *evaluative opinions*: that is, to be *for* or *against* something (Bem 1970). Evaluative opinions are often operationalized as *attitudes*: an individual disposition towards an attitudinal object (e.g., person, institution, issue, event, bill, policy position). Attitudes can also be held towards complementary attitudinal objects. A classical example are attitudes towards liberal and conservative values: positive attitudes towards one set of values imply negative attitudes towards the other, so that individuals can be placed on attitudinal scales ranging from the most liberal to the most conservative positions. Attitudes are, contingently on other factors, important determinants of behavior (Ajzen 1989; Ajzen and Fishbein 2005). The traditional method for estimating people's attitudes is the administration of surveys or polls (e.g., the ANES poll in the US, or the Eurobarometer in Europe). Besides polling, different behavioral traces can be observed to infer the attitudinal positions of individuals. A classical example is found in the work of Poole and Rosenthal (1985, 1984, 2000), in which they estimate attitudes of parliamentarians in the US towards liberal and conservative values using Bayesian inference on roll call data. Recently, Barberá (2015) used similar methods to infer the liberal or conservative attitudes of millions of US Twitter users with statistical inference on observed friendship—who follows whom—networks. This method postulates a model for the formation of a friendship network based on homophily (people with similar attitudes establish network ties; Lazarsfeld et al. (1954), and that the opinions of users are accounted for with a single latent *ideological* variable. The method then uses an observed network structure to perform an inference to estimate the ideological parameter. Importantly, these methods do not rely on textual data, making them language-independent. Empirical validation using external data (such as self-declared political affiliation) has shown that the recovered attitudinal variable in the US coincides with attitudes towards liberal and conservative values.

These methods for the inference of attitudes using social networks, however, have been much less successful in European and other settings (Barberá and Rivero 2015). This is due to the multi-issue, multi-party socio-political underlying systems that structure public debate (Hix et al. 2006; Benoit and Laver 2012), and ultimately online social networks such as Twitter. In this article we use methods inspired in network scaling on large social networks, and political science expert survey data with attitudinal positions of very few referential social network nodes. We show that, using both sources of data, it is possible to extract attitudinal positions for several issues for large portions of social networks. The result is an ideological embedding procedure for social networks in which dimensions stand as indicators for attitudes towards different issues, such as taxation, immigration, left–right cleavages, and trade protectionism, to name a few. The resulting spaces are *ideological* in the sense that the position of a user along each dimension provides information about the attitude of that user towards a set of combined issues. These latent ideological spaces reported by (Ramaciotti Morales et al. 2021) have known limits, which we discuss and address in this work. While dimensions can be shown to be related to positive and negative attitudes towards certain issues, they do not have a consistent metrical meaning. For example, a user in a position with value equal to 2 cannot be said to be twice as favorable than a user in a position with value equal 1. In other words, in ideological spaces order is readily meaningful but distances are not. Similarly, there are no evident referential positions in space. For example, a user in the position with value equal to 0 in the left–right scale cannot be said to have a central or apolitical position. To overcome these limits, we further map positions of *ideological space* onto the attitudinal spaces as defined by political surveys, which have scales with meaningful reference points (for example, a left–right ranging form extreme left-wing at 0 to most right-wing at 10) and explicit reference points (for example, a value equal to 5 meaning a central position in a left–right scale ranging from 0 to 10). Through this mapping, we position users in the scales of the survey as if all users in our sample had been placed using the survey. We illustrate this procedure with French Twitter data, and we propose benchmarks to test the validity of the approach. We also show the usefulness of our method by analyzing a fraction of the Twitter social graph. We separate the social graph into communities, and analyze their mean positions and homogeneity along several opinion dimensions. We show that communities that are in the political far-left and far-right extremes are more homogeneous, i.e., their members are less dispersed along the left–right dimension, providing novel empirical evidence to the study of phenomena related to the so-called "echo chambers" (Quattrociocchi et al. 2016).

This article is structured as follows. After discussing the related works that are pertinent to this article, we present the datasets on which we will illustrate our method. We then present the *ideological embedding* method as it was introduced by (Ramaciotti Morales et al. 2021), detailing the

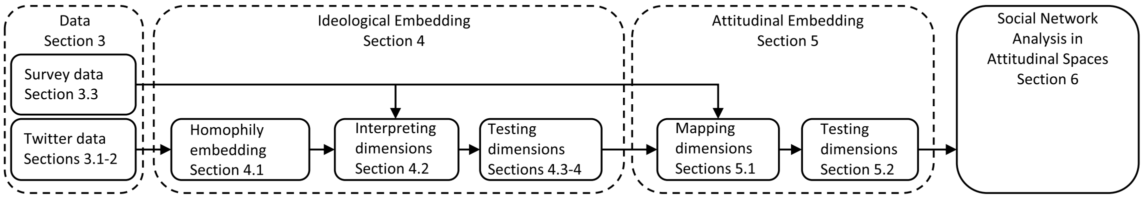


Fig. 1 Schematic representation of the sections containing the main results of this article, pointing at the dependency between sections

embedding method and the use of external attitudinal data to interpret the substance of the inferred dimensions. Next, we present the *attitudinal embedding* method, which consists in projecting the positions of users in *ideological space* onto the scales of the attitudinal spaces defined in external data. We then show different social network analyses that can be performed using these embedded social networks. In particular, we investigate how detected communities are distributed in our attitudinal spaces. Finally, we present our main conclusions and discuss about several possible applications of our method in diverse fields of research, such as the study of populism, polarization, and the effects of Recommender Systems. Figure 1 presents the main results of the articles as they are distributed in the following sections, indicating the dependencies between them.

2 Related work

The broadest field within social network analysis in which this work is inscribed is that of opinion mining (Liu and Zhang 2012; Messaoudi et al. 2022). This field consists of a diversity of techniques and models. Many methods rely on textual analysis (Gentzkow et al. 2019; Groseclose and Milyo 2005), and are thus language- and context-dependent. For example, people express themselves differently on different platforms, and a model capturing ideology from parliamentary debates (Rheault and Cochrane 2020) will necessarily differ from one predicting political slant in tweets (Stefanov et al. 2020). Our work is mainly concerned with a family of methods called ideological scaling. This family consists of methods that produce spatial models that explain choice data (e.g., parliamentarians voting for bills, users liking politicians online), in which dimensions hold some relation with choice, and counts a variety of applications (Imai et al. 2016). Scaling methods rely only on relational traces, producing ideological spatializations from topological relations: e.g., who is friends with whom, or votes for or clicks on what. The NOMINATE method by Poole and Rosenthal (1985) is a landmark and pioneering example. Using voting data from the US congress, Poole and Rosenthal were able to position Democrat and Republican parliamentarians on ideological left–right, and measure distances between parties as a proxy for polarization (McCarty et al. 2016)

Bond and Messing (2015) were among the first to apply ideological scaling methods to large social network choice data. In their pioneering work, they applied the same principle of NOMINATE on data on how users *liked* politicians on Facebook. The resulting spatial models allowed them to position these politicians present on Facebook on some left–right scale, according to how they were perceived by users. Their method also allowed them to position the nearly 6 million that gave *likes* on the online platform. The work presented in this article is related to a similar network scaling method, proposed by Barberá (2015) for one-dimensional scaling. This scaling computes a single latent ideological parameter ϕ_i for every user i, following an homophily probabilistic law adjusted for activity and popularity:

$$P\left(A_{ij} = 1 | \alpha_i, \beta_j, \gamma, \phi_i, \phi_j\right) = \mathrm{logit}^{-1}\left(\alpha_i + \beta_j - \gamma |\phi_i - \phi_j|^2\right), \quad (1)$$

where $A_{ij} = 1$ when user i follows user j, α_i and β_j are the "activity" (tendency to follow others) and "popularity" (tendency to be followed) of users i and j, ϕ_i and ϕ_j are their latent ideological parameters, and γ is a normalization constant. When computed in the bipartite network of the US parliamentarians and their followers on Twitter, using Markov Chain Monte Carlo methods, the ideological parameters ϕ_i were shown to act as indicators of attitudes towards liberals and conservatives. The inference of ideological parameters has been shown (Lowe 2008; Carroll et al. 1997) to be approximated by Correspondence Analysis (CA) (Greenacre 2017), which has also been verified empirically on Twitter data (Barberá et al. 2015). CA also allows for the inference of multi-dimensional ideological parameters, embedding users in ideological spaces where positions are informative of their attitudes (D'Esposito et al. 2014). In recent works (Ramaciotti Morales et al. 2020; Cointet et al. 2021) we have suggested that these multi-dimensional parameters might be related to attitudes towards several issues of public debate, beyond the classical one-dimensional attitude scale from left–right or liberal-conservatives cleavages. Several recent works leverage ideological inference, for example for measuring polarization in politics (Flamino et al. 2021) or around particular issues, such as climate change (Falkenberg et al.

2021). In this article, we further explore the extraction of multi-dimensional attitudinal indicators suggested by Ramaciotti Morales et al. (2020), and later by Ramaciotti Morales et al. (2021), detailing the inference of latent multidimensional parameters. In contrast with opinion inference relying on text analysis or news media citations (Baumann et al. 2020; Kwak et al. 2021), ideological embeddings relaying only social networks structures have the potential to be language- and context-independent, thus allowing, for example, comparisons between countries, and between users with data collected at different moments in time.

While the first sections of this article relate to methods for estimating positions in multidimensional empirical opinion spaces, Sect. 6 seeks to leverage these empirical spaces in social network analysis. In particular, leveraging both geometrical and topological analysis, Sect. 6 seeks to advance the understanding of the link between polarization and topological fragmentation: i.e., the so-called *echo chamber* phenomena (Van Alstyne et al. 1996; Cinelli et al. 2021). The literature on these two— related— phenomena is vast and, while we do not aim at providing an extensive literature review of them, we will provide a short overview of the thread of research within this domain to which our results are addressed. We will then provide further details on the conceptualizations of polarization to which our results provide further advances. Polarization, as an object of research, has existed in the attention of multiple disciplines for several decades (DiMaggio et al. 1996; Bartels 2000; Fiorina et al. 2005), recently regaining additional interest due to new hypotheses and data surrounding digital social media studies (Bail et al. 2018; Tucker et al. 2018; Guerra et al. 2013; Lee et al. 2014). The method for estimating continuous positions along dimensions of opinion spaces contributes to its understanding in several ways. Continuous opinion estimation, as opposed to bi- or multi-polar opinions (Bakshy et al. 2015) allows for the study of individuals that are not *polarized* (when polarization is conceptualized as the alignment or affiliation into one political camp. This distinction is important in studies that rely on comparisons between individuals with different degrees of polarization – e.g., (Osmundsen et al. 2021). Furthermore, while some studies have used ideology scaling to produce single-dimensional opinion models and analyze how individuals cite each other (Barberá et al. 2015) – thus integrating topology (of citation networks) and continuous opinion estimations – these are not meaningfully geometrical in the sense that they are not multidimensional. Our method addresses this by allowing embedding large networks in multi-dimensional spaces. This in turn, opens a path for the formalization of polarization as a property of the distribution of individuals in these spaces, addressing one of the major shortcoming in advancing towards meaningful multidimensional polarization measures (Bramson et al. 2016).

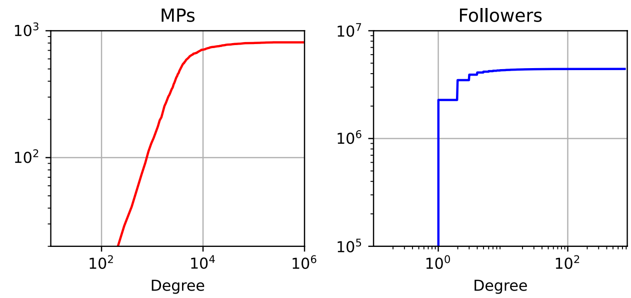


Fig. 2 Cumulative Distribution Function (CDF) of the degree of two types of nodes of the bipartite network of French MPs (in red) and their followers (in blue) (Colour figure online)

3 Social network and reference data

In this section we describe the data that we use in the rest of the article. These data are organized in three types: Sect. 3.1) a set of Twitter users for which we will illustrate the proposed method, Sect. 3.2) the social graph (i.e., the directed *who follows whom* graph) subtended by these users, and Sect. 3.3) external (to Twitter) data with attitudes of some reference points in several opinion scales as given by political survey.

3.1 Selecting a set of Twitter users

To select a set of users on which to apply the methods proposed in this article we use French Twitter networks of friends[1]. We consider the set of the 831 (out of 925) French Members of Parliament (MPs) present on Twitter who are affiliated to 10 parties and their followers[2]. We choose this starting point for collection, as we expect, based on abundant evidence (Barberá 2015; Barberá and Rivero 2015; Briatte and Gallic 2015; Ramaciotti Morales et al. 2020) that the choice of MPs to follow will be revealing of implicit political opinions and preferences. This collection was conducted in May 2019, and resulted in 4.487.430 unique followers. In this bipartite network, some MPs are followed by a handful of users (min. degree = 38) while others are followed by nearly half of the users that follow any MP (max. degree = 2.241.986). On the other hand, many users in this network (2.279.199) follow only one MP, with the most diverse user following 757 MPs (see Fig. 2). To filter out inactive or bot accounts, and accounts without enough ideological

[1] Please refer to the Acknowledgments section to consult the data registry and treatment declaration, related GPDR documents, and access to the respective legal notice.

[2] Obtained from http://www2.assemblee-nationale.fr/deputes/liste/reseaux-sociaux for deputies, and http://www.senat.fr/espace_presse/actualites/201402/les_senateurs_sur_twitter.html for senators.

referential connections, we follow the criteria proposed by Barberá (2015) and consider only followers that follow at least 3 MPs, and that have at least 25 followers. We also removed followers with a repeated set of followed MPs to obtain 368.831 followers to ensure the full rank of the adjacency matrix. Using the bipartite graph of MPs and their 368.831 followers we will illustrate the ideological and attitudinal embedding methods in the next Sects. 4 and 5.

3.2 Social graph data

The previously described bipartite graph between MPs and their followers will be used to mine opinions through our embedding methods. Additionally, we consider the social network formed by these users for which opinions will be inferred. For these 368.831 users, we collected the subtended directed social graph (who *follows* whom). Some users have disabled permissions to have their followers collected, thus resulting in a graph of 230.911 users and 67.217.556 edges (density = 0.00126). Having also the social graph subtended by 230.911 of the initial 368.831 nodes, will allow us to produce social network analyses using both opinion spatial data and topological network data in Sect. 6. We will refer to this friendship network as "social graph", while we will refer to the links between MPs and their followers (from Sect. 3.1) as the "bipartite graph".

3.3 External referential attitudinal data

To interpret the dimensions of ideological spaces, we use external attitudinal data. These external data contain the positions some referential points in attitudinal scales with predefined bounds (ranging for example from 0, *most opposed*, to 10, *most favorable*), and associated with predefined issues of public debate: e.g., European integration, special rights for minorities, anti-elite sentiments. These external data will also provide a referential attitudinal frame onto which to project social network users. Since our data collection strategy is based on accounts of MPs (belonging to known political parties), we choose to use external referential frames for political parties, which will serve as reference points in space. We use the 2019 Chapel Hill Expert Data (CHES) (Bakker et al. 2020), corresponding to the period on which our Twitter data were collected. The CHES is compiled using the responses of a survey administered to 421 political scientists, in which they place European political parties on scales from 0 to 10 (or from 1 to 7 for the scale of attitudes towards EU integration) for 51 different issues. From the 51 issues, we exclude 3 related to attitudes towards Turkey and the conditions needed for EU accession, which are not available for all parties, thus resulting in 48 dimensions. These data

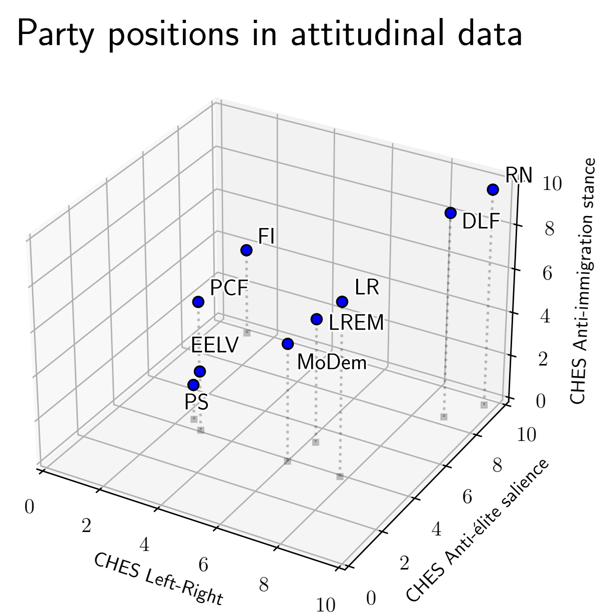


Fig. 3 Position of French political parties in attitudinal reference space provided by the Chapel Hill Expert Survey (CHES) on 3 chose dimensions (out of 51): left–right, Anti-elite salience, and Anti-immigration stance

include some scales of interest for comparison with previous studies, such as a left–right scale, but also include new important emerging societal cleavages observed by (Ramaciotti Morales et al. 2021). Figure 3 illustrates the contents of this external referential attitudinal space by showing the position of French political parties in three chosen dimensions: left–right, Anti-elite salience, and Anti-immigration stance.

4 Ideological embedding

This section describes the *ideological embedding* method, as presented by (Ramaciotti Morales et al. 2021). We first show how to exploit generative social network models relying on homophily to infer latent dimensions on which to embed users. Next, we use external attitudinal data to show that some of these dimensions are ideological in the sense that they act as indicators of attitudes towards several political issues. We provide to types of validations to this ideological embedding: (1) comparing how online text utterances are correctly positioned in ideological space (e.g., users declaring themselves as being right-wingers are positioned accordingly in the emerging left–right scale), and (2) by showing that people that declare sympathy for a political party, are positioned in the spatial vicinity of that party.

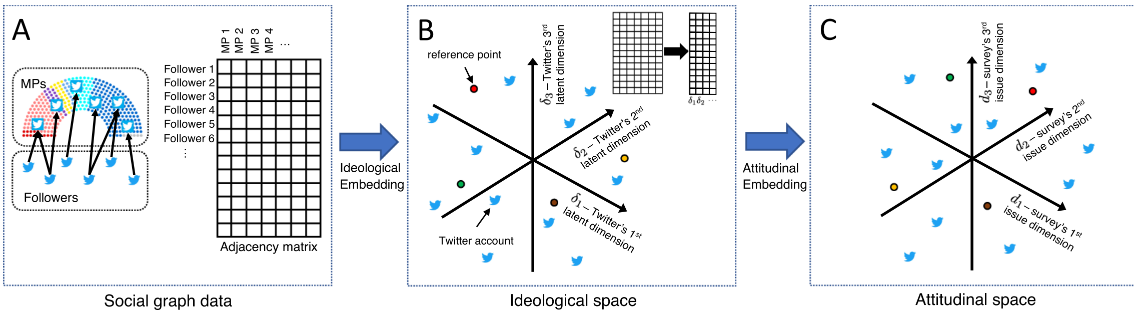
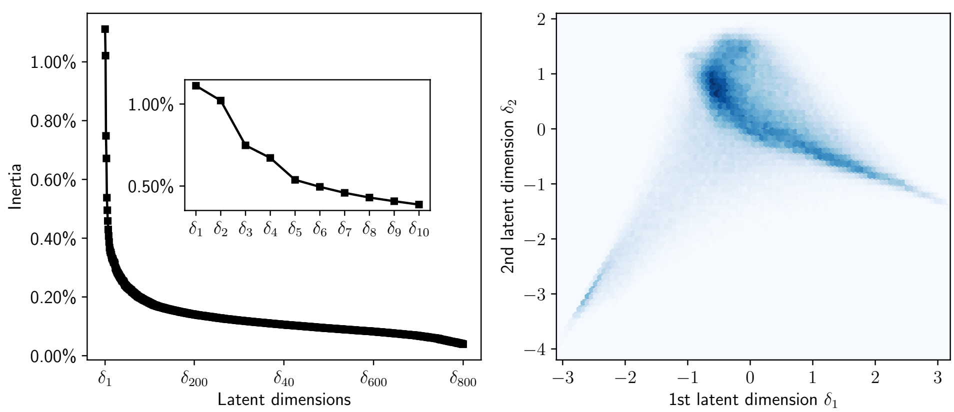


Fig. 4 Illustration of the ideological and attitudinal embedding methods with the data they take as input and what they produce as output. Ideological embedding takes a bipartite network of the parliamentarians (MPs) and their parties **A**, to produce a spatialization in a latent ideological space **B**. Attitudinal embedding takes the ideological positions (**B**) of all users, plus the positions of some reference points in both ideological, to embed all users in this reference space **C**

Fig. 5 Inertia of the principal components of the Correspondence Analysis of the bipartite network of MPs on Twitter and their followers (left), and density of users on the first two components, δ_1 and δ_2 (right)



4.1 Homophily embedding

First, we represent the bipartite subgraph of the 831 MP and their 368.831 followers as an adjacency matrix $A \in \{0,1\}^{|368.831| \times |831|}$ (subfigure A in Fig. 4). Next, we produce a reduced-dimensionality representation of these 831 MPs and the 368.831 followers using the coordinates of accounts in the latent dimensions of the CA of the adjacency matrix (subfigure B in Fig. 4). The map between subfigures B and C using, e.g., an affine transformation between these two multidimensional spaces, is computed using reference points present in both ideological and attitudinal spaces, and are the object of Sect. 5. Lowe (2008) has shown theoretically, and Barberá et al. (2015) empirically, that CA provides an estimation of parameters ϕ of model from Eq. 1, when these parameters are assumed to be multi-dimensional, thus performing the latent space embedding. This first ideological embedding is said to be an homophiliy embedding in the sense that the underlying social mechanism leveraged to infer positions is homophily: in Eq. 1, the closer that two users i and j are in the latent space (i.e., the lower the value $\|\phi_i - \phi_j\|$, for ϕ_i, ϕ_j vector parameters), the higher the probability that an edge will be observed between them. We denote the emerging latent dimensions of this bipartite graph by δ_1 to δ_{831} in decreasing order of inertia associated with each dimension. Figure 5 shows the inertia of the latent dimensions of CA, and the density of the 368.831 followers in the space spanned by the first 2 latent dimensions, i.e., δ_1 and δ_2, of this reduced-dimensionality space. Keeping the same notation δ, we apply a normal standardization to each dimension to ensure that the dimension-wise mean of users is at $\delta = 0$, and that they have equal variance. When most of the inertia is concentrated in the first dimensions, a spatial representation in the first few dimensions is a suitable representation of the topological data in the sense that a random graph computed using such spatialization and the chosen probabilistic model from Eq. 1 would be similar to the original network (Roberts Jr 2000). As seen in Fig. 5, the first two dimensions hold relatively more importance in explaining the topological network data.

4.2 Interpreting the dimensions using external attitudinal data

To link the dimensions of our space with issues of public debate present in the CHES data, we compare the positions

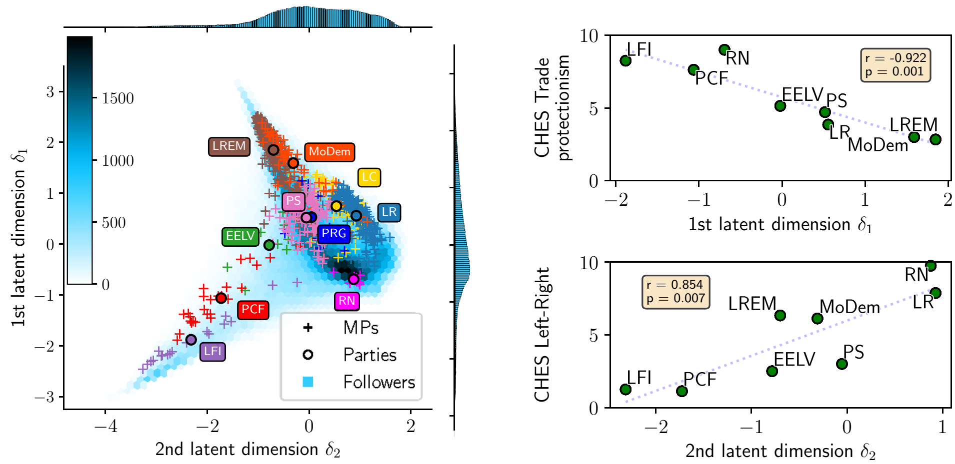


Fig. 6 Positioning of parties in the reduced-dimensionality space spanned by the first two latent dimensions of the ideological space of the bipartite Twitter network, computed as means of the positions of parliamentarians (left), and examples of two external party attitudinal positions (CHES) that are correlated with first two dimensions (right)

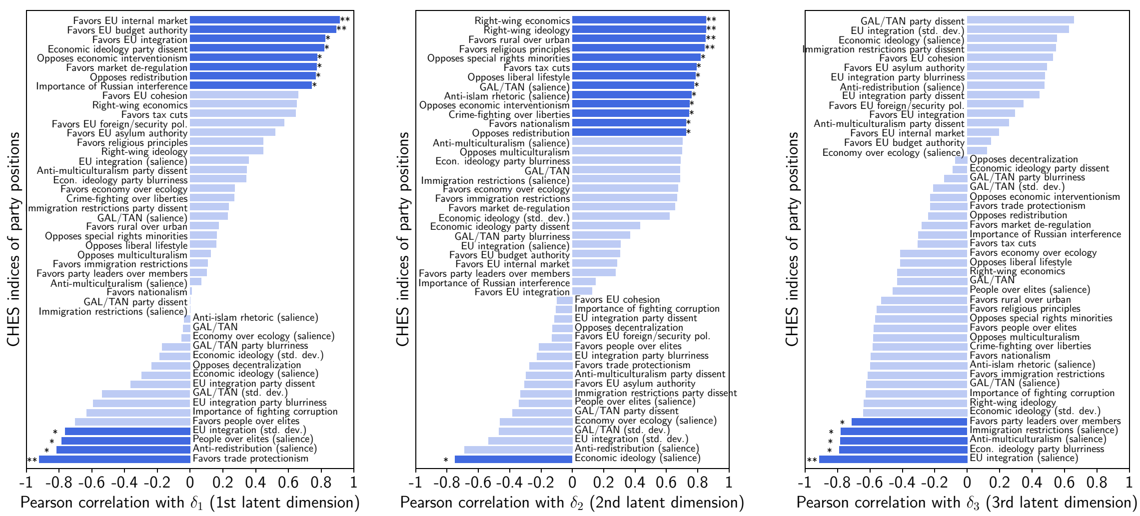


Fig. 7 Pearson correlations of the positioning of parties on Twitter and according to CHES data for the first three latent ideological dimensions, and for all CHES dimensions, each associated with an issue of public debate (*marks 0.05 confidence and **marks 0.01 confidence levels) (Colour figure online)

of political parties according to our latent dimensions, with their positions in the attitudinal CHES dimensions (scoring parties in a 0–10, or 1–7 scale, ranging from *very opposed* to *very favorable* to each issue). To compare a latent dimension with a CHES dimension, we compute the Pearson correlation of party positions on both. We compare each one of the first 15 latent dimensions (as suggested by Fig. 5) with all 48 available CHES dimensions. We find that only the first 3 latent dimensions show statistically significant correlations with CHES dimensions (Fig. 6). These correlations show that each one of the first 3 latent dimensions is associated (up to a correlation significance of $\alpha = 0.05$, marked in dark blue in Fig. 7) with a set of CHES dimensions.

The first latent dimension, δ_1, is positively correlated with positive attitudes towards the EU, suggesting that the higher the value of the δ_1 position of a user, the more positive their views are towards the EU. δ_1 is also correlated with opposition to redistribution, economic interventionism, and market regulation. It is also correlated with high levels of left–right economic policy dissent (inside parties), and with the relevance granted to Russian interference in politics. Negative δ_1 positions are correlated with positive views on economic protectionism, and with a high level of importance is placed on people-elites cleavages and redistribution. These issues are related to attitudes towards globalization in European settings, and we call δ_1 the “Local-Global” (LG) axis. The second latent dimension, δ_2, is associated with positions on issues widely attributable to left–right cleavages in France, including: left–right economic and ideological positions, rural-urban cleavages, religious principles, rights of minorities, authoritarianism, and the balance between fighting crime and civil liberties among others. We call this second dimension the “Left–Right” (LR) axis. Attitudes towards issues related to internationalization (e.g., trade

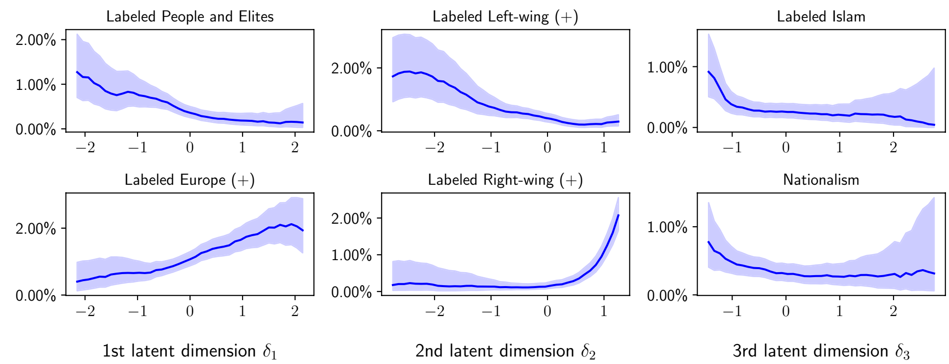


Fig. 8 Proportion of Twitter users labeled as referring to different topics on their text profiles according to their positions along the three axes of ideological space, with $\alpha = 0.05$ confidence intervals. Symbol + indicates whether only those referring to the issue with positive have been considered

protectionism, EU integration) have long been recognized as growing in importance in structuring individual preferences (e.g., in voting behavior; (Grossman and Sauger 2019) when compared with the importance of attitudes towards issues relevant to left–right cleavages (e.g., privatization, taxation, welfare spending). The third latent dimension to exhibit correlations with CHES dimensions, δ_3, is associated with views on the relative importance between leaders and members within parties, with economic policy blurriness, but mostly with the importance granted to European integration, multiculturalism, and immigration restrictions. We call this third dimension the "Immigration and multiculturalism" (IM) axis. This third axis acts as an indicator of the importance granted to these issues (salience): lower positions in δ_3 relate increased importance.

We call the latent space and *ideological space* because it is latent with respect to the model of Eq. (1), which is similar to the *ideal point* model used in roll call data (Clinton et al. 2004). Our dimensions also span an *ideological space* in the sense that they position users according to attitudes towards a set of correlated issues (i.e., issues for which attitudes are not independent). While the notion of *ideology* refers to "fuzzy" (Van Dijk 1998) sets of different concepts (including normative ones relating to power, e.g., the choice of rulers, as described by Lane (1962), or the justification of power, as described by McClosky (1958), we chose to use this term in its descriptive dimension: as an "organization of opinions, attitudes and values" (Adorno et al. 1950), a "structure of attitudes" (Campbell et al. 1960), or as a description of high attitude consistency (Converse 1964). When a single variable is informative of a set of attitudes towards some grouped set of issues because they display high spatial correlation, we call this an ideological dimension.

4.3 Testing the positions of users with text analysis

These associations presented so far were computed and justified using the positions of parliamentarians and their parties. We now seek to validate the definition of our LG, LR, and IM axes using the Twitter profile descriptions of their 368.831 followers, in which users briefly describe themselves. Twitter profiles are short texts in which users describe themselves and are shown on the profile page and profile preview of each user. For each one of the three axes we select two topics, and for each topic we define a minimalist dictionary to classify the profile of followers. Our goal is to show that, even when applying a minimal text analysis, our three ideological axes distribute users (and not only parliamentarians) according to the meaning proposed for these dimensions. For the LG axis we classify users with labels "Europe" (if they include the words "eu" or "europe") and "People and Elites" (if they include the strings "peuple", meaning *people* in French, "élite", or "politicien"). For the LR axis we label users as mentioning the "Left-wing" (if they include the string "gauche", meaning *left* in French) and as mentioning the "Right-wing" (if they include the string "droite", meaning *right* in French). For the IM axis we label users as mentioning "Islam" (if they include the strings "musulman", meaning *muslim* in French, or "islam") and as talking about "Nationalism" (if they include the string "patriot" or "patriotique" or "patriotisme"). In the context of the French public debate, questions of multiculturalism and immigration policy rapidly revolve around a debate on Islam (Hargreaves 2007; Freedman 2017). In the strings that define a topic, we included all possible variants and misspellings. For the labels "Europe", "Left-wing", and "Right-wing" we only included profiles with positive sentiment, to differentiate those that express support for the referred topic (users might use the word "right", e.g., to express criticism of right-winger). We computed sentiments using a BERT-based multilingual model for sentiment analysis[3]. For the other labels we did not filter by sentiments, as we are trying to detect the importance or salience of the label along the dimensions (which does not involve positive or negative opinion). Figure 7 shows that axis IM is, for example, correlated with the salience of the issue of immigration

[3] https://huggingface.co/nlptown/bert-base-multilingual-uncased-sentiment.

restrictions. Figure 8 shows the proportion of users that use these topics in their profiles and the Clopper-Pearson confidence interval of this proportion at a $\alpha = 0.05$ CI. Selection of profiles by positive sentiment is signaled in Fig. 8 with a + symbol. Our results show that, on the sets of users considered for text-based validation (selected by keywords and sentiments when pertinent), ideological dimensions produce an order among users. For example for the LR axis, a user to the left of another has higher probability of being more to the left politically than more to the right, and that this probability increases with distance along that dimension. Users at the extreme left and right edges of the sample have very low probability of being incorrectly positioned: a user describing itself using the word "right" with a positive sentiment, e.g., has very low probability of being in the leftmost edge of the sample.

As many studies suggest, left–right cleavages (corresponding to our LR axis) have a structuring role in behavior (e.g., voting behavior; (Aldrich et al. 2010) and in particular on French Twitter (Briatte and Gallic 2015). Indeed, LR axis is among the first two latent dimensions of our ideological space. In our dataset, however, as suggested by other studies (Miklin 2014; Schön-Quinlivan 2017; Grossman and Sauger 2019), social cleavages related to globalization (e.g., European integration, increasing trade openness, and other related to our first latent dimension, LG) take an even more important role. This is a relevant finding, indicating the declining role of left–right cleavages, now observed second to globalization in its importance to determine social choice in digital arenas.

4.4 Testing the positions of party sympathizers

A second way of looking at the position of the followers is by examining the relation between those that declare sympathies towards a party and their position in space with respect to those of the party itself. While it is difficult to account for partisanship, we propose a simple approach based on positive mentions of parties in the Twitter profile description. To do so, we propose again a minimalist approach to classifying users as sympathizers of a given party, based on a few strings for each one of the ten political parties in our dataset:

- EELV (*Europe Écologie - Les Verts*): "eelv", "les verts";
- LFI (*La France Insoumise*): "insoumis", "lfi";
- LREM (*La République en Marche*): "lrem", "en marche";
- MoDem (*Mouvement Démocrate*): "modem", "mouvement démocrate";
- LC (*Les Centristes*): "centristes";
- PCF (*Parti Communiste Français*): "pcf", "communiste";
- PRG (*Parti Radical de Gauche*): "radical de gauche", "parti radical", "prg";
- PS (*Parti Socialiste*): "ps", "socialiste";
- RN (*Rassemblement National*): "rn", "rassemblement national";
- LR (*Les Républicains*): "lesrepublicains", "lr".

As before, we also consider singular and plural, masculine and feminine declensions when possible, as well as upper- and lower-case versions of our keywords. We also consider hashtag for keywords made of a single word (e.g., "pcf" and "#pcf"). Finally, as in the previous section, we filter out possible negative references using the same method for estimating sentiments. Figure 9 shows the position of party sympathizers in comparison with that of the ensemble of followers and the position of the political parties computed as the mean position of their MPs.

5 Attitudinal embedding

The ideological space described in the previous section has important properties, but also limitations. Correlation with attitude dimensions from external data such as CHES, and later validation using text analysis, shows that dimensions act as indicators in the sense that they produce a relative order of users along them. This order is identified with a set of group issues that provide the substance of the ideological dimension. In ideological spaces, however, the notion of distance is less clear, and we lack important reference points. To illustrate these points, let us examine the results along the LR ideological axis. Concerning distance, for example, a user at a position with value equal to 2 on the LR axis (thus on the rightmost edge) cannot be meaningfully said to be half as radical or extreme than another user at a position with a value equal to -4 on the LR axis (thus on the leftmost edge of our sample). This is because ideological space has a provable ordering property but not a well-defined metric. Similarly, regarding reference points, a user at a position with value equal to 0 on the LR axis cannot be meaningfully said to be a centrist user, and it can only be said to be in the mean of our sample in that dimension. These distinctions are important, in part because whether a user is a political radical (and to what degree) or moderate, is at the heart of research questions regarding online behavior (Osmundsen et al. 2021). Ideological spaces are also not comparable: if we take two ideological embeddings from different datasets (at two moments in time, or between networks extracted in two different countries), their positions cannot be readily compared to extract general conclusions regarding comparative order. For example if we collect a bipartite graph of the followers of MPs a month later, and if this bipartite graph has changed, we cannot compare directly the positions of a user that is present on both moments. Finally, because ideological dimensions act as attitudes towards a grouped set of issues, it is difficult to extract conclusions regarding

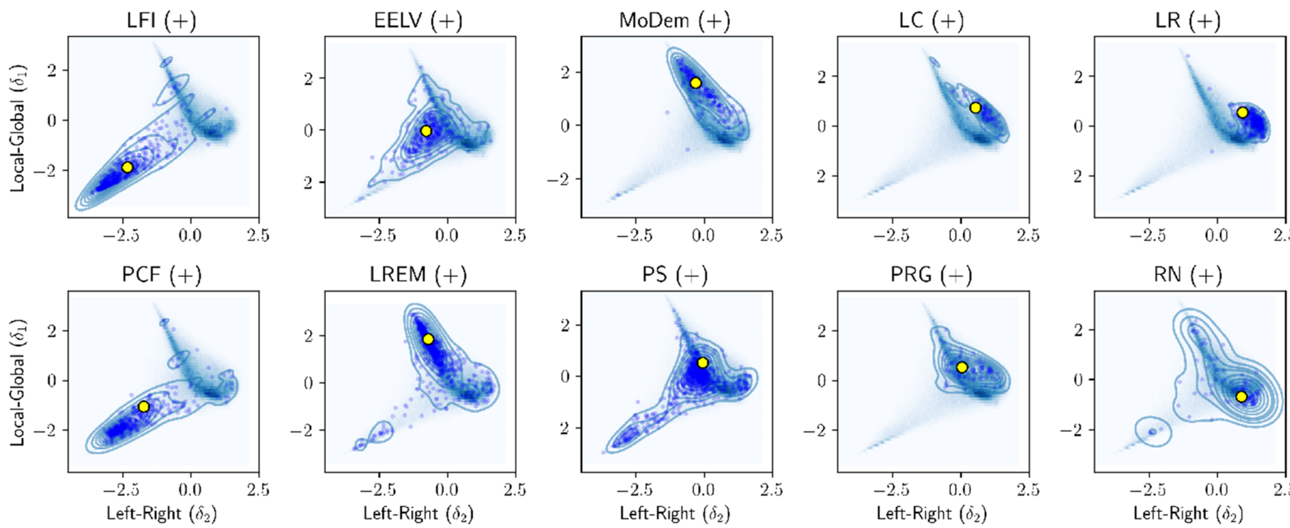


Fig. 9 Position of the users identified as sympathizers (mentioning them with positive sentiment) of each one of the ten parties in our dataset (identified with blue dots), and the position of the parties computed as the mean position of its MPs (shown in yellow dots (Colour figure online)

particular individual issues. For example, in Fig. 6, LFI (*La France Insoumise* party) is to the left of PCF (*Parti Communiste Français*) in the latent so-called LR dimension, but it is less straightforward to compare these two parties in two attitudinal dimensions that are included in the definition of the LR axis, such as their attitudes towards tax cuts, authoritarianism, or nationalism.

Exploiting the identified relations between the dimensions of our ideological space and the attitude dimensions of instruments such as CHES, we propose to tackle these aforementioned shortcomings of the ideological embedding method by mapping ideological space positions (subfigure B in Fig. 4) onto the attitudinal dimensions predefined by an instrument such as CHES (subfigure C in Fig. 4), using parties as reference points to compute the mapping. We call this procedure *attitudinal embedding*. The attitudinal dimensions of an external instrument such as the CHES have several advantageous properties. To illustrate this, let us take again left–right cleavages as an example. The CHES Left–Right attitudinal dimension specifies 3 spatial reference points: 0 being the extreme left for parties, 10 being the extreme right for parties, and 5 being a centrist position for parties. We emphasize the fact that these references are for parties, as it is conceivable that some users are more extreme than parties. By mapping all users in our dataset onto this CHES Left–Right dimension, we seek to obtain an approximation to the position that each user would have gotten, had it been evaluated by the CHES instrument (i.e., had it been known, examined, and positioned by the same experts, which is impossible for many reasons, including economic ones).

Relying on external attitudinal dimensions also solves the problem of comparison: two ideological embeddings taken at two moments in time can be compared if projected onto the same CHES dimension, and two ideological embeddings taken on two countries can be compared, if the instrument has been applied in those two countries. The CHES dataset would allow this in theory, as it is administered in 32 countries: users from two different countries embedded using the way in which they follow local MPs, could be projected onto a common CHES dimension for comparison.

5.1 Mapping ideological space onto attitudinal reference frames

The results from the previous section, in particular the correlations shown in Fig. 6 and more systematically in Fig. 7, suggest that at least some ideological dimensions could be approximated by affine subspaces of the attitudinal space defined by the 48 attitudinal dimensions available in the CHES dataset. Thus, to map ideological spaces onto this attitudinal reference frame or attitudinal space of 48 dimensions, we seek to establish an affine transformation between ideological and attitudinal spaces (of correspondingly 831 and 48 dimensions). Determination of this affine transformation may be seen as an optimization problem using the positions of the 8 political parties that are present in both spaces (see the right subfigure in Fig. 6). Let $P = 8$ be the number of political parties serving as guidance for the fit of the affine transformation, $M = 48$ the number of dimensions of the CHES attitudinal space, and N the number of

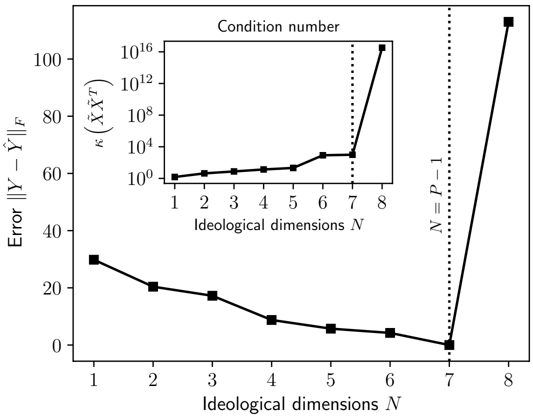


Fig. 10 Error in estimating position of $P = 8$ political parties in attitudinal reference CHES space ($\hat{Y}$) when using an affine transformation fitted with a varying number N of ideological dimensions, compared with the actual positions (Y), and the condition number of matrix inverted during fitting

latent ideological dimensions δ to be considered for the optimization of the affine transformation $T_{\mathrm{aff}} : \mathbb{R}^N \to \mathbb{R}^M$. Let $Y \in \mathbb{R}^{M\times P}$ be the position of parties in the CHES attitudinal space, and $X \in \mathbb{R}^{N\times P}$ the position of parties in the ideological space. The optimization problem is then to determine an optimal affine transformation T^*_{aff} that minimizes some error between Y and $\hat{Y} = T^*_{\mathrm{aff}}X$. We posit the transformation equation as an augmented matrix one (also called *homogeneous coordinates*), for $\widetilde{T}_{\mathrm{aff}} : \mathbb{R}^{N+1} \to \mathbb{R}^{M+1}$:

$$\underbrace{\begin{pmatrix} Y \\ \hline 1 \end{pmatrix}}_{\widetilde{Y}\in\mathbb{R}^{(M+1)\times P}} = \underbrace{\left[\begin{array}{c|c} A & B \\ \hline 0\cdots 0 & 1 \end{array}\right]}_{\widetilde{T}_{\mathrm{aff}}\in\mathbb{R}^{(M+1)\times(N+1)}} \cdot \underbrace{\begin{pmatrix} X \\ \hline 1 \end{pmatrix}}_{\widetilde{X}\in\mathbb{R}^{(N+1)\times P}}, \tag{2}$$

where $A \in \mathbb{R}^{M\times N}$ and $B \in \mathbb{R}^M$. We choose as error metric the Frobenius norm of $Y - \hat{Y}$:

$$\|Y - \hat{Y}\|_F = \sqrt{\sum_{d=1}^{M}\sum_{p=1}^{P}\left|Y_{dp} - \hat{Y}_{dp}\right|^2}. \tag{3}$$

Next, we tackle the question of the number of dimensions N that should be used for optimizing $\widetilde{T}_{\mathrm{aff}}$, as for any chosen value the error is minimized by the pseudo-inverse $\widetilde{T}^*_{\mathrm{aff}} = \widetilde{Y}\widetilde{X}^T\left(\widetilde{X}\widetilde{X}^T\right)^{-1}$ (Penrose 1956; see Dokmanić and Gribonval 2017, for further details). We know that at least the first three dimensions of the ideological space δ_1, δ_2 and δ_3 (that we named LG, LR and IM) are correlated with some CHES dimension. To explore the gain in error reduction we compute it for different and growing values of N. The result of this exploration (see Fig. 10) is that adding ideological dimensions, beyond the first three identified in the previous section, contributes to reducing the error (up until $N = 7$). The error is of the order of 10^{-12} when $N = 7$, which is also the number of ideological dimension for which the system of (2) is determined: it is easy to check from (2) that there are $M(N + 1)$ unknowns and MP equations, and that the system is under-determined for $N < P - 1$, over-determined for $N > P - 1$, and determined for $N = P - 1$. As seen in Fig. 10, over-determined systems produce much greater errors, accompanied by the deterioration of the condition number of the matrix that is inverted in the fit process.

Using the affine transformation fitted for $N = 7$, we map the position of all of our 368.831 users onto the CHES attitudinal reference space. In Fig. 11 we show the spatial distribution of these users, and of the MPs and their parties using four attitudinal dimensions of the CHES data: CHES left-right, CHES Anti-elite salience, CHES EU integration, and CHES Importance of ecology. All parties are inside the bounds of the CHES survey (which is answered by experts positioning parties within these predefined bounds). It is noteworthy that not all MPs are within these bounds, which is natural considering that some have more extreme positions than those of their parties, and that Twitter users might perceive it that way. This supposes that the position of parties are mean of the position of their MPs. Likewise, not all of their followers are within the bounds of the boundaries, as it is possible that many users are more extreme in their positions that the positions held by parties. Before extracting conclusions from the distribution of users in attitudinal space, we proceed to test their positions using text analysis.

5.2 Testing the positions of users with text analysis

As before, we use the self-descriptions written by users to validate that the dimensions are correctly positioning users along attitudinal dimensions. We begin by comparing the already selected labels from Sect. 4.3 (see Fig. 8) with the corresponding dimensions in the CHES attitudinal reference space. We observe (see Fig. 12) that the users featuring these labels are positioned coherently in the new CHES attitudinal space[4].

Next, we show that there is a variety of newly available attitudinal dimensions related to new issues that can now be retrieved. To produce validations for additional dimensions, we extracted terms from the Twitter profile self-descriptions of users, and ranked them by term frequency. We then proceeded to associate these terms with new labels potentially relevant for CHES dimensions whenever

[4] In comparison with Fig. 8, these new validations from Fig. 12 have wider confidence intervals for the labels "Islam" and "Nationalism" for values close to 0 in the CHES Multiculturalism salience dimensions. This is due to the fact that there are very few total users in that region of attitudinal space, but that we are including them in the figure for the sake of spanning the [0,10] interval that serves as reference for those CHES dimensions.

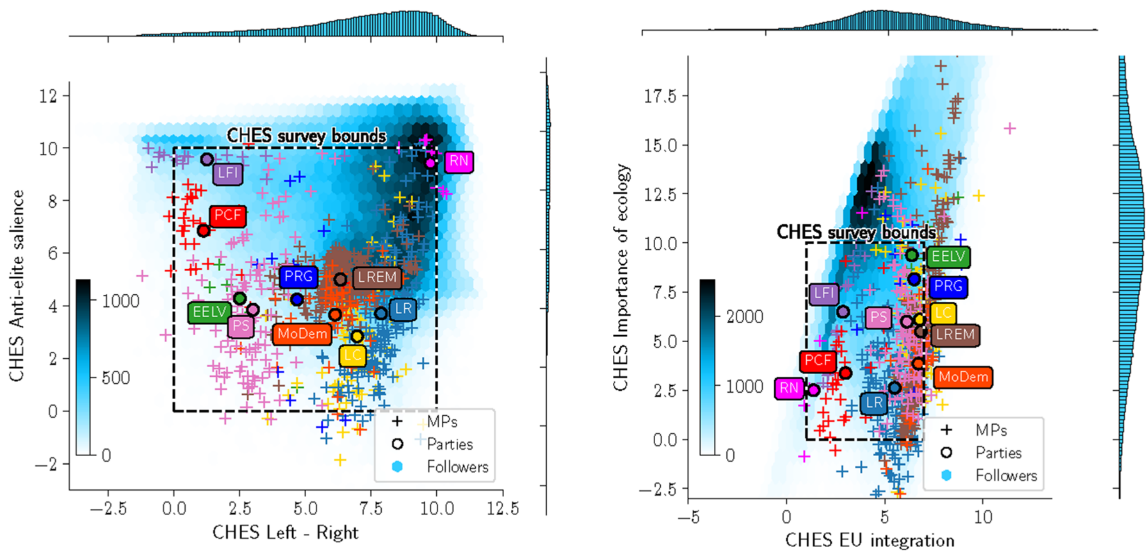


Fig. 11 Spatial distribution of the MPs, their followers, and their parties using four attitudinal dimensions of the CHES data: left–right, Anti-elite salience, EU integration, and Importance of ecology

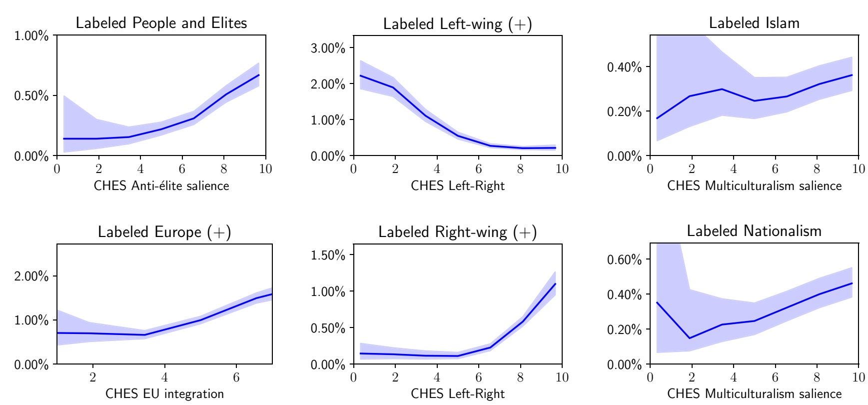


Fig. 12 Proportion of Twitter users labeled as referring to different chosen topics on their bios and according to their positions along corresponding dimensions of attitudinal space. Symbol + indicates that we consider only users indexed with the label and whose descriptions express a positive sentiment

possible, using a minimalist approach as in Sect. 4.3. This led to the identification of 29 labels that could be directly linked to 24 CHES attitudinal dimensions (from the available 48). For example, we identified the label "Ecology", because 8.180 users included that word in their Twitter profiles, and because as a concept, ecology, is pertinent to two CHES dimensions: "Importance of ecology" (or salience of the issue) and "Economic growth over ecology" (meaning attitudes more favorable towards economic growth when confronted as an alternative to protecting the environment). We present the best 8 cases for which our labeling strategy offers the sharpest contrast (related to 5 new labels, see Fig.13) with their respective CHES dimensions. These 8 cases are related to the following additional labels:

- "Ecology": defined by the words "écologie" or "écologiste";
- "Tradition": defined by the word "tradition";
- "Agriculture": defined by the words "agriculture" and "agriculteur";
- "Social Security": defined by "sécurité sociale" and "droits sociaux" (meaning *social rights*);
- "Entrepreneur": defined by "entrepreneur".

Figure 13 shows the distribution of the proportion of users indexed with these different labels, according to pertinent related CHES attitudinal dimensions. When the CHES dimension is intended to measure positive or negative attitudes towards an attitudinal object, we further filter by sentiment, indicating the sentiment used to retain users with symbols + and –. For example, the "CHES Opposition to multiculturalism" is intended to distinguish cleavages between favorable and opposite view towards the attitudinal object, namely "multiculturalism". When the CHES dimension is intended to measure the importance

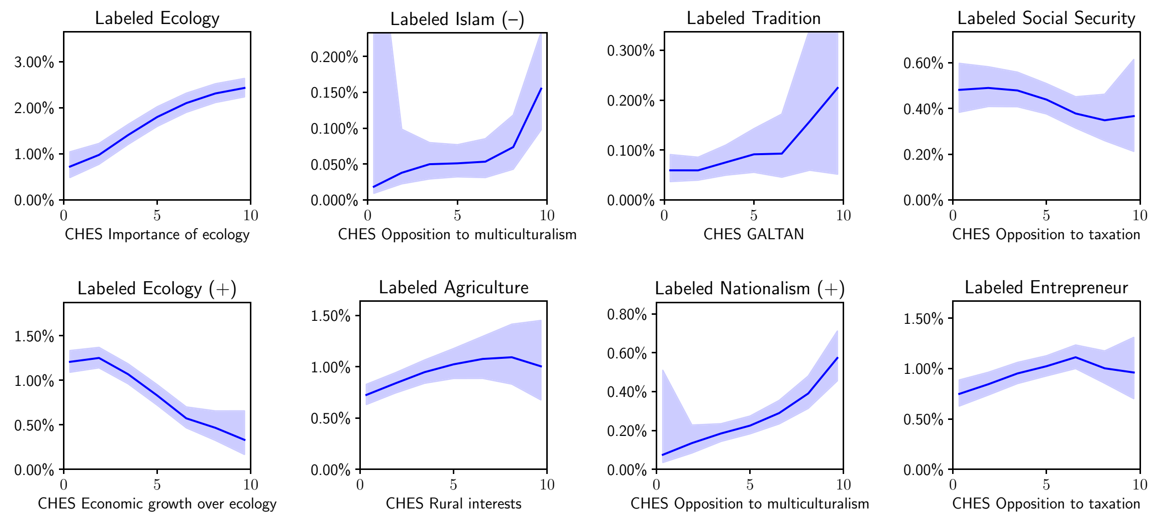


Fig. 13 Proportion of Twitter users labeled as referring to different topics on their profiles according to their positions along selected dimensions of the CHES attitudinal space. Symbols + and – indicate that the users with the label have been further filtered to keep those whose text expresses a positive or negative sentiment

granted to an issue, or its salience, we do not filter by sentiment. This is the case, for example, of “Ecology” and CHES Ecology salience. A second case of use without sentiment is when the keywords already presuppose a stance. This is the case, for example of the label “Entrepreneur”. Many users define themselves as entrepreneurs on Twitter (2.185 in our dataset). We expect that people that are not entrepreneurs will not use the word them in their text profiles. Also we expect that people describing themselves as entrepreneurs might do so without negative or positive sentiment. The CHES GALTAN dimension[5] refers to a socio-political cleavage dimension between Green-Alternative-Libertarian and Traditional-Authoritarian-Nationalist individuals or parties, introduced in 2002 in the context of research looking for cleavages beyond left–right stances (Hooghe et al. 2002). In the CHES survey, the question regarding the positioning of parties in the GALTAN dimension is framed as 0 being most liberal, 10 being most traditional, and 5 being the center.

The recovered CHES dimensions in Fig. 13 confirm the main hypothesis behind our attitudinal embedding method: looking only at friendship networks (to the exclusion of textual data), in this case a bipartite network MPs and their followers, it is possible to infer the attitudes of these followers not only on left–right scales (which is the current state of the art), but also on a larger number of dimensions relating to several issues of public debate that are participating in social choice on internet platforms (in this case Twitter). In what follows, we will focus on a small set of CHES attitudinal dimensions for which the validation is the strongest (“CHES Left-Right”, “CHES Anti-elite salience”, “CHES EU integration” and “CHES Importance of ecology”), to illustrate how these attitudinal spaces can be leveraged in extracting conclusions of social facts from social networks network data.

6 Social network analysis in attitudinal spaces

Having computed and validated the embedding of the network in an attitudinal space where dimensions have (1) reference positions (such as a extreme-left, extreme-right and center), (2) metrical consistency across space, and (3) dimensions explicitly related to single issues or cleavages, we now turn to the analysis of the 230.911 users for which we also know the subtended social graph, as described in Sect. 3.2. In particular, we are interested in the relation between topological communities, and the attitudinal stances of the members of these communities along some previously identified CHES attitudinal dimensions. To identify communities in the social graph of the directed friendship network on Twitter, we infer a degree-corrected stochastic block

[5] The full definition of CHES attitudinal dimensions can be seen in the survey’s *codebook*: https://www.chesdata.eu/2019-chapel-hill-expert-survey.

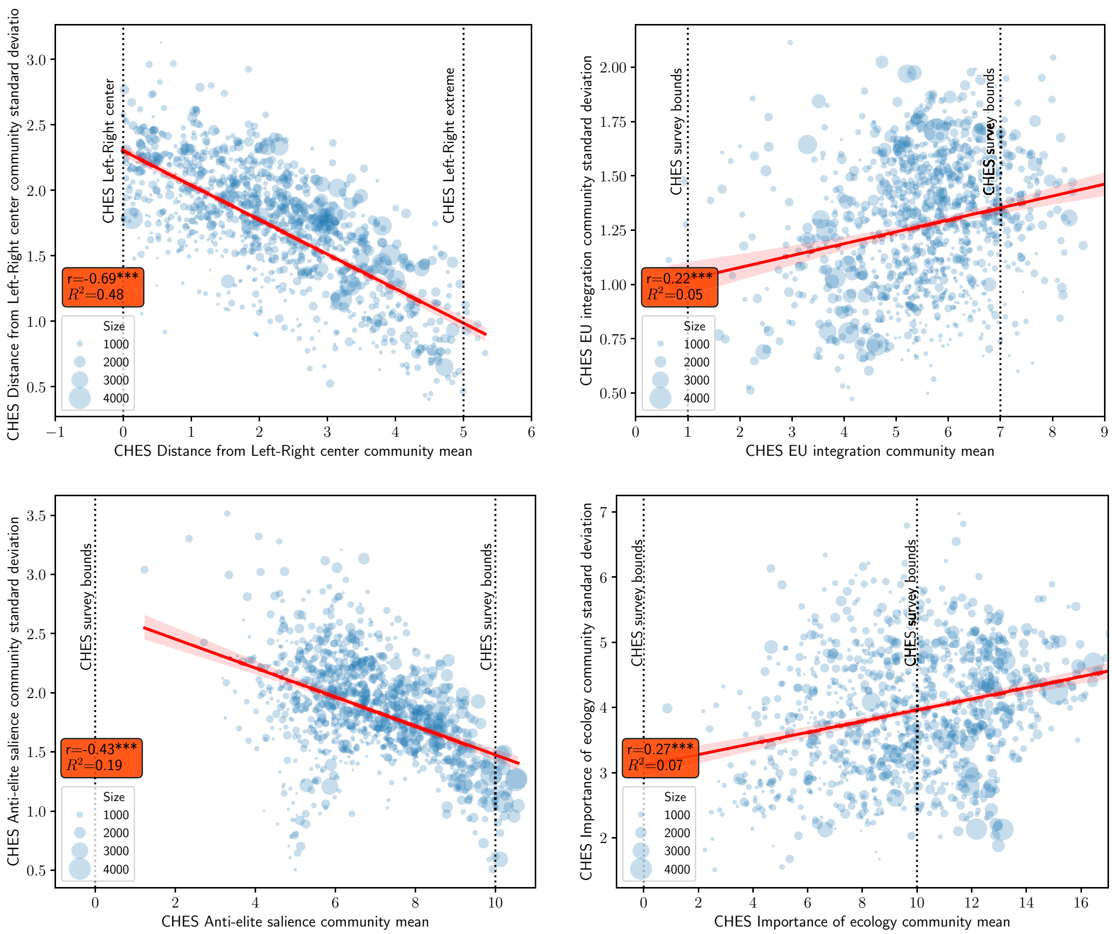


Fig. 14 Scatterplots of the 1.067 communities of the Twitter social graph for four selected CHES attitudinal dimensions, according to the mean and standard deviation of the members of the communities (***marks a p-value < 0.001 for Pearson correlation r) (Colour figure online)

model by minimizing description length using *graph-tool*[6] (Peixoto 2014). The result is the partition of the 230.911 nodes of the network into 1.067 non-overlapping communities. Next, we select four CHES attitudinal dimensions ("CHES Left–Right", "CHES Anti-elite salience", "CHES EU integration" and "CHES Importance of ecology") and compute, for each community and for each dimension, the mean position of the members of the community, and the standard deviation of the positions of the members. For the "CHES Left–Right" dimension, we consider the distance from the central position (defined as 5 on the 0–10 scales on the CHES survey), as we are interested on how extreme stances relate to community structures. Figure 14 shows a scatterplot of the 1.067 community of the social graph, for the four selected CHES dimensions, according to the mean and standard deviation of their members. Red curves show the best linear model for the communities position and standard deviation along each one of the four selected CHES dimensions.

These observations have interest in the context of the hypothesized but ill-defined *echo chambers* in social networks (Quattrociocchi et al. 2016; Baumann et al. 2020; Cinelli et al. 2021). Broadly speaking, *echo chamber* refers to a situation in which a group of users develops few interactions with contents and other users outside their group. This phenomenon holds importance as these relatively isolated users might develop extreme views—for example, through selective exposure (Bryant and Miron 2004; Kwak et al. 2010)—fragmenting populations into groups with sufficiently different world-views to the point that it may obstruct social deliberation and coordination regarding common issues (e.g., climate change; Williams et al. 2015). Having the ability to inspect the community structure of the social graph in some important attitudinal dimensions provides new means for quantifying the degree to which a network is fragmented along different issues. Figure 14, for

[6] https://graph-tool.skewed.de/static/doc/inference.html#graph_tool.inference.minimize_blockmodel_dl.

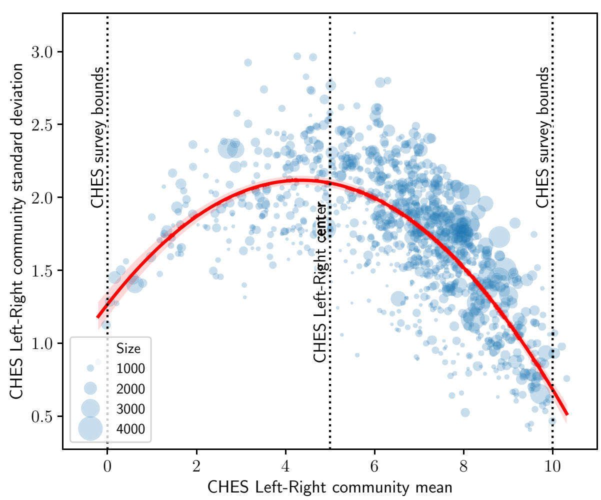


Fig. 15 Scatterplots of the 1.067 communities of the Twitter social graph on the CHES left–right dimension, according to the mean and standard deviation of the members of the communities. Extreme political communities are more homogeneous in their stances

example, shows that communities of users with extreme-left and extreme-right views, are also more homogeneous in their left–right stances. Communities of users with central left–right positions have a tendency to include a set of broader stances, showing a clear relation between *political polarization* (here understood as distance from the political left–right center) and political homogeneity. This can be seen more clearly in the distribution of communities along the CHES left–right axis (and not just according to distance from the center), as seen in Fig. 15, to which we have fitted a polynomial of degree 2. While there is no immediate null model against which to compare this relation between mean and standard deviation, this parabolic shape is not an artifact of the bounded region that supports the position of communities. Given the range of standard deviations across the whole set of communities, we expect that the boundedness of the mean positions would only impact the shape of the curve on the extremes near positions 0 and 10.

If a strong relation between extreme mean positions communities and their homogeneity is a measure of fragmentation, our empirical network is less fragmented along the issue of the European Union, elites, and ecology, than along left–right divides. For attitudes towards European integration or the importance of the ecology, mean attitudes are comparatively less related with homogeneity of communities. The situation is yet slightly different regarding attitudes towards elites. Communities of users that attribute little importance to the societal cleavage between people and elites, are formed by diverse users that have different stances on this issue. Communities of users that attribute great importance to this cleavage, are however much more homogeneous on their views towards this issue.

One remarkable fact is that curves in Fig. 14 display one direction towards which communities become more homogeneous. For example, communities grow more homogeneous on attitudes towards elites the more they are towards the anti-elite side of the spectrum. The fact that some curves have positive or negative slope, is not significant, as it depends on the definition of the concept being captured by the dimension. In particular, if it is defined positively or negatively. For example, we could create a new variable called “pro-elite salience” inverting the order of communities along the abscissa. Because standard deviation is symmetrical, the positions along the ordinate would not change, thus inverting the slope of the curve.

7 Discussion and conclusions

In this article we have proposed two results: (1) a new method for embedding users in political opinion spaces, and (2) an application showing that the empirical network surrounding political debate on Twitter in France is fragmented mainly along left–right divides and less along attitudes towards the European Union, ecology, and sentiments towards elites.

We have shown that network topology (without textual meta-data of nodes) and external attitudinal data can be used to embed social networks in ideological spaces where dimensions stand for indicators of attitudes towards sets of issues of public debate. We have illustrated this procedure using the social network formed by the followers of French parliamentarians on Twitter and leveraging the position of political parties in attitudinal dimensions of political surveys. This allowed us to embed the network in an ideological space spanned by three dimensions, positioning users according to their views on globalization and European integration, left–right positions, and views regarding immigration and multiculturalism. This so-called ideological space has the important property of ordering users: a user A to the left of user B in the left–right ideological dimension has high probability of being politically to the left of B. The greater the distance, the higher this probability is. Importantly, users far from moderate positions can be shown to be correctly positioned with high probability. This ideological embedding method already expands on the state of the art by allowing to identify several dimensions, associated with several issues of public debate that might be participating in social choice in internet platforms, where previous methods considered only one-dimensional differences. This method also has some limitations that we discussed in length before presenting a new improved method in Sect. 5. These are mainly three: (1) ideological spaces do not have reference points in space (e.g., they lack central positions), (2) there is no consistent metric across space, and (3) dimensions act as indicators of attitudes towards a set of grouped issues (thus the name *ideological*). We were able to address these issues by proposing a second method called attitudinal embedding. Leveraging the relation between positions of a few reference points present in both ideological space and attitudinal reference spaces such as political surveys, we were able to map entities from the former onto the latter. Attitudinal embedding solves the aforementioned limitations and thus pave the way towards interesting applications, of which we showed one by examining both, the structure of the social graph and the attitudinal position of the nodes. Computing a community partition of the social graph and measuring the mean position and standard deviation of these communities in several attitudinal dimensions, we were able to show that communities with extreme political positions are more homogeneous than those with less extreme stances.

At this point we stress the main contributions and significance of our work in a summarized fashion. Our work shows that it is possible to infer several indicators of opinions for users related to issues of public debate, by exploiting only topological traces, namely social graphs, to the exclusion of textual data. Independence from textual meta-data means that our method can be applied in any context, and even on users that do not express themselves in written text, audio sound, or with images. It also means that this method can be used to compare users from different settings, countries, and different moments in time. Furthermore, since the method does not require users to provide answers by themselves, this method does not suffer from issues relating to question framing, or by the possible lack of truthfulness on the side of the respondents. This is a significant improvement with respect to the state of the art, and paves the way for a greater understanding of opinion dynamics and related phenomena. Our attitude estimators are inherently probabilistic: from the probabilistic homophily model at its origin in Eq. (1), to the affine transformation that produces a mapping by interpolation in attitudinal space, to possibly additional undetermined sources or noise (e.g., a user might follow a politician contrary to its own beliefs just to be updated on political issues, or even by error). While this might have obvious limitations in applications where the opinions of a single user are studied, it is certainly a powerful tool in studying the opinions of aggregates of users: the opinion and its dynamics for selected social groups, or even the whole of set of users.

Ideological and attitudinal spaces have many additional potential applications. They can be used to study ideological trajectories in time, and to study issues related to the meaning of ideological axes (Ramaciotti Morales and Muñoz Zolotoochin 2022), such as the relative importance of left–right divides and polarization regarding globalization (Grossman and Sauger 2019). This method can also be applied to interactional data traces from other platforms: for examples to Facebook (Cointet et al. 2021) and YouTube data (Ramaciotti Morales et al. 2021). They can also be used to study the dimensional structure (number and relative importance of dimensions) in different digital arenas (Benoit and Laver 2012), and party systems online (Bakker et al. 2012). A different line of applications involves the study of the effect of Recommender Systems in phenomena such as polarization, now conceptualized in geometrical spaces (Ramaciotti Morales and Cointet 2021), beyond traditional connectionist approaches. Having estimations of the stances of users to which recommendations are made can be leveraged to understand the effects of algorithmic recommendation in large socio-political digital systems (Bakshy et al. 2015). Additionally, many recent studies analyzing the spread of misinformation online leverages political stances of users as an independent variable (Osmundsen et al. 2021). Finally, the study of polarization processes can benefit from the proposed method, especially because they allow to identify not only polarized (i.e., extreme) individuals, or individuals with clear partisan affiliation, but also individuals all along the spectrum subtended by different dimensions capturing attitudes towards different issues (Bramson et al. 2016). All these suggested applications, and others, may benefit from the inferences of stances offered by the ideological and attitudinal embedding methodologies here presented and tested.

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Data availability Data declared the 19 March 2020 and 15 July 2021 at the registry of data processing at the *Fondation Nationale de Sciences Politiques* (Sciences Po) in accordance with General Data Protection Regulation 2016/679 (GDPR) and Twitter policy. For further details and the respective legal notice, please visit https://medialab.sciencespo.fr/en/activities/epo/.

Declarations

Conflict of interest The authors declare no conflicts of interest.

Human and animal rights Our study did not involve experimentation with human subjects, and all data used is publicly available through Twitter's API.

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