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From production networks to geographical economics

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Abstract

Although standard economics textbooks are seldom interested in production networks, modern economies are more and more based upon supplier/customer interactions. One can consider entire sectors of the economy as generalised supply chains. We will take this view in the present paper and study under which conditions local failures to produce or simply to deliver can result in avalanches of shortage and bankruptcies and in localisation of the economic activity. We will show that a large class of models exhibit scale free distributions of production and wealth among firms and that regions of high production are localised.

JEL classification: D85; L14; L22; O18

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1. Networks of firms

Firms are not simply independent agents competing for customers on markets. Their activity involves many interactions, some of which involve a degree of cooperation. Interactions among firms might include

- information exchange (Davis and Greve, 1996; Battiston et al., 2003a,b),
- loans (Stiglitz and Greenwald, 2003 chapter 7; Delli Gatti et al., 2005)

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- common endeavours (Powell et al., 1996)
- partial ownership (Kogut and Walker, 2001; Battiston et al., 2007a)
- and of course economic transactions allowing production (Bak et al., 1993; Battiston et al., 2007b, and the present paper).

Economic activity can be seen as occurring on an economic network ("the economic web"): firms are represented by vertices and their interactions by edges. The edges are most often asymmetric (think for instance of provider/customer interactions). The availability of empirical data has provoked research on the structure of these networks; many papers discuss their "small world properties" and frequently report scale free distribution of the connections among firms.

The long term interest of economic network research is rather the dynamics creating or occurring on these nets: how are connections evolving, what are the fluxes of information, decisions (Battiston et al., 2003a,b), economic transactions, and so on, but dynamic studies lag behind statistical approaches because of conceptual difficulties and because time series of individual transactions are harder to obtain than time aggregated statistics.

Business to business connections are a recent hot topic, especially on the Internet; the practice is probably as old as long distance trade (say antiquity), and its importance was been recognised early by 19th century economists. Even the role of linkages in favouring economic development, a central topic of the present paper, is already discussed in, for example Marshall (1890).

The connections among different industries and countries have received a lot of attention since the pioneering work of Leontieff (1966). These previous approaches concerned aggregated exchanges and are in the domain of macro-economics. The units of models that we present here are individual firms, and we want to establish how their exchanges shape time and spatial properties of the global economy.

Our problematics is largely inspired from earlier efforts by Bak et al. (1993), Stiglitz and Greenwald (2003) and Delli Gatti et al. (2005), to determine the role of local random events on the distribution of production and wealth dynamics.

In Bak et al. (1993) for instance, production networks are defined by edges that represent supplier/customer connections among firms engaged in batch production activity. The authors describe the distribution of production avalanches triggered by random independent demand events at the output boundary of the production network.

The papers by Stiglitz and Greenwald and Delli Gatti et al. are about the consequences of firms' failures to pay their debts in a network where edges represent inter-firm loans. Bad debt might propagate in such loan-connected networks, resulting in avalanches of bankruptcies and in scale free distribution of wealth among firms.

These papers are not based on any empirical description of the network structure, but assume a very simple interaction structure: star structure in the case of Stiglitz and Greenwald and Delli Gatti et al., periodic lattice in Bak et al.

The simplifications introduced in the present paper are largely inspired from Bak et al., Stiglitz and Greenwald and Delli Gatti et al.

We start from a very simple lattice structure, and we study the consequences of simple local processes of orders/production (with or without local failure)/delivery/profit/investment on the global dynamics: evolution of global production and wealth in connection to their distribution and spatial patterns. However, it is worth emphasising from the beginning that while the structure of our production network is inspired by Bak et al., the dynamics of supply and demand differ completely from that work, and in particular, our model does not include the mechanisms that lead to the so-called "self-organized criticality."

The kind of questions that we want to answer concern the dynamics of production and wealth:

- Should we expect the equivalent of a "laminar" (regular) flow of production or a turbulent flow dominated by avalanches?
- What is the influence of the local processes on the overall distribution of firms size? narrow or scale free?
- Will the spatial repartitions of wealth and production display homogeneity with minor fluctuations or on the opposite, stable patterns of economic activity?

Our main finding with respect to previous economic literature is the emergence of highly productive regions in the network, although our assumptions are quite distinct from the standard assumptions of geographical economics (e.g. Fujita and Thisse, 2002). A comparison with this strain of literature is presented in the conclusion section.

We should also mention the long and still on-going debate about the origin of highly skewed wealth distributions across economic agents (Cowell, 1999). This debate started with the empirical works of Pareto (1896) and later, with the so-called "law of proportional effect", proposed by Gibrat (1931) to explain the emergence of the lognormal distribution. A related debate concerns the relation between inequality and economic growth (Aghion and Williamson, 1998). The seminal work of Champernowne (1953) introduces a model in which idiosyncratic shocks proportional to wealth of individuals lead to a lognormal distribution as unique invariant distribution. Moreover, the work shows that, under an alternative set of assumptions about the distribution of idiosyncratic shocks, the Pareto form can be obtained as a unique invariant distribution.

It should be noticed that, in Champemowne's work as well as in many subsequent models in the economic literature (Simon, 1955; Nirei and Souma, 2003; Gabaix, 1999), the emergence of broad wealth distributions is associated with laws of motion for wealth that can be approximated with variants of multiplicative stochastic processes. Indeed, a combination of random multiplicative and additive processes can give rise to power law distributions as proved in very general way by Kesten (1973). More precisely, the lognormal distribution arises from a pure multiplicative stochastic process, while the power law arises when adding to the multiplicative stochastic process a simple mechanism such as a lower reflecting barrier (representing, e.g., a bankruptcy threshold below which a firm disappears and a new one is created) or a reset event (Sornette and Cont, 1997).

However, the weak point of such a modelling approach is, in our view, that interaction between firms is not considered at all. We know that firms do not exist in isolation and that their interactions do matter for their own survival and growth. We might then expect that such relations should play a role in the statistical properties observed at a macroscopic scale. In this paper we take this issue seriously, and we investigate the distribution of wealth as well as the patterns of output that emerges from a networked economy where interaction is the primary vehicle of growth. Instead of idiosyncratic shocks on the return of firms, in our model we assume local spontaneous failures in the production process.

In the spirit of complex systems analysis, our aim is not to present specific economic prediction, but primarily to concentrate on the generic properties (dynamical regimes, transitions, scaling laws) common to a large class of models of production networks.

A minimal model of a production network will first be introduced in Section 2. Simulation results are presented in Section 3. Section 4 is a discussion of the genericity of the obtained results. We also summarise the results of several variants of the simplest model. The conclusion is a discussion of possible applications to geographical economics and a comparison with previous approaches.

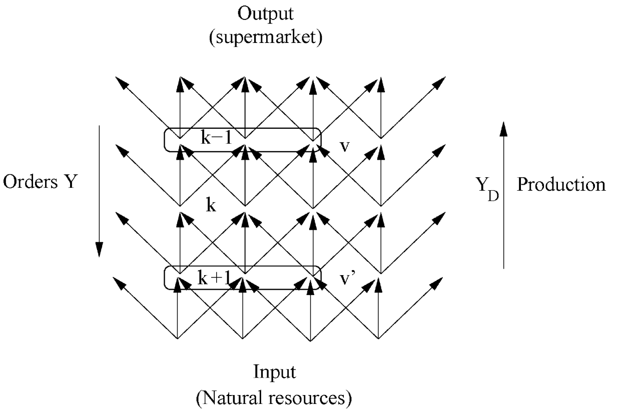

Fig. 1. Firms are located at the nodes of the lattice. Production (Y^D) flows from the resource input layer ($k = l$) to the output layer ($k = 0$); orders (Y) flow backward. The v and v' are, respectively, the output and input neighborhoods of a firm at level k.

2. A simple model of a production network

We can schematise the supplier/customer interactions among firms by a production network, where firms are located at the vertices and directed edges represent the delivery of products from one firm to its customers (see Fig. 1).

We have chosen a simple periodic lattice with three input connections of equal importance and three output per firm. The network is oriented from an input layer (say natural resources) towards an output layer (say the shelves of supermarkets). The transverse axis (horizontal on the figure) can be thought as representing either geographical position or some product space while the longitudinal axis (vertical on the figure) relates to production flow. We here use a one-dimensional transverse space to facilitate the representation of the dynamics by two-dimensional patterns, but there is no reason to suppose geographical or product space to be one-dimensional in the real world.

In real economies, the network structure is more heterogeneous with firms of unequal importance and connectivity. Furthermore some delivery connections go backwards. Most often these backward connections concern equipment goods; neglecting them as we do here implies considering equipment goods dynamics as much slower than consumption goods dynamics. Anyway, since these backward connections enter positive feedback loops, we have no reason to suppose that they would qualitatively disrupt the dynamics that we further describe.

At each time step two opposite flows move across the lattice: orders are first transmitted upstream from the output layer; production is then transmitted downstream from the input layer to the output layer.

- Orders at the output layer

  We suppose that orders are only limited by the production capacity[2] A_{0i} of the firm in position 0,i, where 0 indicates the output layer and i the transverse position in the layer:

$$Y_{0i} = qA_{0i}. \tag{1}$$

[2] A number of simplifying assumptions of our model are inspired from Delli Gatti et al., especially the assumption that production is limited by production capacity, not by market.

Y_{0i} is the order in production units and q a technological proportionality coefficient relating the quantity of product Y to the production capacity A, combining the effect of capital and labour. q is further taken equal to 1 without loss of generality. Eq. (1) implies constant returns to scale.

- Orders

  Firms at each layer k, including the output layer, transfer orders upstream to get products from layer $k+1$, allowing them to produce. These orders are evenly distributed across their three suppliers upstream. But any firm can only produce according to its own production capacity A_{ki}. The planned production Y_{ki} is then a minimum between production capacity and orders coming from downstream:

$$Y_{ki} = \min\left(qA_{ki}, \sum_{v} \frac{Y_{(k-1)i}}{3}\right); \tag{2}$$

  v stands for the supplied neighborhood, here supposed to be the three firms served by firm k, i (see Fig. 1).

  We suppose that resources at the input layer are always in excess, and here too, production is limited only by orders and production capacity.

- Production downstream

  Starting from the input layer, each firm then starts producing according to inputs and to its production capacity, but production itself is random, depending upon alea. We suppose that at each time step some catastrophic event might occur with constant probability $\mathcal{P}$ and completely destroy production. Failures result in cancelling production at the firm where they occur, but they also reduce production downstream since firms downstream have to reduce their own production by lack of input. These failures to produce are uncorrelated in time and location on the grid. Delivered production Y^d_{ki} by firm k, i then depends upon the production delivered upstream from its delivering neighborhood v'_i at level $k+1$:

$$Y^d_{ki} = \left(\sum_{i' \in v'_i} Y^d_{(k+1)i'} \frac{Y_{ki}}{\Sigma_{i'' \in v_{i'}} Y_{ki''}}\right) \varepsilon(t). \tag{3}$$

  - Whenever any of the firms $i' \in v'_i$ at level $k+1$ is not able to deliver according to the order it received, it delivers downstream at level k to its delivery neighbourhood v'_i in proportion of the initial orders it received, which corresponds to the fraction term;
  - $\varepsilon(t)$ is a random term equals to 0 with probability $\mathcal{P}$ and 1 with probability $1-\mathcal{P}$.

  The propagation of production deficit due to local independent catastrophic event is one of the collective phenomenon we are interested in.

- Profits and production capacity increase

  Production delivery results into payments without failure. For each firm, profits are the difference between the valued quantity of delivered products and production costs, minus capital decay. Profits Π_{ki} are then written

$$\Pi_{ki} = p_k Y^d_{ki} - c_k Y^d_{ki} - \lambda A_{ki}, \tag{4}$$

  where p_k is the unit sale price, c_k is the unit cost of production, and λ is the capital decay constant due to interest rates and material degradation. The production cost c_k at level k includes the cost of supply from level $k-1$, p_{k-1}, plus the cost of manufacturing the product by firm k, i. We simplify the model and the notation by supposing that the price increase, written p, between two levels and the manufacturing cost c are constant. Their difference is the same as the difference between p_k and c_k, which is the reduced variable that influences profits. Eq. (4) can then be used without k indices for prices and costs. In the rest of the paper, p and c will always, respectively, refer to inter-level price increase and to manufacturing costs, excluding supply costs.

We suppose that all profits are re-invested into production. Production capacities of all firms are thus upgraded (or downgraded in case of negative profits) according to

$$A_{ki}(t+1) = A_{ki}(t) + \Pi_{ki}(t). \tag{5}$$

- Bankruptcy and re-birth.

  We suppose that firms whose capital becomes negative go into bankruptcy. Their production capacity goes to zero, and they neither produce nor deliver. In fact we even destroy firms whose capital is under a minimum fraction of the average firm (typically 1/50). A re-birth process occurs for the corresponding vertex after a latency period; re-birth is due to the creation of new firms that use the business opportunity to produce for the downstream neighborhood of the previously bankrupted firm. New firms are created at a unique capital, a small fraction of the average firm capital (typically 1/25).[3]

The dynamical system that we defined here belongs to a large class of nonlinear systems often called reaction–diffusion systems (see, e.g. Kuramoto, 1984) from chemical physics or Turing systems in morphogenesis. The reaction part here is the autocatalytic loop of production and capital growth coupled with capital decay and death processes. The diffusion part is the diffusion of orders and production across the lattice.

We can then a priori expect a dynamical behaviour with spatio-temporal patterns, well-characterised dynamical regimes separated in the parameter space by transitions or crossovers, and scale free distributions since the dynamics is essentially multiplicative and noisy. These expectations guided our choices of quantities to monitor during simulations.

3. Simulation results

3.1. Methods and parameters choice

Unless otherwise stated, the following results were obtained for production networks with 2000 nodes per layer and five layers between the input and the output. Patterns are displayed for smaller lattices.

Initial wealth is uniformly and randomly distributed among firms:

$$A_{ki} \in [1.0, 1.1]. \tag{6}$$

One time step corresponds to the double sweep of orders and production across the network, plus updating capital according to profits. The simulations were typically run for 5000 time steps or more.

[3] Adjusting these capital values relative to the average firm capital $\langle A \rangle$ is a standard hypothesis in many economic growth models: one supposes that in evolving economies such processes depend upon the actual state of the economy and not upon fixed and predefined values.

The figures displayed further correspond to

- a capital threshold for bankruptcy of $\langle A \rangle/50$,
- an initial capital level of new firms of $\langle A \rangle/25$.

Production costs c were 0.8 and capital decay rate $\lambda = 0.2$. In the absence of failures, stability of the economy would be ensured by sales prices $p = 1.0$. In fact, only the relative difference between these parameters influences stability, but their relative magnitude with respect to the inverse delay between bankruptcy and creation of new firm also qualitatively influence the dynamics.

Since the dynamics of wealth and production is essentially exponential in time, we adjusted the sale price parameter to a breakeven regime, ensuring that A and Y_d variables would not reach very large values within the large simulation times necessitated by the observation of asymptotic regimes. This choice of the breakeven price to run the simulations has, of course, nothing to do with a search for equilibrium! We model dynamical systems that have no reason to be at equilibrium.

In the limits of low probability of failures, when bankruptcies are absent, we expect a linear relation between failure probability $\mathcal{P}$ and equilibrium price p, written

$$p = c + \lambda + \frac{l}{2}\mathcal{P}, \tag{7}$$

where l is the total number of layers. (The 1/2 comes from the fact that the integrated damage due to an isolated failure is proportional to the average number of downstream layers). In addition, first we had to adjust the sale price by trial and error before running simulations.

Even at breakeven prices, large time variations of production and wealth are present as seen in Figs. 2 and 3; all distribution and local data are then presented in terms of their relative amplitude with respect to the largest firm (in other words, the firm with the largest economic performance). Because production and wealth distribution involve four orders of magnitude in the asymptotic regime, we had to adjust grey levels of patterns (Figs. 5–7 and 11) to make "small" firms visible on the patterns by using a transform such as

$$\text{Grey level} = 1 - \frac{(\text{factor} + 1) * A_{ik}}{\text{factor} * A_{ik} + A_{\max}}, \tag{8}$$

where $A_{\max}$ is the wealth of the largest firm and the *factor* takes values such as 10 or 50.

Most simulations were monitored online; we directly observed the evolution of the local patterns of wealth and production that our choice of a lattice topology made possible; most of our understanding comes from these direct observations. However, we can only display global dynamics or static patterns in this manuscript.

3.2. Time evolution

The simplest way to monitor the evolution of the system is to display the time variations of some of its global performance. Fig. 2 displays the time variations of total wealth A and of the fraction of active firms for a 2000×5 lattice, with a probability of failure of 0.05 and a compensation sale price of 1.08. Time lag between bankruptcy and new firm creation is either 1 (for the left diagram) or 5 (for the right diagram).

The features that we here report are generic to most simulations at breakeven prices. During the initial steps of the simulation, here say 1000, the wealth distribution widens due to the influences of failures. Bankruptcy cascades do not occur, as observed by checking the number of active firms,

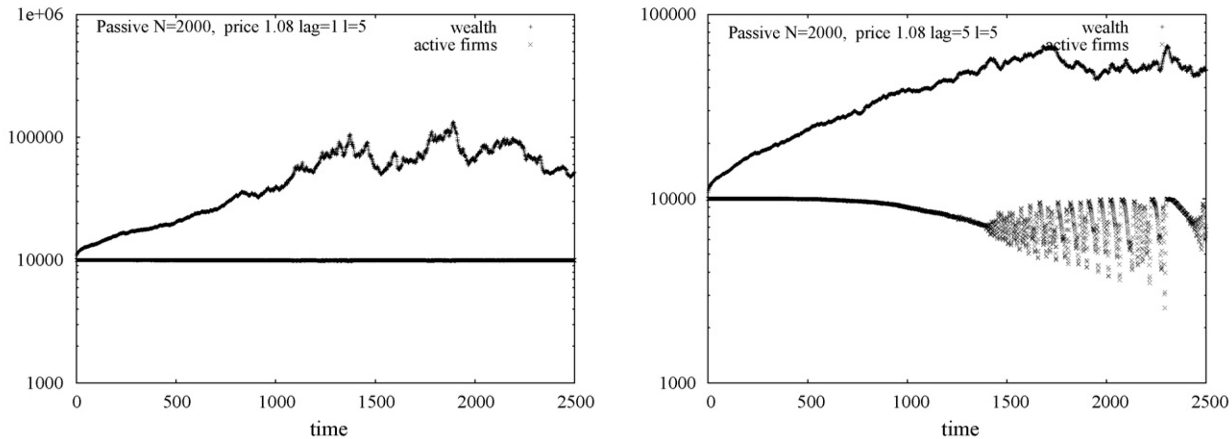

Fig. 2. Time evolution of total wealth ("+") and number of active firms ("×"). The network has five layers, 2000 firms per layer, $\mathcal{P} = 0.05$ (the failure probability). The left diagram corresponds to a small time lag equal to 1 between bankruptcy and firm re-birth, the right one to a larger time lag equal to 5. Vertical scale is logarithmic. The large wealth fluctuations do not correlate with avalanches of bankruptcies when they occur.

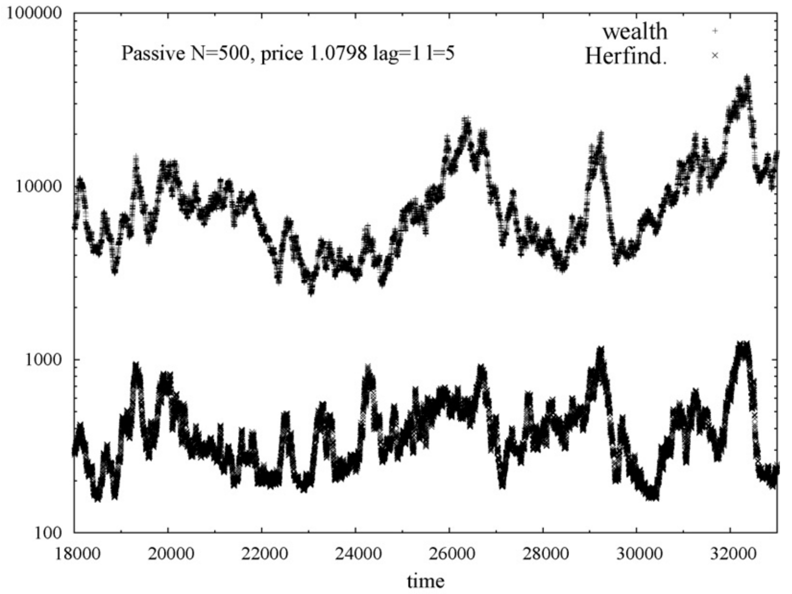

Fig. 3. Time evolution of wealth ("+") and 10,000 times the Herfindahl index ("×") for network with five layers, 2000 firms per layer when the time lag is 1. The large wealth fluctuations correlate Herfindahl index at the output layer.

until the lowest wealth values reach the bankruptcy threshold. All quantities display relatively small fluctuations.

Later, for $t > 1000$ one observes large wealth variations; their amplitude goes up to a factor 3 change in total wealth.

For the larger time lag (5) between bankruptcy and firm re-birth, when bankruptcies become frequent, they can cascade across the lattice and propagate in both network directions. As a consequence, avalanches of bankrupcies are observed as seen on the right diagram of Fig. 2. A surprising feature of the dynamics is that avalanches of bankruptcies are not correlated with the level of total production. Even when only one-tenth of the firms are active, the total production is still high. In fact, in this model, most of the total production is dominated by large firms, and avalanches that concern mostly small firms are of little consequence for the global economy.

On the other hand, in both cases, large wealth variations are positively correlated with concentration of the economy. Since the output layer is a good indicator of the upstream economic activity, we monitored the Herfindahl index of firms at the output level defined by

$$H = \sum_i \left(\frac{A_{0i}}{\Sigma_i A_{0i}} \right)^2. \tag{9}$$

We found that the Herfindahl index varies between 0.01 and 0.1, which are surprisingly high values for a network of width 2000. The positive correlation between total wealth and Herfindahl index tells us that production is at its highest when the concentration of economic activity is also. Further evidence will be given in the next sections, but we still lack a convincing explanation for this observation.

Anyway, the large measured Herfindahl index explains part of the magnitude of the wealth variation: although the number of firms is large, we are actually observing fluctuations of a much smaller set of objects, say the inverse Herfindahl index. The second amplifying factor is that the dynamics is exponential.

3.3. Dynamical regimes and phase transitions

As earlier discussed, the dynamics display different regimes, growth or collapse, and presence or absence of avalanches. The dynamical regimes are separated by transitions in the parameter space.

Let us start with the growth versus collapse transition. Drawing the breakeven manifolds, for instance in the failure probability $\mathcal{P}$ and sale price p plane allows comparison of the influence of other parameters on the transition. The growth regime is observed in the low $\mathcal{P}$ and high p region, the collapse regime in the high $\mathcal{P}$ and low p region.

Fig. 4 displays three breakeven manifolds corresponding to different lattice depths. They are computed according to a dichotomy algorithm, checking for either increase or decrease of wealth during 5000 iteration steps with a precision on price of 2×10^{-4}.

At low failure probability, the breakeven lines follow Eq. (7). At higher values of $\mathcal{P}$, interactions among firms failures are important, hence the nonlinear increase of compensating prices.

Breakeven manifold is a simple test of the economic performances of the network: when performances are poor, the compensating sales price has to be larger. We checked, for instance, that increasing the bankruptcy threshold and new firms' initial capital increases global economic performance. On the other hand, increasing the time lag between bankruptcy and the apparition of new firms increases breakeven sale prices in the non-linear region.

The transition line separating the regimes with and without avalanches mostly depends upon the depth of the network and upon the time lag between bankruptcy and rebirth, at least at the breakeven price. The frequency of failures, tested from 0.01 to 0.05 only changes the time of occurrence of avalanches of bankruptcies, not the transition line. A rule of thumb for the transition

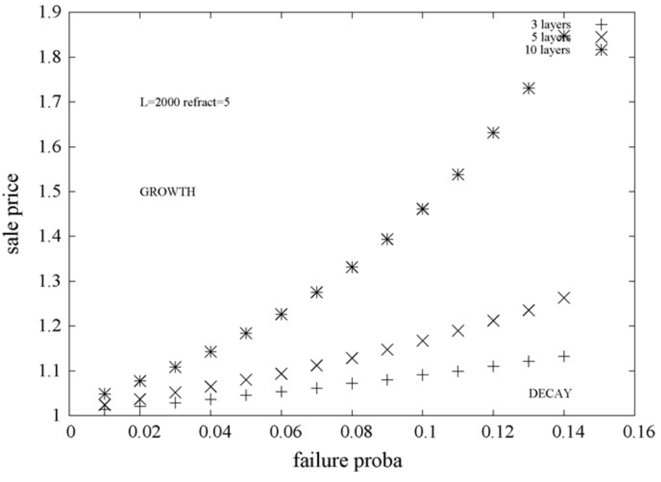

Fig. 4. Regime diagram in the sale price vs. probability of failure plane. The time lag between bankruptcy and re-birth is 5. The two regions of growth and economical collapse at large times are separated by lines whose position are fixed by simulation parameters. We here varied the production network depth: the "+" line was obtained for a three layers net, the "×" line for a five layers net, and the "*" line for a 10 layers net.

Fig. 5. Patterns of wealth (lower pattern) and production (upper) after 2500 iterations steps with the parameter set-up of Fig. 3 (left; time lag = 1), for a 100×5 lattice. The input layer is the lower layer of each pattern and wealth is coded by grey level (Black is the largest wealth, white is low wealth). We observe alternation of highly productive regions (black), with less active regions (in lighter grey).

Fig. 6. Patterns of wealth (upper pattern) and production (lower pattern) after 2500 iterations steps with the parameter set-up of Fig. 3 (right) (time lag is 5). The same alternance of active and less active regions is observed, but with a larger time lag (5), we also get large zones of bankrupted firms signaled by crosses (×).

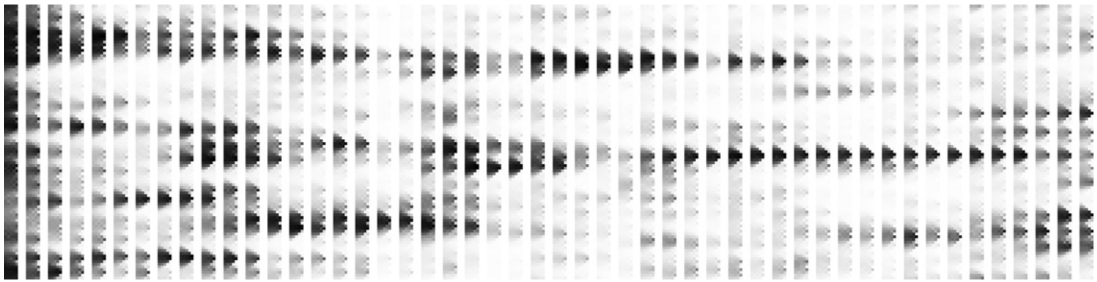

Fig. 7. Successive patterns of wealth from left (at time 200) to right at time 10,000). These patterns are rotated by 90 degrees with respect their orientation in Figs. 5 and 6. Input layer is the left layer of each pattern. Following the horizontal axis we see that patterns are metastable, with life times of the order of a few thousands (parameter set-up: time lag = 1, 100 × 5 lattice).

line, measured when the average fraction of bankrupted firms reaches 10 percent, is

$$\text{depth} \times \text{delay} \simeq 13 \pm 1. \tag{10}$$

3.4. Wealth and production patterns

Like most reaction–diffusion systems, the dynamics is not uniform in space and display patterns. The wealth and production patterns displayed after 5000 time steps in Figs. 5 and 6 were obtained for a failure probability $\mathcal{P} = 0.05$. They reflect wide distributions and spatial organisation. In these diagrams, production flows upward as in Fig. 1. The upper diagram displays wealth A and the lower one production, Y_{d}. Both wealth and production are coded by a grey level according to Eq. (8). Black is maximum production, white is minimum production and crosses signal bankrupted firms.

The important result is that although production has random fluctuations and diffuses across the lattice, the inherent multiplicative (or autocatalytic) process of production plus re-investment, coupled with local diffusion, results in a strong metastable local organisation: the dynamics clusters rich and productive firms in "active regions" separated by "poor regions" (light grey).

These patterns are evolving in time, but are metastable on a long time scale (a few thousand time steps), as visible from the series of wealth patterns displayed in Fig. 7. Most patterns have many active regions, but some (e.g., at time 3600 and 6000) have few: they correspond to larger wealth distribution and to peaks of total wealth (Fig. 8).

The relative importance of active (and richer) regions can be checked by a Zipf plot (Zipf, 1949). We again use the output layer as representative of the upstream wealth and checked the largest regional wealth as a function of their rank order.

All three Zipf plots display some resemblance to standard Zipf plots of individual wealth, firm size and city size. For the models discussed here, the size decreases following very approximately a power law. Its exponent is the inverse exponent of the CDF. The apparent exponent is one when

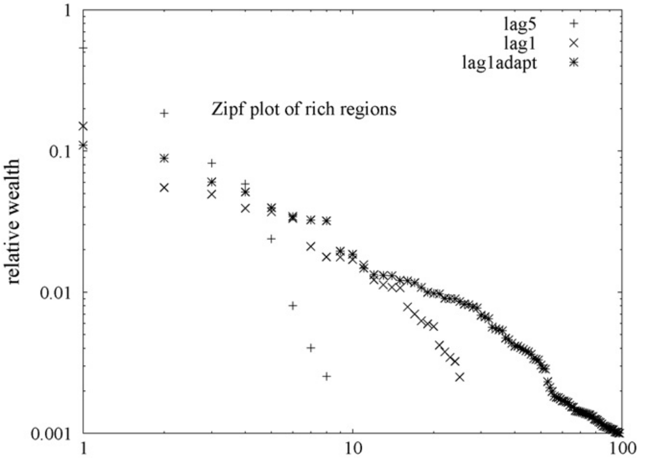

Fig. 8. Zipf plot of wealth of the most active regions for the standard and adaptive firms models (cf. Section 4.1). The vertical axis displays the production relative to the total production. The "+" signs correspond to the standard model with time lag = 5, the "×" to time lag = 1, and the "*" to the adaptive firms model with time lag = 1.

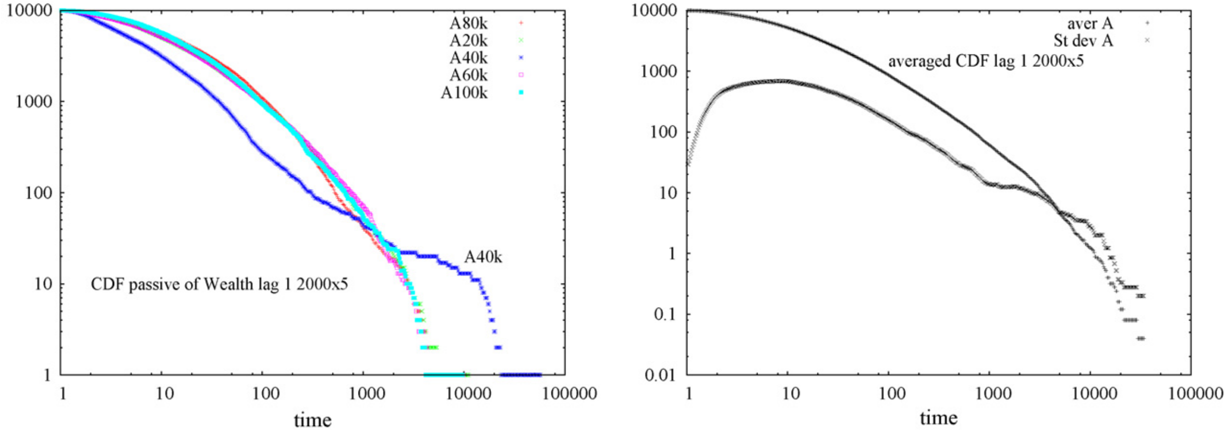

Fig. 9. Cumulative distributions of wealth in the absence of avalanches, after several ten thousands iteration steps. The left diagram is a series of histograms at different times: even after long evolution, the distribution may still fluctuate by large amounts. The right diagram shows the time-averaged distribution of net worth and standard deviation of net worth. Parameter setting is the same as for the previous figures.

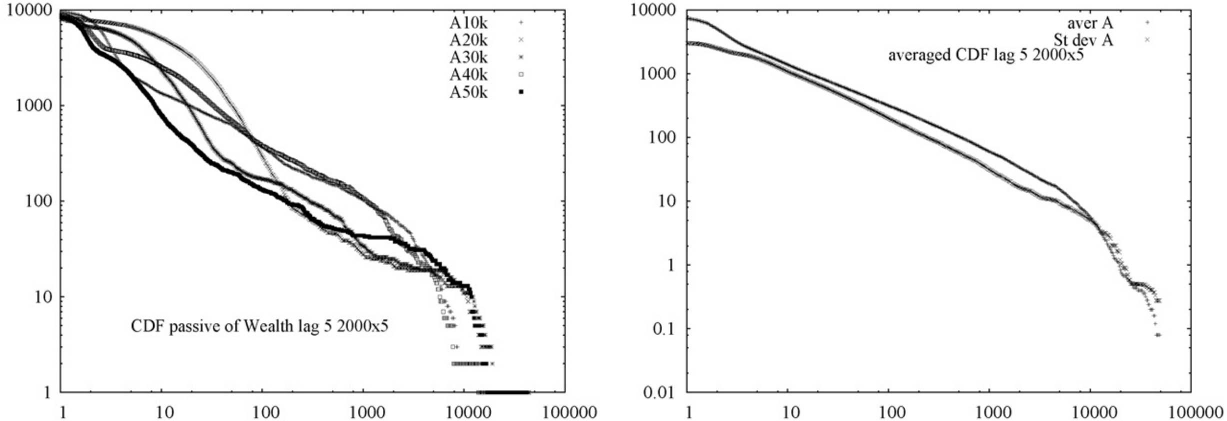

Fig. 10. Cumulative distributions of wealth in the avalanche regime (the time lag is 5) after several ten thousands iteration steps (left diagrams). Even after such long evolutions, the distribution fluctuates by large amounts. The right diagram shows the time-averaged distribution of net worth and standard deviation of net worth. Parameter setting is the same as for the previous figures.

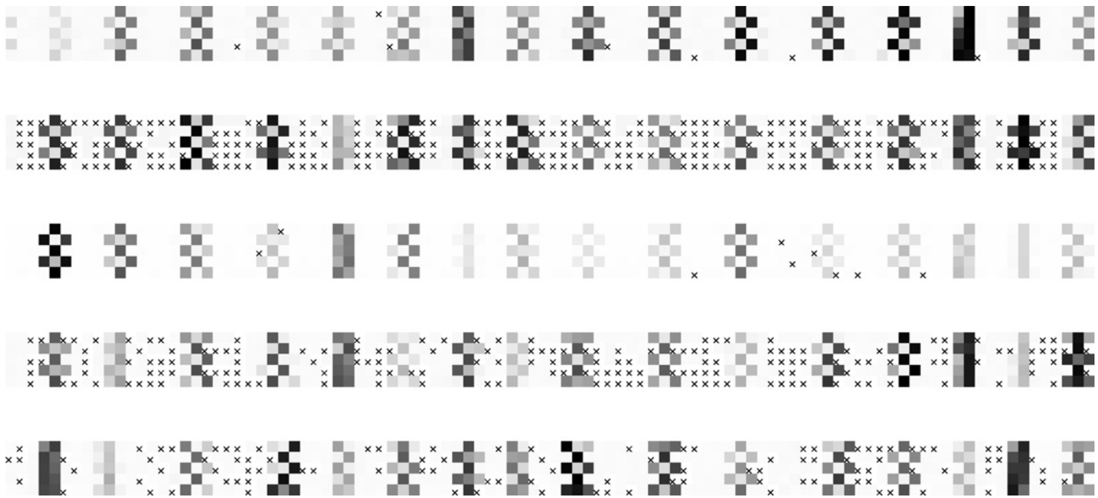

Fig. 11. Series of wealth and patterns for a network of "adaptive" firms. The conventions and parameters are the same as for Figs. 5 and 6, for a 100 × 5 lattice. Time lag is 1. Patterns are taken at time 2000 (for the lower pattern) and 4000, 6000, 8000, 10,000 upward. The position of the rich regions are stable, but their relative amplitude varies.

the time lag is 1. It is much higher when the time lag is 5. We also check Zipf plots of integrated production regions; they display the same characteristics.

In conclusion, the patterns clearly display some intermediate scale organisation in active and less active zones: strongly correlated active regions are responsible for most part of the production. The relative importance of these regions obeys a Zipf distribution.

3.5. Wealth histograms

The multiplicative random dynamics of capital and the direct observation of wealth and production led us to predict a scale free distribution[4] of wealth.

The complementary cumulative distribution functions (cdf) of wealth observed in Figs. 9 and 10 indeed span a wide range (3.5 orders of magnitude for a 10,000 firms network) and do not display any characteristic scale. The wealth data were taken for a 2000×5 lattice after several tens of thousands time steps to check the stability of the distribution. After such large iteration times, the distribution is asymptotic, but it still displays large fluctuation reflecting the large influence of the richest region. Production histograms (not shown) display the same features.

When the time lag is 1 (Fig. 9), the asymptotic distribution resembles a log-normal distribution, but large deviation can be observed for certain time values (e.g., at time 40,000 on the figure). The large deviation corresponds to a maximum of the Herfindahl index as earlier discussed. The large shoulder at high wealth values is due to one single active region. The diagrams on the right are averaged histograms over 24 time steps distant by 1000 iterations, taken after 25,000 iteration steps. The magnitude of the standard deviation reflects the relative instability of the richest region with respect to the rest of the distribution.

When the time lag is 5 (Fig. 10), in the avalanche regime, even larger fluctuations from the asymptotic distribution are observed. They correspond to an increased predominance of the most active regions as reflected by the observed shoulders. Furthermore, since the system is in the avalanche regime, the total number of active firms is not constant: the standard deviation is high, even in the low wealth region.

In conclusion, the observed statistics largely reflect the underlying region structure: at intermediate levels of wealth, the different wealth peaks overlap (in wealth, not in space!); we can observe smoother cdfs. The large wealth extreme of the distribution reveals the fine structure of regions.

4. Conclusions

The simple model of production networks that we proposed exhibits some remarkable dynamical properties:

- Scale free distributions of wealth and production.
- Regional organisation of wealth and production: a few active regions are responsible for most production.
- Avalanches of shortage and resulting bankruptcies occur for larger values of the time lag between bankruptcy and firm re-birth, but even when most firms are bankrupted, the global economy is little perturbed.

[4] What we mean here by scale free is that no characteristic scale is readily apparent from the distribution as opposed, for instance, to Gaussian distributions. Power law distributions are scale free.

We also have a clear picture of the parameter regions corresponding to the various dynamical regimes exhibited by the production network. A further step is to ensure whether these properties are specific to our particular simplifying assumptions or whether they are generic to a larger class of models.

In fact the first two features, scale free distributions and patterns already have a long history in science. Scale free distributions as a result of random multiplicative processes were already described and applied to economics and social sciences by Mandelbrot (1951), Champernowne (1953) and Simon (1955), as also mentioned in the introduction.

Spatial patterns as a consequence of nonlinear dynamics coupled to diffusion were already proposed by Turing (1952) as models for morphogenesis. The specific patterns that we observed, active spots whose position along the spatial axis is uncorrelated (as opposed to the stripes described by Turing), is analogous to the stable peaks of excitation observed in neural tissues described, for instance, by Ermentrout (1998). In fact, running simulations of our model in the absence of failures (i.e., taking $\mathcal{P} = 0$) shows that these patterns are stable. A uniform distribution of wealth remains stable, but initial inhomogeneities give rise to patterns with the same cone shape, as shown by simulations starting from a uniform distribution of wealth A with the exception of a narrow homogenous richer zone.

A simple picture arises from the above observation: under the influence of the growth dynamics, noise generates the spatial inhomogeneities that are shaped by the diffusion dynamics.

4.1. Variant models

"Econophysicists" (Bouchaud and Mezard, 2000; Sornette and Cont, 1997; Solomon, 2000, etc.) studied Generalised Volterra–Lotka systems as models generating scale free distributions:

$$\frac{\mathrm{d}A_i}{\mathrm{d}t} = A_i \eta_i(t) + \sum_j J_{ij} A_j - \sum_j J_{ji} A_i, \tag{11}$$

where A stands for individual wealth of agents and $\eta_i(t)$ is a multiplicative noise. Agents are involved in binary transactions of "intensity" J_{ij}. Mean field formal solutions display scale free distribution of wealth, while simulations display patterns on lattice structures (Souma et al., 2001).

We checked three variants of our basic model. We started by varying production costs, taking into account

- influence of capital inertia: production costs do not instantly readjust to orders; capital and labour have some inertia, which we modeled by writing that productions costs are a maximum function of actual costs and costs at the previous period;
- influence of the cost of credit: production failures increase credit rates.

The preliminary simulations confirm the genericity of our results. The fact that cost dynamics does not change the generic properties of the model is a good indication that price dynamics would not either, since the influence of costs and price changes on profits are symmetrical.

The third variant is a model with "adaptive firms." The lattice connection structure supposes a passive reactive behaviour of firms, but if a firm is consistently delivering less than the orders it receives, its customers should order less from it and look for alternative suppliers. Such adaptive behaviour leading to an evolutive connection structure would be more realistic.

We then checked an adaptive version of the model by writing that orders of firm *i* are proportional to the production capacity *A* of the upstream firms connected to firm *i*. Simulations gave qualitative results similar to those obtained with fixed structures.

We observe that adaptation strongly re-enforces the local structure of the economy. The general picture is the same scale free distribution of production and wealth with metastable patterns. Due to the strong local character of the economy,

- avalanches of production are observed (see Fig. 11), even when time lag is short (time lag of 1), and
- the activity distribution among zones is again like "winner takes all" (Fig. 8). As opposed to passive firms patterns, a spatial periodicity of the active zones is observed. This spatial periodicity is also observed in the absence of noise when starting from a uniform distribution of wealth with the exception of a single narrow homogenous richer zone (Turing type instability).

4.2. *Relevance to spatial economics*

Can we consider the above set of models as a "new path" in spatial economics?

The recognition of the importance of space in economic organisation is not new; Von Thunen model of agglomeration dates from 1826 and Alfred Marshall's discussion of the advantages to produce in industrial districts appeared in his "Principle of Economics" in 1890. In terms of Marshall's three advantages, knowledge spillovers, thick markets for specialised skills, and linkages, our model rests mostly on the third one: the importance of backward and forward linkages among firms that translates into our network formalism.

The actual formalisation of agglomeration and the emergence of industrial districts is difficult and actually started in the 1980s with Fujita et al. (2000) and Fujita and Thisse (2002) models. Let us compare their assumptions to ours:

- production factors are differentiated, for instance in Capital and Labour, while we only consider here one factor, production capacity *A*;
- transportation costs are explicitly taken into account, while they are implicitly taken into account in our model by limiting economic connections to a neighborhood of three firms;
- their models are equilibrium models with adjustment through prices, while ours is essentially a growth model with fixed prices;
- increasing returns are an essential ingredient of their models, while they are not explicitly present in ours.

In our growth model, the circular causation (more orders, more trade generating more profit and then more capacity to produce allowing to process more orders) plays the role of instability generator played by increasing returns in theirs.

Prices are parameters in our model. This corresponds to a constant mark-up assumption. Such a simple assumption is often used in the economics of growth and can be derived from a bargaining dynamics; Osborn and Rubinstein (1990), Section 2.4.1) show that a bargaining solution to the division of a given sum is a fixed rate depending upon risk aversion of the players. The same kind of reasoning can be applied to our model: it results in a constant mark-up.

The boundary conditions that we use for the input layer (infinite supply, or rather the absence of rationing) and the output layer (infinite demand, or rather no limit to consumption other than production) make sense in a growth perspective: the time increase in consumption feeds the increasing labour part in the *A* production factor.

We certainly do not pretend that our simple models capture all the subtleties of spatial economics, nor even that they demonstrate that increasing returns are not important. The real world is far more complicated than models and does not follows Occam's Razor: the fact that increasing returns are not a necessary ingredient to generate localisation of the economy is not a proof that they did not play any role. On the other hand, the fact that strong localisation of economic activities followed periods of intense economic development (e.g., during and after the Industrial Revolution as discussed for instance in Bairoch, 1997) is an argument to discuss economic localisation within a growth framework.

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