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SLOWER IS FASTER: FOSTERING CONSENSUS FORMATION BY HETEROGENEOUS INERTIA

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We investigate an extension of the voter model in which voters are equipped with an individual inertia to change their opinion. This inertia depends on the persistence time of a voter's current opinion (ageing). We focus on the case of only two different inertia values: zero if a voter just changed toward a new opinion, and ν otherwise. We are interested in the average time to reach consensus, i.e. the state in which all voters have adopted the same opinion. Adding inertia to the system means to slow down the dynamics at the voter's level, which should presumably lead to slower consensus formation. As an unexpected outcome of our inertial voter dynamics, there is a parameter region of ν where an increasing inertia leads to faster consensus formation. These results rest on the heterogeneity of voters which evolves through the described ageing. In a control setting of homogeneous inertia values, we only find monotonously increasing consensus times. In this paper, we present dynamical equations for the mean field case which allow for analytical insight into the observed slower-is-faster effect.

Keywords: Voter model; heterogeneity; social inertia; consensus times.

1. Introduction

Decision making means selection among alternatives. It is one of the fundamental processes in economics and in social systems. If these systems consist of many interacting elements — which we will call *voters* from now on — the system dynamics may be described on two different levels: the *microscopic* level, where the decisions of the individual voters occur, and the *macroscopic* level, where a certain collective behavior can be observed [24].

Based on incomplete information, how does a voter take his decision on a particular subject? A "simple" utility maximization strategy may fail, because in many social situations, such as public votes, the private utility cannot easily be quantified, i.e. voters do not exactly know about it. So, voters have to involve supplemented strategies to take their decisions. In order to reduce the risk of making the wrong decision, it seems to be appropriate just to copy the decisions of others. Such an *imitation* strategy is widely found in biology, and also in cultural evolution. Different species, including humans, imitate the behavior of others of their species to become successful or just to adapt to an existing community [7].

In order to understand the intrinsic properties of systems comprising many such individuals, a number of models have been developed that take the spread of opinions as sample application. Early approaches in the social sciences showed that the existence of positive social influence (i.e. imitation behavior) tends to establish homogeneity (i.e. consensus) among individuals [10, 1]. The "voter model" (VM), rigorously defined by Liggett [18], confirms these results. Other works showed that mobility might lead to the segregation of opinions [23] and that selection of interaction partners ("bounded confidence" [5, 14]) can lead to stable diversity of opinions, even when considering positive social influence.

Here, we focus on the VM — a paradigmatic model to simulate such imitation behavior. Because of its simplicity, it allows for many analytical calculations [18, 22] and, therefore, gives a comprehensive understanding of the dynamics involved. Application areas of the VM range from coarsening phenomena [6] to spin glasses [18, 8], species competition [21, 4] and opinion dynamics [17]. Among the most prominent properties of the VM, the conservation of magnetization has extensively been studied [9, 2, 28] and compared to other prototypical models, such as the Ising model with Kawasaki dynamics [13].

Based on the VM, investigations were conducted to study interesting emergent phenomena and relevant applications. Such works comprise the possibility of minority opinion spreading [11, 30], dominance in predator–prey systems [21], forest growth with tree species competition [4] and the role of bilingualism in the context of language competition [3]. The question of consensus times and their scaling for different system characteristics was particularly addressed in several studies [18, 22, 26, 2, 29].

In this paper, we study a modified version of the VM introduced recently [27]. There, we assume that an individual voter has a certain inertia ν_i to change his opinion. ν_i increases with the persistence time τ_i, which is the time elapsed since the last change of opinion. The longer the voter already stays with his current opinion, the less he may be inclined to change it in the next time step. We show that this slowing-down of the dynamics at the microscopic level of the voters can lead to accelerated formation of consensus at the macroscopic level. In this paper, we extend the previous results by presenting a reduced description of the model which is based on only two levels of inertia. We show that this reduction still explains the origin of the faster consensus formation and thus complements the results presented in Ref. 27. Moreover, we emphasize the relevance of our approach to the research on social dynamics.

At variance with the standard VM, our extension considers the current opinion of voters as an important decisive factor. The voters do not only act based on the frequencies in their neighborhood, but take their own current opinion into particular account. This general idea can also be compared to the models of continuous opinion dynamics [5, 14, 19]: already in the basic continuous models, the current opinion of a deciding individual is of high importance. More precisely, it is as decisive as the average opinion in the considered neighborhood because the updated opinion is the average of both. The concept of bounded confidence emphasizes this importance because individuals do only approach opinions that are not too far away from their own current one. Therefore, bounded confidence can also be interpreted as a kind of inertia that tends to let individuals keep their own current opinion. However, the parameter regulating the confidence interval is generally kept constant in time whereas, in our model, individuals change their decision behavior depending on their history.

Our new parameter ν, which reduces the probability of state changes in the VM, can have different interpretations in the various fields of application of the VM: It may characterize molecules that are less reactive, the permanent alignment of spins in a magnet, etc. In economics, changes may be discarded due to transition costs or "sunk" costs. In social applications, there are at least two interpretations for the parameter ν: (i) within the concept of *social inertia*, which deals with a habituation of individuals and groups to continue their behavior regardless of possible advantages of a change; (ii) to reflect a (subjective or objective) conviction regarding a view or an opinion. Originally, the latter point served as a motivation for us to study the implications of built-in conviction in a simple imitation model like the VM. Will the systems, dependent on the level of conviction, still reach a consensus state, or can we observe the segregation of opinions? What do the ordering dynamics and the emergent opinion patterns look like?

Our investigations focus on the average time to reach consensus, i.e. the number of time steps the system evolves until it reaches an equilibrium state in which all voters have the same opinion. Taking into account the inertia introduced into the VM, we would assume that the time to reach consensus would be increased because of the slowed-down voter dynamics. Counterintuitively, we find that increasing inertia in the system can *decrease* the time to reach consensus. This result resembles the "faster-is-slower" effect reported in a different context by Helbing *et al.* [15]. In their work on panic situations, they explain why rooms can be evacuated faster if people move slower than a critical value through the narrow exit door. When individuals try to get out as fast as they can, this results in clogging effects in the vicinity of the door, which decreases the overall evacuation speed. Note that although the phrase "slower-is-faster" is appropriate for both findings, our effect has to be clearly distinguished from the one described by Helbing *et al.* In their generalized force model, an individual increase in the desired velocity would have a contrary effect on the microscopic level, i.e. all individuals would get slower and thereby the macroscopic dynamics would be decelerated. In our case, microscopic changes produce the counterintuitive effect only on the macroscopic level.

This paper is organized as follows. In the following section, we introduce the model. In Sec. 3 we present simulation results of our main finding — the "slower-is-faster" effect on reaching consensus through inertial voters. Section 4 investigates deeper, under which circumstances the effect can be observed, and introduces a theoretical framework, which allows to understand the phenomenon. Finally, conclusions are drawn in Sec. 5.

2. The Model

2.1. *The standard voter model*

In the original VM [16, 18, 6], N voters are positioned at the sites of a regular, d-dimensional lattice, the topology which defines the number of neighbors for each voter. Every voter has one of two possible opinions: $\sigma_i = \pm 1$. A time step consists of N update events, in each of which one voter is picked at random and adopts the opinion of one of the voters he is connected with. Thus, the probability that voter i adopts opinion σ, which we will denote as $W_i^V(\sigma)$, is equal to the density of opinion σ in its neighborhood. Hence,

$$W_i^V(\sigma, t) \equiv W_i^V(\sigma|\sigma_i, t) = \frac{1}{2}\left(1 + \frac{\sigma}{k}\sum_{j\in\{i\}} \sigma_j(t)\right), \qquad (1)$$

where k is the number of neighbors each voter has, and $\{i\}$ is the set of its neighbors. Note that this equation can also be applied to networks of different topology, as we will do later on.

The dynamics is a fluctuation-driven process that, for finite system sizes, ends up in one of two absorbing states, i.e. consensus in one of either opinions. The time to reach consensus, T_κ, depends on the size of the system and the topology of the neighborhood network. For regular lattices with dimension $d = 1$, $T_\kappa \propto N^2$, for $d = 2, T_\kappa \propto \ln N$, and for $d > 2$, $T_\kappa \propto N$. A critical dimension, $d = 2$, was found, below which the system coarsens. For any dimension larger than 2, the system can get trapped in disordered configurations in infinite systems [25].

Let $P_\sigma(t)$ be the global density of voters with opinion σ at time t. The average opinion of the system (also called "magnetization", analogous to studies of spin systems in physics) can be computed as

$$M(t) = P_+(t) - P_-(t). \qquad (2)$$

The order parameter, most often used in the VM, is that of the average interface density ρ. It gives the relative number of links in the system that connect two voters with different opinions and can be written as

$$\rho(t) = \frac{1}{4}\sum_i \sum_{j\in\{i\}} \big(1 - \sigma_i(t)\sigma_j(t)\big). \qquad (3)$$

In the mean field limit, which we will study in more detail later on, we assume that the change in the opinion of an individual voter only depends on the average densities of the different opinions in the whole system. Therefore, we replace the local densities (1) by global ones, which leads to the adoption probabilities

$W^V(\sigma|-\sigma,t) = P_\sigma(t)$. For the macroscopic dynamics, we can compute the change in the global density of one opinion as

$$\begin{aligned} P_\sigma(t+1) - P_\sigma(t) &= W^V(\sigma|-\sigma,t)P_{-\sigma}(t) - W^V(-\sigma|\sigma,t)P_\sigma(t) \\ &= P_\sigma(t)P_{-\sigma}(t) - P_{-\sigma}(t)P_\sigma(t) \\ &\equiv 0, \end{aligned} \tag{4}$$

i.e. the density of each opinion is conserved for every state of the system. In the simulations, consensus is reached by finite-size fluctuations only.

2.2. *The voter model with social inertia*

In contrast to the standard VM described above, we consider that voters additionally are characterized by a parameter ν_i, an inertia to change their opinion. This extension leads us to the *inertial voter model*, in which we have to distinguish between the probability that voter i changes his opinion,

$$W_i(-\sigma_i|\sigma_i,\nu_i) = (1-\nu_i)\, W_i^V(-\sigma_i), \tag{5}$$

and the complementary probability of sticking to his previous opinion, $W_i(\sigma_i|\sigma_i,\nu_i) = 1 - W_i(-\sigma_i|\sigma_i,\nu_i)$. In this setting, ν_i represents the strength of "conviction" that voter i has regarding his opinion.

We consider that the longer a voter has been keeping his current opinion, the less likely he will change to the other one. For the sake of simplicity, we consider that the inertia grows with the persistence time as

$$\nu_i(\tau) = \begin{cases} \nu_0, & \text{if } \tau = 0, \\ \nu, & \text{if } \tau > 0. \end{cases} \tag{6}$$

At time $t = 0$, and in every time step after voter i has changed his opinion, the persistence time is reset to zero, $\tau_i = 0$, and the inertia has the minimal constant value ν_0.[a] Whenever a voter keeps his opinion, his inertia increases to ν. We will study two distinct scenarios later on: (i) fixed social inertia, where $\nu_0 = \nu$ is a constant value for all voters; (ii) $\nu_0 < \nu$, a scenario in which inertia grows for larger persistence times.

It would be expected that including inertial behavior in the model would invariably lead to a slowdown of the ordering dynamics. We will show that, contrary to this intuition, these settings can lead to a much faster consensus.

3. Numerical Results

We performed extensive computer simulations in which we investigated the time to reach consensus, T_κ, for systems of N voters. We used random initial conditions with equally distributed opinions and an asynchronous update mode, i.e.

[a]Note that the results of this paper are qualitatively robust against changes in the concrete function $\nu_i(\tau)$. For more details on this see Ref. 27.

on average, every voter updates his opinion once per time step. The numerical results correspond to regular d-dimensional lattices (von Neumann neighborhood) with periodic boundary conditions, and small-world networks with a homogeneous degree distribution.

3.1. *Fixed social inertia*

We first consider the case of a *fixed* and *homogeneous* inertia value, $\nu_0 = \nu$. In the limit $\nu \to 0$, we recover the standard VM, while for $\nu = 1$ the system gets frozen in its initial state. For $0 \leq \nu < 1$, the time to reach global consensus will be affected considerably, i.e. the system will still always reach global consensus, but this process is decelerated for higher values of ν. This can be confirmed by computer simulations which assume a constant inertia equal for all voters (see left panel of Fig. 1).

In the right panel of Fig. 1, we depict the evolution of the interface density ρ for both the standard VM and the inertial VM with $\nu_0 = \nu = 0.5$. Differences between these cases can be seen in the very beginning and at about 10^3 time steps, right before the steep decay of disorder in the system. There, the ordering process is slower than in the VM without inertia. The distributions of T_κ at different ν values are very similar and show the log-normal like form known from the standard VM ($\nu = 0$).

This behavior can be well understood by analyzing Eq. (5). It can be seen that the ratio between the opinion changes in the standard VM and the inertial

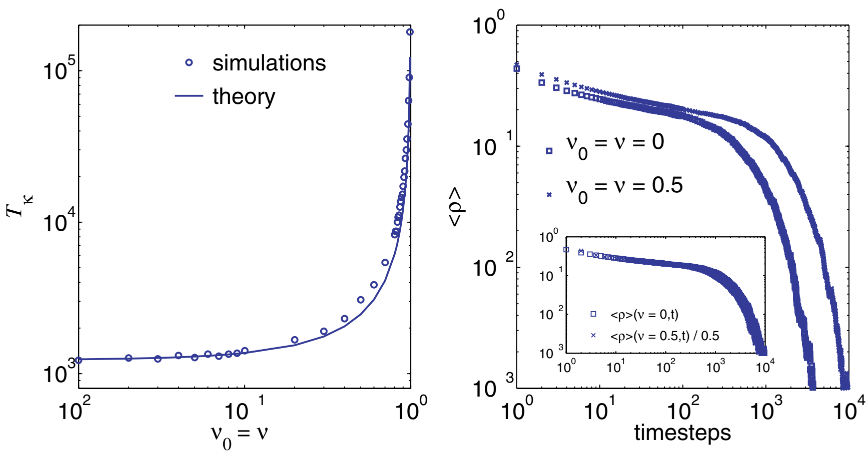

Fig. 1. Left: Average times to consensus T_κ in the voter model with a fixed and homogeneous inertia value, $\nu_0 = \nu$. The line corresponds to the theoretical prediction $T_\kappa(\nu) = T_\kappa(\nu = 0)/(1-\nu)$, whose details are given in the text. Right: Comparison of the development of the average interface density ρ in the voter model and the model with fixed inertia. The curves correspond to the mean values obtained out of 500 realizations. Right panel, inset: Collapse of the curves where the time scale has been rescaled according to $t \to t/(1-\nu)$. The system size is $N = 30 \times 30$ in both panels and the voters are placed in a two-dimensional regular lattice.

VM is given by $W(-\sigma|\sigma,\nu)/W^V(-\sigma|\sigma,0) = 1-\nu$. Consequently, it is possible to infer that the characteristic time scale for a VM with fixed inertia will be rescaled as $t \to t_V/(1-\nu)$. As can be seen in Fig. 1, there is good agreement between this theoretical prediction and computer simulations in both: the average time to consensus (left panel), and the time evolution of the interface density (inset of right panel).

3.2. *Evolving social inertia*

We now turn our attention to the case where the individual inertia values evolve with respect to the persistence time according to Eq. (6). Without loss of generality, we fix $\nu_0 = 0$. Other choices simply decelerate the overall dynamics as described in the previous subsection. Note that increasing ν raises the level of social inertia in the voter population. Figure 2 shows the average time to reach consensus as a function of the parameter ν, namely the maximum inertia value reached by the voters when the system is embedded in regular lattices of different dimensions. In Fig. 2, it is apparent that, for lattices of dimension $d \geq 2$, the system exhibits a noticeable reduction in the time to reach consensus for intermediate values of the

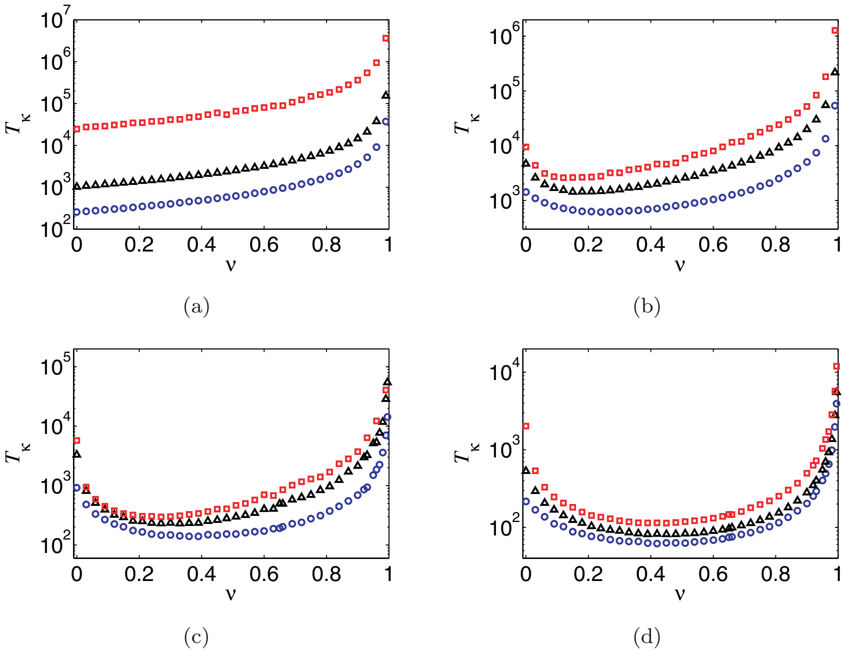

Fig. 2. Average time to reach consensus T_κ as a function of the maximum inertia value ν. Panels (a), (b), (c) and (d) show the results for different system sizes in one-, two-, three- and four-dimensional regular lattices, respectively. The results are averaged over 10^4 realizations. The system sizes for the different panels are the following: (a) $N = 50$ ($\circ$), $N = 100$ ($\triangle$), $N = 500$ ($\square$); (b) $N = 30^2$ ($\circ$), $N = 50^2$ ($\triangle$), $N = 70^2$ ($\square$); (c) $N = 10^3$ ($\circ$), $N = 15^3$ ($\triangle$), $N = 18^3$ ($\square$); (d) $N = 4^4$ ($\circ$), $N = 5^4$ ($\triangle$), $N = 7^4$ ($\square$).

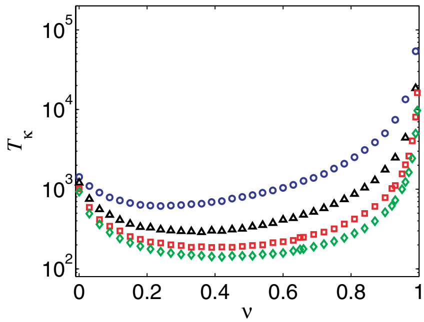

Fig. 3. Dependence of the average time to consensus on the control parameter ν. The symbols represent different rewiring probabilities ω, when the network topology is a small-world one. The curves correspond to $\omega = 0$ ($\circ$), $\omega = 0.03$ ($\triangle$), $\omega = 0.1$ ($\Box$) and $\omega = 0.9$ ($\diamond$).

control parameter ν. We observe that there is a critical value of ν such that the average consensus time reaches a minimum. Especially when compared to the results of the previous section, this result is against the intuition that a slowdown of local dynamics would lead to slower global dynamics. Furthermore, it is apparent that the larger the dimension of the lattice, the more pronounced is the phenomenon. Figure 2(a) shows the results for a one-dimensional lattice, where the phenomenon is not present at all. For this network topology, it is found that all the curves collapse according to a scaling relation, $T_\kappa(\nu, N) = T_\kappa(\nu)/N^2$.

In Fig. 3, we plot T_κ as a function of the maximum inertia value ν for different small-world networks [31]. Starting with a two-dimensional regular lattice, two edges are randomly selected from the system, and with probability ω their end nodes are exchanged [20]. With this procedure, the number of neighbors remains constant for every voter. It can be seen that the phenomenon of lower consensus times for intermediate inertia values is also present in small-world networks. Furthermore, increasing the rewiring probability ω leads to larger reductions of the consensus times at the optimal value ν_c. This implies that the formation of spatial configurations, such as clusters, is not the origin of this slower-is-faster effect.

Finally, we show the results on a fully connected network, i.e. where every voter has $N - 1$ neighbors. The results are presented in Fig. 5. As can be seen, the time to reach consensus is significantly decreased for intermediate values of ν.

4. Analytical Approach

As already mentioned, the results of Figs. 3 and 5 indicate that the spatial clustering plays no important role in the voters' ageing[b] and, therefore, in the qualitative

[b]By "ageing" we mean the possibility of building up higher persistence times, which in turn lead to increasing inertia values.

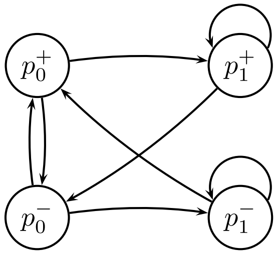

Fig. 4. Illustration of the four fractions p_l^σ and the possible transitions of a voter.

behavior observed. This finding allows a quantitative approach to the dynamics in the mean field limit, i.e. we now use the *global* densities of opinions to calculate the probability $W_i(-\sigma_i|\sigma_i, \nu_i)$ in Eq. (5).

Let us first introduce $p_l^\sigma(t)$ as the fraction of voters with opinion σ and inertia state l, i.e. $l = 1$ if they are inertial ($\tau > 0$) and $l = 0$ if they are not inertial ($\tau = 0$). Thus, voters with opinion +1 who changed their opinion in the last update step would contribute to the quantity $p_0^+(t)$; without an opinion change they would contribute to $p_1^+(t)$. The global density of an opinion σ at time t is given by

$$P_\sigma(t) = p_0^\sigma(t) + p_1^\sigma(t). \tag{7}$$

Figure 4 illustrates the possible transitions of voters from one fraction to another.

For the mean field limit, the evolution equations have the forms

$$\begin{aligned} p_0^\sigma(t+1) - p_0^\sigma(t) &= W(\sigma|-\sigma,0)p_0^{-\sigma} + W(\sigma|-\sigma,\nu)p_1^{-\sigma} \\ &\quad - (W(-\sigma|\sigma,0) + W(\sigma|\sigma,0))p_0^\sigma, \end{aligned} \tag{8}$$

$$p_1^\sigma(t+1) - p_1^\sigma(t) = W(\sigma|\sigma,0)p_0^\sigma - W(-\sigma|\sigma,\nu)p_1^\sigma. \tag{9}$$

In these equations, the global changing and sticking probabilities are easily found by using Eqs. (1) and (5):

$$\begin{aligned} W(-\sigma|\sigma,0) &= W^V(-\sigma) = P_{-\sigma}(t), \\ W(\sigma|\sigma,0) &= W^V(\sigma) = P_\sigma(t), \\ W(-\sigma|\sigma,\nu) &= (1-\nu)\,W^V(-\sigma) = (1-\nu)\,P_{-\sigma}(t), \\ W(\sigma|\sigma,\nu) &= 1-(1-\nu)\,W^V(\sigma) = P_{-\sigma}(t) + \nu P_\sigma(t). \end{aligned}$$

After some steps of straightforward algebra, the former expressions can be written in the example of the +1 opinion as

$$p_0^+(t+1) - p_0^+(t) = P_+(t)\big[p_0^-(t) + (1-\nu)p_1^-(t)\big] - p_0^+(t), \tag{10}$$

$$p_1^+(t+1) - p_1^+(t) = P_+(t)p_0^+(t) + P_-(t)\,p_1^+(t)(\nu-1), \tag{11}$$

and the equivalent terms are found for opinion −1.

The global density of the +1 opinion evolves as the sum of Eqs. (10) and (11), which yields, after some more straightforward algebra, the change in the global density

$$P_+(t+1) - P_+(t) = \nu \left[p_0^-(t)\, p_1^+(t) - p_0^+(t) p_1^-(t) \right]. \tag{12}$$

For $\nu = 0$, i.e. the standard VM, we obtain the general conservation of magnetization which we already have seen in Eq. (4). For $\nu > 0$ everything depends on the quantities $p_l^\sigma(t)$. If there is no heterogeneity of social inertia in the system, i.e. if at some time either $p_0^+(t) + p_0^-(t) = 1$ or $p_1^+(t) + p_1^-(t) = 1$, then there also is no dynamics in the magnetization. The same holds if the two products in the square brackets of Eq. (12) are equally high. This is true if $P_+ = P_-$ and the ratio of inertial voters is the same within the two global densities, i.e. if $p_0^+(t) = p_0^-(t)$.

In the remaining configurations of these four quantities, there is a dynamics in the magnetization of the system. This implies that even if the global densities of the opinions are the same ($P_+ = P_- = 0.5$), we can find an evolution toward full consensus at one of the opinions. Interestingly, the opinion whose density is increasing can be the *minority* opinion in the system. In general, at every time step an opinion σ has an increasing share of voters in the system whenever its internal ratio of inertial voters reaches the inequality

$$\frac{p_1^\sigma}{p_0^\sigma} > \frac{p_1^{-\sigma}}{p_0^{-\sigma}}. \tag{13}$$

However, the complete process is nonlinear and, therefore, it is not possible to derive the final outcome of the dynamics from Eq. (13).

Note that condition (13) is evidence of the important role of the heterogeneity of voters in the dynamics of the system. More precisely, the main driving force of the observed "slower-is-faster" effect is the voters' heterogeneity with respect to their inertia.

In order to have an analytical estimation of the effect of social inertia on the times to consensus, we initialize the system in a situation just after the symmetry is broken. In particular, we artificially set the initial densities to differ slightly, i.e. we set $p_0^+(0) = 1/2 + N^{-1}$ and, hence, $p_0^-(0) = 1/2 - N^{-1}$.[c] Then we iterate according to Eqs. (10) and (11). Furthermore, we assume that the consensus is reached whenever for one opinion $p_0^\sigma(t) + p_1^\sigma(t) \le N^{-1}$ holds.[d] This is due to the fact that for a system of size N, if the frequency of the minority state falls below N^{-1}, the absorbing state is reached. As we are interested in the effect of different inertia levels, we again use ν as control parameter and compare the results with computer simulations of the identical setup of our inertial VM. In Fig. 5, the lines

[c] We also calculated the theoretical predictions for breaking the symmetry in the other way, namely by setting $p_0^+(0) = 1/2 - N^{-1}$ and $p_1^+(0) = N^{-1}$. Here, again, opinion +1 is favored, but this time just by a higher fraction of inertial voters. The initial densities of opinions are equal. Qualitatively, this procedure leads to the same theoretical predictions.

[d] With the described initial condition, +1 can be the only consensus opinion.

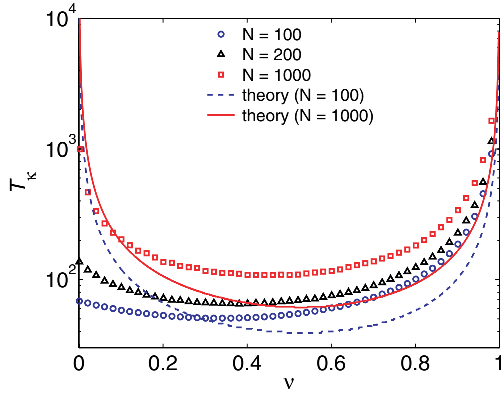

Fig. 5. Averaged times to consensus T_κ as a function of the value of social inertia ν. Symbols show the simulation results for different system sizes; intersected lines, the results of the theoretical estimation. A fully connected network of voters was used.

correspond to this theoretical analysis, where a qualitative agreement can be seen with the simulation results.

5. Conclusions

The time for reaching a fully ordered state in a two-state system such as the voter model is a problem that has attracted attention from different fields in the last few years. In this paper, we have studied the effect of social inertia in the VM based on the assumption that social inertia grows with the time for which the voter has been keeping his current opinion. We focused our study on how the times to consensus vary depending on the level of inertia in the population (ν).

Counterintuitively to the expectation that increasing inertia may lead to increasing times to reach consensus, we found that, for intermediate values of ν, this inertia mechanism causes the system to reach consensus *faster* than in the standard VM. We showed that this phenomenon is robust against the exact topology of the neighborhood network as we found it in regular lattices and small-world networks. In the former it holds that the larger the dimension, the more noticeable the effect. Furthermore, we found that the phenomenon also appears in random and fully connected networks.

Simply, this intriguing effect can be understood as follows. Due to fluctuations, one of the opinions is able to acquire a slight majority of voters. Therefore, voters of this opinion change less likely and, hence, the average inertia of this opinion will be higher than the other. Since inertia reduces emigration but not immigration, the majority will become even larger. This development is enforced by higher values of ν and constitutes a clear direction of the ordering dynamics which intuitively can lead to a faster reaching of consensus. However, for high values of ν, this development is outperformed by the high level of average inertia in the complete system, i.e. also within the minority population of voters, which slows down the overall time scale of the ordering dynamics (cf. Fig. 1).

Interestingly, this phenomenon implies that individuals reluctant to change their opinion can have a counterintuitive effect on the consensus process, which has been studied for some particular cases [12]. Furthermore, an inertial minority can overcome a less inertial majority in the same fashion as was previously discussed in Refs. 11 and 30.

Whereas, in a recent paper, we derived the complete macroscopic dynamics of a system with slowly increasing inertia [27], here we discussed a reduced model based on only two levels of inertia. Albeit simple, this model can still give rise to the observed "slower-is-faster" phenomenon. It also allows a theoretical approach to unveiling its origin, namely the described ageing mechanism that breaks the magnetization conservation. This is different from the standard VM, where magnetization is always conserved. We showed that the breaking of magnetization conservation holds only when the voters build up *heterogeneity* with respect to their inertia to change opinion. Therefore, once the symmetry between (a) the global densities of the two possible states and/or (b) the proportions of inertial voters is broken, the favored state (opinion) achieves both (i) reinforcement of its average inertia and (ii) fast recruitment of the less inertial state. Both effects contribute to a faster deviation from the symmetric state. For some parameter ranges, these mechanisms out weigh the increasing in the time to reach consensus generated by the high inertia of the state that disappeared in equilibrium.

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