Refined Risk Assessment and Banking Stability

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Keywords: financial intermediation, macroeconomic risks, risk assessment, risk premiums, banking regulation, rating

Classifications: JEL Codes: D40, E44, G21

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1 Introduction

With the experience of an ongoing global financial crisis, there is widespread consensus that banks should manage their balance-sheet risks more carefully. The hope that a more refined assessment of the exposure to risks will reduce the systemic risks of the banking industry had been epitomized in the Basel II regulatory framework and is now incorporated in its successor Basel III. From the viewpoint of an individual institution, it is intuitively clear that a more refined risk assessment is always beneficial because credit risks can be priced in more accurately. From a systemic viewpoint, however, the extent to which it is beneficial to a whole economy when all banks adopt more refined techniques is less obvious. This issue is the theme of the present paper.

We consider a competitive banking system in which risk-neutral banks offer their intermediation services to a population of entrepreneurs who have three investment opportunities: a production project, bank equity, and a risk-free asset. Production is subject to macroeconomic productivity shocks. Banks are delegated monitors which offer loan contracts and compete for equity and deposits. To offer intermediation services, banks are required to meet minimum capital requirements which are prescribed by regulation. Risk premia on loans are determined in the markets for loans and deposits. In order to attract equity, banks must earn sufficiently high returns on equity.

The paper investigates the systemic effects on a banking system if banks can – and have to – apply refined risk assessment techniques. To do so, we compare two polar cases, a simple and a sophisticated banking system. In the simple system, banks are unable to distinguish between different production projects. They attribute the same default risk to each project so that each entrepreneur is in the same rating category and is offered the same loan contract. In the sophisticated system, refined risk-assessment allows banks to distinguish between different production projects. Rating categories are infinitely fine so that loan-interest rates are conditioned on individual default risks of entrepreneurs. As banks are risk-neutral, they charge actuarially fair loan-interest rates. The paper analyzes the impact of refined risk assessment on market conditions and the systemic default risk of a banking system and draws policy implications.

Our main findings are the following. First, by charging actuarially fair interest rates, a sophisticated system prices in individual default risks. This reduces loan demand because it is less attractive for entrepreneurs with low-quality projects to apply for loans. An economy with a sophisticated banking system thus finances fewer production projects and needs less funds. Therefore aggregate repayments to banks are lower than in an economy with a simple system.

Second, since issuing equity is more costly than raising deposits, banks operate with
the regulatory minimum in both systems. As sophisticated banks need less funds, deposit rates are lower so that their liabilities are lower than in the simple system.

Third, the sophisticated banking system accumulates more equity in adverse macroeconomic environments. As banks earn only liquidation values of defaulting entrepreneurs, the simple banking system’s advantage of higher aggregate repayments does not outweigh its higher financing cost, if, due to adverse shocks, large numbers entrepreneurs default. However, sophisticated banks are less likely to default only if regulatory capital is sufficiently high. Otherwise, refined risk assessment will decrease bank stability as the default probability of simple banks is lower.

Fourth, an economy with a sophisticated banking system invests more funds into the alternative asset. As typically aggregate consumption in an economy with a simple banking system is lower, the economy with a sophisticated system invests its funds more efficiently. This shows that refined risk assessment may entail a tradeoff between stability and efficiency.

Our stability results relates to three issues discussed in public policy debates after the recent financial crisis. First, the failure of credit rating agencies in the recent subprime crisis. This failure has led to a thorough examination of the conflicting interests within such agencies and raised the question of how such conflicts can be mitigated by regulatory policies (see Bolton, Freixas & Shapiro 2009). Second, even if ratings from credit agencies or banks themselves are, on average, not distorted, it is unclear whether more refined techniques, adopted across all banks, will foster the stability of the banking system. This is the subject of this paper. Finally, from a historical perspective, bank equity ratios across countries have been very low before the current crisis. There are a number of convincing arguments that higher equity requirements can be justified on welfare grounds, e.g., see Hellwig (2009) and Admati at al. (2010). Our paper argues that refined risk-assessment will only increase stability if equity ratios are sufficiently high. From this viewpoint, requiring higher bank equity ratios and forcing banks to refine their risk assessment techniques are complementary measures. They tend to increase stability if taken together, but may not be successful if only the latter is implemented.

The approach taken in this paper is complementary to the work of Gehrig & Stenbacka (2001), who show that the uncoordinated screening behavior of competing financial intermediaries creates a financial multiplier, which may be responsible for macroeconomic fluctuations. In analyzing the systemic effects of screening activities, this paper contributes to the literature on screening by banks, as comprehensively surveyed in Freixas & Rochet (2009). Our analysis suggests that the new regulatory policy for banking (Basel III), which requires banks to adopt more refined techniques in risk assessment will only
work capital requirements are adjusted appropriately.

Over the past decades, a large body of literature has investigated the consequences of modern techniques in rating and risk assessment for capital markets, cf. Carey & Stulz (2007). Krahnen & Weber (2001), for example, developed a set of intuitive rating principles. Yet, the aggregate consequences of applying these principles are to a large extent unknown.\(^1\) This calls for a more detailed analysis of how refined risk assessment techniques affect the banking system and the macroeconomy, as we set out in this paper.

The paper is organized as follows: In the next section, we introduce the model and discuss the general framework. In Section 3, we examine simple banks, and in Section 4, we develop the mirror-image of the analysis for sophisticated banks. In Section 5, we compare both systems, and explain our main results. Section 6 concludes. All technical proofs are relegated to an appendix which includes an explicit example.

2 The Model

2.1 Consumers and entrepreneurs

Consider an economy with two periods in which a single good that can be used for investment and consumption. The population of agents consists of a continuum indexed by \(i \in [0, 1]\). Each agent has \(W\) units of the good in the first period. Agents are divided into two classes. One fraction of agents, indexed by \([0, \eta]\) with \(0 < \eta < 1\), are potential entrepreneurs. The other fraction, indexed by \((\eta, 1]\), are consumers. Entrepreneurs and consumers differ in that only the former have access to investment technologies.

Consumers have identical intertemporal preferences over consumption which are represented by a standard utility function \(u(c_1, c_2)\), where \(c_1\) and \(c_2\) denote first-period and second-period consumption, respectively. Given the endowment \(W\) in the first period and a deposit interest rate \(r^d\), each consumer saves the amount \(s(r^d)\) in the first period. Aggregate savings of all consumers are denoted by \(S(r^d)\) and given by \(S(r^d) = (1-\eta)s(r^d)\).

Entrepreneurs are assumed to be risk-neutral and consume in the second period only. Each entrepreneur has to decide whether to undertake a production project that converts period-1 goods into period-2 goods, to provide equity for banks, or to invest her funds in an alternative asset with return \(r_A\) \((r_A > 0)\).\(^2\) The alternative asset may be thought of as a safe outside option, such as bonds of a fiscally sound government or an investment into

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\(^1\)A limited partial equilibrium comparison with exogenously fixed deposit rates and identical financing costs for both banking systems has been performed in Gersbach & Wenzelburger (2010).

\(^2\)For tractability, we assume that consumers are not allowed to invest in the alternative asset. This can be justified by liquidity services of deposits. However, without affecting the qualitative results, the model could be extended to the case in which consumers hold a portfolio of deposits and other assets.
another sector of the economy that is not modeled explicitly.

Entrepreneurs are heterogeneous regarding the quality of their production project which is private information. The funds required for each production project are fixed at $W + I$. To undertake the production project, an entrepreneur must therefore borrow $I$ additional units of the good from banks. The output of entrepreneur $i$’s production project in period 2 amounts to

$$y = q(1 + i)f(W + I),$$

where $f$ denotes a standard atemporal neoclassical production function, $(1 + i)$ is the quality of the production project, and $q > 0$ represents an exogenous macroeconomic productivity shock in the economy. Entrepreneurs are thus ordered by quality which is the index $i$. Since $W$ and $I$ will remain fixed throughout the paper, we write $f = f(W + I)$. The distribution of shocks $q$ is assumed to be a continuous density function $h(q)$ with support on a compact interval $[q, q]$, where $0 < q < q$. Entrepreneurs are price-takers and operate under limited liability. Given a loan interest rate $r^c$, the expected profit for a producing entrepreneur $i$ is

$$\Pi(i, r^c) := \int_{q}^{\overline{q}} \max\{q(1 + i)f - I(1 + r^c), 0\} h(q)\,dq. \tag{1}$$

As in Stiglitz & Weiss (1981), $\Pi(i, r^c)$ is non-decreasing in quality levels $i$ and non-increasing in loan rates $r^c$. As each entrepreneur $i$ is risk-neutral, she will prefer to undertake the production project if its expected profit is as least as high as the gross return on the alternative investment opportunities, i.e., if

$$\Pi(i, r^c) \geq W(1 + \max\{g, r_A, r^d\}), \tag{2}$$

where $g$ is the return on bank equity, $r_A$ the return on the alternative asset, and $r^d$ the deposit rate.

### 2.2 Banking sector

The rationale for the existence of banks in our model rests on the assumption that depositors can neither observe the quality of loan applicants and nor verify whether or not funded applicants invests. This informational asymmetry is standard in the banking literature and necessitates financial intermediation, e.g., see Hellwig (1994). To alleviate this adverse-selection problem, we assume that there exist $n$ banks, indexed by $j = 1, \ldots, n$ ($n > 1$). Banks monitor borrowers and their monitoring is assumed to be efficient in

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3 All results of the paper carry over to discrete distributions of shocks.
the sense that they are able to secure both the investment of an entrepreneur and the liquidation value in case of default.⁴

We next spell out the details of financial intermediation. In particular, in order to isolate the effects of refined risk assessment on an economy, we stipulate five characteristics of the competitive and regulatory environment which both banking systems (simple and sophisticated) have in common.

First, banks need to raise equity by issuing equity contracts. An equity contract specifies the share of dividends a holder is entitled to. Entrepreneurs who provide equity become shareholders of a bank, so that bank owners are risk-neutral and consume only in the second period. The objective of banks therefore is to maximize expected profits accruing to their shareholders.⁵ In terms of the physical good, the cost of issuing equity is assumed to be \( \beta \) per unit \(( 0 < \beta < 1)\). Their monitoring cost is \( \gamma \) per unit of loan \(( \gamma > 0)\). Monitoring of entrepreneurs guarantees that production takes place and that banks receive either the repayment specified in the loan contract or the liquidation value in case of default.

Second, capital regulation specifies the amount of equity banks are required to have in order to be allowed to offer intermediation services.⁶ Third, both banking systems are perfectly competitive, that is, banks are deposit and loan contract takers. Banks can write deposit contracts in which \( 1 + r^d \) is the repayment offered for one unit of resources. The nature of loan contracts will be discussed in the next section.

Fourth, in order to provide intermediation services, banks require physical capital. Physical capital may be thought of as buildings, IT systems, and payment systems. We assume that each bank in either system needs the same amount of physical capital \( K \) in period 1, where \( K \) denotes the aggregate physical capital stock. To simplify the exposition, we assume that physical capital fully depreciates over both periods. Physical capital along with monitoring costs prevent banks from simply investing deposits and bank equity into the alternative asset without granting loans.

Finally, depositors assume that their repayments are safe. This is justified when deposits are implicitly guaranteed by a government. Throughout this paper, we assume that banks are bailed out by future (unmodeled) generations and thus, no costs are borne by the current generation. In Section 5, we will discuss alternative ways of distributing bailout costs such as taxes on second-period consumption.

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⁴See Gersbach & Uhlig (2006) for the underlying banking model and the details of the agency conflict that provide the rationale for the occurrence of financial intermediation.

⁵We could also assume that banks are owned by a group of investors who are not modeled explicitly. The crucial assumption is that expected profit maximization is in the interest of shareholders.

⁶This assumption corresponds to capital requirements in the sense of Basel I. In the concluding section, we outline how our findings will be affected by risk-sensitive capital requirements.
2.3 Simple and Sophisticated Banking

We distinguish between a simple and a sophisticated banking system that differ only in their ability to assess the quality of an entrepreneur.

1. Simple Banking System. The essential feature of the simple banking system is that banks are unable to detect the quality of an entrepreneur and are therefore unable to condition loan contracts on the quality $i$ of an entrepreneur. They offer all entrepreneurs the same loan contract with interest rate $r^e$, so that $1 + r^e$ is the repayment required from entrepreneurs for one unit of borrowed funds.

2. Sophisticated Banking System. In a sophisticated banking system, banks are able to detect the quality of an entrepreneur. They offer entrepreneur-specific loan contracts, denoted by $C(r^e_i)$, where $r^e_i$ is the loan interest rate demanded from entrepreneur $i$.

Throughout this paper, we assume that the refined risk-assessment technology incurs no extra costs. This gives such techniques the best possible chance to improve the stability of the sophisticated banking system. Extra costs of refined risk assessment, e.g., fixed costs for credit-worthiness tests, would increase the wedge between entrepreneur-specific loan rates and deposit rates and could be incorporated, at the cost of additional notational complexity.

For tractability, we assume that the banking system operates under unlimited liability in the sense that banks fully internalize the default risk that would materialize in losses. This assumption can be justified in several ways. For example, one may assume that the non-pecuniary cost of defaults for managers are so high that banks behave as if they were maximizing expected profits.\(^7\)

Since all banks within a particular system are identical, each bank will obtain the same share of total equity, deposits and borrowers.\(^8\) The financial intermediation process in either system is the following. In the first period, banks offer equity contracts, deposit contracts and loan contracts, where the latter are specified by either $r^e$ (simple banking) or a menu of loan contracts $\{r^e_i\}_{i=0}^\eta$ (sophisticated banking), respectively. Each bank has

\(^7\)It is shown in Gersbach & Wenzelburger (2008) that limited liability in general reduces the spread between deposit and loan rates and thus increases the default risk of banks. Hence, limited liability would increase default risk of banks in both systems, but not the relative comparison between the two systems. Details on this extension are available upon request.

\(^8\)We assume throughout the paper that aggregate uncertainty is canceled out when depositors and entrepreneurs randomly choose banks. The exact construction of individual randomness so that this statement holds can be found in Alós-Ferrer (1999). We could also rely on weaker forms of the strong law of large numbers developed in Al-Najjar (1995), where independence of individual random variables can be assumed and aggregate stability is the limit of an economy with finite characteristics.
to raise enough equity to fulfill regulatory requirements. Entrepreneurs decide which contracts to accept and resources are exchanged. In the second period, funded entrepreneurs undertake their production project subject to a macroeconomic shock and pay back loans with limited liability. Banks repay depositors and shareholders.

We are now ready to investigate the properties of the two banking systems. To this end, we will compare how equity develops in both banking systems. We are particularly interested in the distribution and downside risk of bank equity in the second period.

3 Competitive Equilibria for Simple Banks

In this section we consider simple banks who are unable to detect the quality of an production project and therefore face an adverse selection problem as they are unable to rate entrepreneurs individually. However, since banks are able to secure the funds of entrepreneurs they finance, entrepreneurs with low-quality projects are deterred from applying for loans. This ability sorts entrepreneurs out into two groups: entrepreneurs with sufficiently high quality projects apply for loans, those with low-quality projects are better off by either providing equity to banks or investing into the alternative asset. The sorting of entrepreneurs prevents credit rationing and thus ensures the functioning of credit markets in equilibrium. Sorting devices of this kind have been introduced in two classical papers by Bester (1985, 1987), following Stiglitz & Weiss (1981).

As all banks are assumed to be identical, the equilibrium conditions will be formulated in terms of the whole banking system rather than for individual banks. Intuitively, a competitive equilibrium consists of a marginal entrepreneur $i^*$ and an equity level $e^*$ together with a deposit interest rate $r^d^*$, a loan interest rate $r^c^*$, and an expected return on equity $g^*$, such that

(i) it is optimal for banks to offer intermediation services;

(ii) entrepreneurs take optimal investment decisions: applying for loan contracts, investing into the alternative asset or into bank equity;

(iii) the market for loans is balanced.

3.1 Prerequisites

Banks receive deposits $S(r^d)$ from consumers which have to be payed back with interest at the end of the second period. Depending on expected return on equity $g$ and a loan interest rate $r^c$, equation (2) determines a marginal quality level $i^*$, so that each entrepreneur
Given equity-issuance cost $\beta$ and monitoring cost $\gamma$, the balance sheet of the banking system amounts to

$$ (1 - \beta)e_1 + S(r^d) = (\eta - i_*)I(1 + \gamma) + K. \quad (3) $$

In (3), banks’ available funds $(1 - \beta)e_1 + S(r^d)$ balance loan demand $(\eta - i_*)I$ with monitoring costs and physical capital $K$ used by the system. Regulatory capital requirements stipulate that banks’ equity $e_1$ lies above a percentage $\alpha$ of the credit volume, i.e. $e_1 \geq \alpha(\eta - i_*)I$, where $0 \leq \alpha \leq 1$ is some small positive number (e.g. $\alpha = 8\%$ as in Basel I). Inserting (3), we see that for a given deposit rate $r^d$, the required regulatory capital

$$ e_{\text{reg}}(r^d) = \frac{\alpha}{(1 - \alpha + \alpha\beta + \gamma)}(S(r^d) - K). \quad (4) $$

Since all producing entrepreneurs have to pay the same loan interest rate $r^c$, repayments $P = P(q, i_*, r^c)$ to the banking system after a macroeconomic productivity shock $q$ are

$$ P(q, i_*, r^c) = \int_{i_*}^\eta \min\{q(1 + i)f, I(1 + r^c)\} d_i. \quad (5) $$

Let

$$ R(i, r^c) := \int_{\frac{q}{2}}^\eta \min\{q(1 + i)f, I(1 + r^c)\} h(q) dq \quad (6) $$

denote the expected repayment of entrepreneur $i$ who has received a loan $I$ at the interest rate $r^c$. Expected repayments to the banking system are

$$ \int_{i_*}^\eta R(i, r^c) d_i \quad (7) $$

and the expected return on equity $g = g(e_1, r^d, r^c)$ of a simple banking system computes as

$$ g(e_1, r^d, r^c) = \frac{1}{e_1} \left( \int_{i_*}^\eta R(i, r^c) d_i - S(r^d)(1 + r^d) \right) - 1. \quad (8) $$

Banks, on the other hand, may decide not to finance entrepreneurs and invest all their funds $(1 - \beta)e_1 + S(r^d)$ into the alternative asset. In this case no financial intermediation is undertaken and the return on equity is

$$ g_A(e_1, r^d) = \frac{1}{e_1} \left( ((1 - \beta)e_1 - K)(1 + r_A) + S(r^d)(r_A - r^d) \right) - 1. \quad (9) $$

$^9$This should be seen as the aggregate regulatory equity level for the banking system. With symmetric banks, the required capital level for an individual bank then is $\frac{e_{\text{reg}}(r^d)}{n}$. 

8
Formally, a competitive equilibrium in which simple banks finance entrepreneurs is now defined as follows:

**Definition 1**

A competitive equilibrium with financial intermediation by simple banks is a list

$$(i_*, e_1, r^d_*, r^c_*, g_*)$$

such that the following conditions hold:

$$\Pi(i_*, r^c_*) = W(1 + g_*),$$  \hspace{1cm} (10)

$$(1 - \beta)e_1 + S(r^d_*) = (\eta - i_*)I(1 + \gamma) + K,$$  \hspace{1cm} (11)

$$\alpha(\eta - i_*)I \leq e_1 \leq i_*W,$$  \hspace{1cm} (12)

$$g_* \geq \max\{g_A(e_1, r^d_*), r_A\} \text{ for all } e_1 \geq e_{\text{reg}}(r^d_*).$$  \hspace{1cm} (13)

Equation (10) determines the marginal quality level $i_*$ and thus the demand for loans. In equilibrium, all entrepreneurs with sufficiently high quality levels $i \geq i_*$ undertake their production project, while all entrepreneurs with insufficient quality levels $i < i_*$ invest either in bank equity or in the alternative asset. Equation (11) is the savings/investment balance of the banking system. Equation (12) ensures that entrepreneurs are able to provide the level of equity banks require to finance production and that capital requirements are met. Capital requirements imply that in equilibrium, the deposit rate $r^d_*$ may not exceed expected return on equity $g_*$. Otherwise banks receive no equity as potential shareholders would be better off as depositors.

Equation (13) represents the equilibrium in the equity market and ensures that banks are willing to act as financial intermediaries. If the equilibrium return on equity $g_*$ were lower than $r_A$, no entrepreneur would offer bank equity as investing into the alternative asset is more attractive. If $g_*$ were lower than some $g_A(e_1, r^d_*)$, then the alternative asset would be more profitable than financing production projects.

There are two types of equilibria: equilibria with investment into the alternative asset, $i_*W > e_1$, and equilibria without investment into the alternative asset, $i_*W = e_1$. The first type requires the return on equity to be equal to the return on the alternative asset, $g_* = r_A$ because entrepreneurs are risk-neutral. The second type requires $g_* > r_A$, implying that no entrepreneur invests into the alternative asset. As the first type is the more plausible one, we restrict attention to equilibria with investment into the alternative asset.
3.2 Existence of equilibria for simple banks

To ensure existence of competitive equilibria with financial intermediation, we make the following assumptions.

Assumption 1

(1) Banks are subject to a capital adequacy rule, stipulated by the equity-loan ratio \( \alpha \). Their fixed cost is \( K \), their relative cost of issuing equity is \( 0 < \beta < 1 \), and their monitoring cost is \( \gamma > 0 \) per unit;

(2) Aggregate endowments of depositors are not sufficient to finance all entrepreneurs, i.e., \( \eta(1 - \alpha)I > (1 - \eta)W \). The aggregate savings function, \( S(r^d) \), is non-decreasing with \( K < \beta S(0) \) and satisfies

\[
\frac{S(r^d)(1 + r^d)}{S(r^d)} \leq 1 - \frac{K}{S(0)}, \quad r^d \in [0, r_A];
\]

(3) The return on the alternative asset, \( r_A \), satisfies \( \frac{(1 - \eta)Wr_A}{1 + r_A} < K \);

(4) The productivity of entrepreneurs is such that \( \Pi(0, 0) > W(1 + r_A) \).

Assumption 1 is assumed to hold for both systems and will later enable us to isolate the effects of refined risk assessment on the systemic stability of a banking system. The first condition stipulates capital requirements and financial intermediation costs.

Condition (2) stipulates three characteristics of the depositors. First, depositors cannot finance all production projects, so that an intermediation problem exists: since \( \alpha I \) is the required minimum amount of bank equity per entrepreneur, the amount \( \eta(1 - \alpha)I \) is needed to finance all projects. Second, fixed costs can always be covered with a small percentage of deposits. Third, interest-rate elasticity of the aggregate savings function is bounded from above.

Condition (3) places an upper bound on the return on the alternative asset, \( r_A \), by stating that discounted interest payments on aggregate wealth of depositors does not exceed fixed cost \( K \). In Lemma 2 of the appendix it is shown that this ensures that financing production projects is more profitable for banks than investing into the alternative asset. This together with the elasticity condition guarantees uniqueness of competitive equilibria with financial intermediation.

Condition (4) states the entrepreneur with the lowest quality level will undertake her production project if the loan interest rate is zero.

Empirically, interest-rate elasticities of savings are relatively small.
In the proof of the following Proposition 1 it will be shown that in equilibrium banks operate with the regulatory capital level in which 
\[ e_{1*} = \alpha(\eta - i_*)I \]. At this regulatory capital level, it follows from (11) that the marginal entrepreneur must be 
\[ i_* = \eta - \frac{S(r_d) - K}{(1 - \alpha + \alpha\beta + \gamma)I} \],
where \( r_d \) is the equilibrium deposit rate. Moreover, for equilibria with investment into the alternative asset, it turns out that the marginal entrepreneur \( i_* \) must lie in between two boundary values 
\[ \underline{i} := \eta - \frac{S(r_A) - K}{(1 - \alpha + \alpha\beta + \gamma)I} \quad \text{and} \quad \overline{i} := \eta - \frac{S(0) - K}{(1 - \alpha + \alpha\beta + \gamma)I}, \] (14)

We are now in a position to establish existence and uniqueness of a competitive equilibrium with financial intermediation.

**Proposition 1**

Let the hypotheses of Assumption 1 be satisfied and assume, in addition, that productivity of entrepreneurs is such that 
\[ (i) \quad (1 + \frac{i_* + \eta}{2})E[q]f < (W + \alpha I + \frac{S(r_A)}{\eta}) (1 + r_A), \]
\[ (ii) \quad (1 + \frac{\overline{i}}{2})E[q]f > (W + \alpha I)(1 + r_A) + \frac{S(0)}{\eta - \gamma}. \]

Then there exists a unique competitive equilibrium with financial intermediation by simple banks, given by \( (i_*, e_{1*}, r^d_*, r^e_*, g_*) \). The capital requirement is binding, \[ e_{1*} = \alpha(\eta - i_*)I \], and \[ e_{1*} < i_* W \] with \( i_* \in [\underline{i}, \overline{i}] \). Interest rates satisfy \[ 0 < r^d_* < g_* = r_A < r^c_* \].

The important feature of the simple equilibrium is that banks operate with the regulatory minimum capital \( e_{1*} = \alpha(\eta - i_*)I \). The reason for this feature here is that issuing equity is more costly than raising deposits.\(^\text{11}\)

The additional assumptions in the proposition concern the productivity of entrepreneurs. Condition (i) states at the deposit rate \( r^d = r_A \), output of entrepreneur \( \frac{i_* + \eta}{2} \) will on average not exceed the sum of the opportunity cost of equity \( (W + \alpha I) \) and the opportunity cost of deposits \( \frac{S(r_A)}{\eta - \gamma} \) per entrepreneur. Condition (ii) states that at a deposit rate \( r^d = 0 \), output of entrepreneur \( \overline{i} \) will on average lie above the sum of the opportunity cost of equity and deposits.

The following corollary establishes the special case of an inelastic savings function that allows for a more tractable solution.

\(^{11}\)In the context of the current crisis, the literature discusses a variety of other reasons why equity holding of banks (e.g., see Hellwig 2009) may be minimal. These could be integrated into our model.
Corollary 1
Under the hypothesis of Proposition 1, suppose that the savings function is inelastic such that \( S(r^d) = \overline{S} \) for all \( r^d \geq 0 \). Then the marginal entrepreneur \( i_* = \eta - \frac{S-K}{(1-\alpha+\beta+\gamma)} \) is independent of \( r_*^c \), the loan interest rate \( r_*^c \) is determined by

\[
\Pi(i_*, r_*^c) = W(1 + r_A),
\]

and the deposit rate is

\[
r_*^d = \frac{1}{S} \left( \int_{i_*}^{\eta} \mathcal{R}(i, r_*^c)di - e_{1*}(1 + r_A) \right) - 1. \quad (15)
\]

3.3 Default of the simple banking system
Let \( P_*(q) := P(q, i_*, r_*^c) \) denote repayments to banks in equilibrium. It follows from (8) that in a simple equilibrium with investment into the alternative asset, average repayments to banks are

\[
\mathbb{E}[P_*] = e_{1*}(1 + r_A) + S(r_*^d)(1 + r_*^d). \quad (16)
\]

Hence, second-period equity of a simple banking system becomes

\[
e_2 = G_*(q) := P_*(q) - S(r_*^d)(1 + r_*^d) = P_*(q) - \mathbb{E}[P_*] + e_{1*}(1 + r_A). \quad (17)
\]

This shows that second-period equity is equal to the sum of expected gross return on initial equity and the difference between realized and expected aggregate repayments. From the indifference condition (10), we know that \( \Pi(i_*, r_*^c) = W(1 + r_A) > 0 \). This implies that in equilibrium, the expected profit of the marginal entrepreneur \( i_* \) is positive. Thus \( i_* \) and hence all entrepreneurs \( i \geq i_* \) will fully repay their obligations with positive probability.

We infer from (17) that second-period equity is on average positive. Hence, \( G_*(q) \) is positive for all shocks \( q \) for which repayments \( P_*(q) \) are not too far below their average. The banking system defaults if its second-period equity is negative, \( G_*(q) < 0 \). Assuming shocks \( q \) sufficiently close to the lowest shock \( q \) occur with positive probability, the banking system defaults with positive probability if and only if

\[
P_*(q) < S(r_*^d)(1 + r_*^d).
\]

In this case, since \( P_*(q) \) is non-decreasing in \( q \), there exists a unique critical shock \( q < q_{crit} < \overline{q} \) for which \( P_*(q_{crit}) = S(r_*^d)(1 + r_*^d) \). The default probability of the simple banking system is now given by

\[
\pi_{default} := \text{Prob}(G_*(q) < 0) = \int_q^{q_{crit}} h(q)dq. \quad (18)
\]
Note that (18) may be interpreted as a value-at-risk formula for the simple banking system. In Lemma 4 of the appendix, we show that the default risk of the simple banking system is positive, $\pi_{\text{default}} > 0$, whenever the lowest shock $q$ satisfies

$$\frac{I}{W + I(1 + \alpha \beta + \gamma) + \frac{\kappa}{\eta - r}} > \frac{q}{\mathbb{E}[q]}.$$  

(19)

4 Competitive Equilibria for Sophisticated Banks

We now turn to the case in which banks are sophisticated in their rating abilities so that they are able to detect the quality level $i$ of an individual entrepreneur. They can thus determine entrepreneur-specific default probabilities. We assume throughout that sophisticated banks are subject to the same minimum capital requirements and that their costs of issuing equity, their fixed cost, and their monitoring costs are the same as for simple banks. Rating of entrepreneurs in practice is costly. On purpose, we assume that sophisticated banks can screen entrepreneurs at no extra costs. This gives a sophisticated banking system the best possible chance to enhance stability.\footnote{Screening costs could be added to monitoring costs. As higher monitoring costs tend to increase instability, the assumption of no extra screening costs is the best-case scenario for sophisticated banks.}

4.1 Prerequisites

The key difference to the simple banking system is the requirement that sophisticated banks charge an actuarially fair interest rate for each loan. Actuarially fair loan-interest rates are interest rates at which expected repayments to banks are equal across entrepreneurs. The expected repayment of entrepreneur $i$ as defined in (6) is

$$\mathcal{R}(i, r^c_i) = \int_q^{q_i} \min\{q(1+i)f, I(1+r^c_i)\} \, di,$$

where $r^c_i$ is the loan-interest rate and $I$ the loan size. If entrepreneur $i \in [0, \eta]$ is required to repay some given amount $R_0$ on average, the interest rate $r^c_i$ is actuarially for $i$, if

$$\mathcal{R}(i, r^c_i) = R_0.$$  

(20)

Since $\mathcal{R}(i, r^c_i)$ is non-decreasing in quality levels, actuarially fair interest rates $r^c_i$ will be non-increasing in $i \in [0, \eta]$.\footnote{Without loss of generality, we restrict our analysis to the case in which (20) can be met by all entrepreneurs, assuming that $R_0 \leq \mathbb{E}[q](1 + i_{\text{low}})f$ with $i_{\text{low}} = 0$. This assumption can be weakened, to allow for an lowest-quality entrepreneur $i_{\text{low}} > 0$ who is just able to meet (20). All entrepreneurs $i < i_{\text{low}}$ with expected repayments below $R_0$ are deterred from applying for loans by setting $r^c_i = r^c_{i_{\text{low}}}$ for all $i \in [0, i_{\text{low}}]$.} Sophisticated banks will thus reward high-quality en-
trepreneurs with low interest rates and make it less attractive for low-quality entrepreneurs to apply for loan contracts.

We assume, in addition, that debt-equity ratios are the same across loans of a sophisticated bank.\(^\text{14}\) Since the loan size \(I\) is fixed, this implies that each loan is backed by the same amount of equity and deposits. As a consequence, expected return on equity is equal across entrepreneurs who undertake a production project. To see this, suppose that all entrepreneurs above some level \(i^o\) apply for loans. Analogously to (3), the balance sheet of a sophisticated banking system reads

\[
(1 - \beta)e^o_1 + S(r^{do}) = (\eta - i^o)I(1 + \gamma) + K,
\]

where \(r^{do}\) is a deposit rate. The required amounts of equity and deposits per loan are \(\frac{e^o_1}{\eta - i^o}\) and \(\frac{S(r^{do})}{\eta - i^o}\), respectively. Given \(r^*_i\), the expected return on equity \(\frac{g^o_i}{\eta - i^o}\) for a loan granted to entrepreneur \(i \in [i^o, \eta]\) is then given by

\[
g^o_i = \frac{1}{e^o_1} \left[ (\eta - i^o)R(i, r^*_i) - S(r^d)(1 + r^d) \right] - 1. \tag{21}
\]

With actuarially fair interest rates for all entrepreneurs, we infer from (20) that the return on equity must be equal across entrepreneurs. Hence

\[
g^o_i = g^o \quad \text{for all } i \in [i^o, \eta], \tag{22}
\]

where \(g^o = g^o(e^o_1, r^d, r^*_0)\) will be referred to as the expected return on equity of the sophisticated banking system.

With these preparations, the equilibrium notion for a sophisticated banking system extends naturally from Definition 1 for a simple banking system. Intuitively, a competitive equilibrium with financial intermediation for a sophisticated banking system is a list, consisting of a marginal entrepreneur \(i^*_0\), an equity level \(e^o_{i^*_0}\), a deposit rate \(r^{do}_{i^*_0}\), a menu of loan-interest rates \(r^c_i = r^{co}(i), i \in [0, 1]\), and an expected return on equity \(g^o_{i^*_0}\), such that

(i) it is optimal for banks to offer intermediation services at actuarially fair interest rates \(r^c_i = r^{co}(i), i \in [0, \eta]\);

(ii) entrepreneurs take optimal investment decisions: applying for loan contracts, investing into the alternative asset, or buying bank equity;

(iii) the market for loans is balanced.

\(^{14}\)This corresponds to the capital requirements of the first Basel Accord.
Formally, a competitive equilibrium with financial intermediation for a sophisticated banking system is defined as follows:

**Definition 2**

A competitive equilibrium with financial intermediation by sophisticated banks is a list

\[
\left(i_o\,^*, e_{1\,^*}, r_{do\,^*}, \{r_{co\,^*}(i)\}_{i=0}^\eta, g_{\,^*}\right)
\]

such that

\[
\mathcal{R}(i, r_{co\,^*}(i)) = \mathcal{R}(i_o\,^*, r_{co\,^*}(i_o\,^*)), \quad i \in [0, \eta],
\]

(23)

\[
\Pi(i_o\,^*, r_{co\,^*}(i_o\,^*)) = W(1 + g_{\,^*}),
\]

(24)

\[
(1 - \beta) e_{1\,^*}^o + S(r_{do\,^*}) = (\eta - i_o\,^*)I(1 + \gamma) + K,
\]

(25)

\[
\alpha(\eta - i_o\,^*)I \leq e_{1\,^*}^o \leq i_o\,^*W,
\]

(26)

\[
g_{\,^*} \geq \max\{g_A(e_1, r_{do\,^*}^o), r_A\} \quad \text{for all} \quad e_1 \geq e_{reg}(r_{do\,^*}^o).
\]

(27)

Condition (23) states that banks charge actuarially fair interest rates so that on average, they receive the same repayment on each loan. Condition (24) is the indifference condition for the marginal entrepreneur \(i_o\,^*\). Since \(\Pi(i, r^c)\) is non-decreasing in quality levels \(i\), non-increasing in interest rates \(r^c\) and \(r_{co\,^*}(i)\) is non-increasing in \(i\), all entrepreneurs \(i < i_o\,^*\) either provide equity or invest in the alternative asset. All entrepreneurs \(i \geq i_o\,^*\) receive loans and undertake their production project. As before, (25) is the equilibrium condition for the loan market. Equation (26) ensures that equity provided by entrepreneurs is sufficient and that banks fulfill minimum capital requirements. As for simple banks, the minimum capital requirement implies that the expected equilibrium return on equity, \(g_{\,^*}\), must be higher than the deposit rate, \(r_{do\,^*}\), since otherwise entrepreneurs would become depositors leaving banks without equity. Equation (27) ensures that financing entrepreneurs is at least as profitable as investing into the alternative asset. If the expected return on equity \(g_{\,^*}\), were lower than the return on the alternative asset, \(r_A\), no equity would be supplied. No bank could operate in this case, since minimum capital requirement were violated.

Analogously to the simple system, a sophisticated banking system admits two types of equilibria, equilibria with investment in the alternative asset, \(i_o\,^*W > e_{1\,^*}^o\), and equilibria without investment in the alternative asset, \(i_o\,^*W = e_{1\,^*}^o\). We restrict attention to equilibria with investment in the alternative asset as these are empirically more relevant.
4.2 Existence of equilibria for sophisticated banks

With the prerequisites of the previous section, existence and uniqueness of competitive equilibria with financial intermediation by sophisticated banks are established in the following proposition. As the focus is on equilibria with investment into the alternative asset, it turns out that the marginal entrepreneur $i_o^*$ lies in between the boundary values $\bar{i} \leq \tilde{i}$, defined in (14).

**Proposition 2**

Let the hypotheses of Assumption 1 be satisfied and assume, in addition, that productivity of entrepreneurs is such that

(i) \[(1 + \bar{i})\mathbb{E}[q]f < \left(W + \alpha I + \frac{S(r_A)}{\eta - \tilde{i}}\right)(1 + r_A);\]

(ii) \[(1 + \tilde{i})\mathbb{E}[q]f > (W + \alpha I)(1 + r_A) + \frac{S(0)}{\eta - \tilde{i}}.\]

Then there exists an unique competitive equilibrium with financial intermediation by sophisticated banks

\[\left(i_o^*, e_1^*, r_{do}^*, \{r_{co}^*(i)\}_{i=0}^\eta, g_\alpha^*\right).\]

The marginal entrepreneur is \[i_o^* = \eta - \frac{S(r_{do}^*) - K}{(1 - \alpha + \alpha \delta + \gamma)I}\] with $i_o^* \in [\bar{i}, \tilde{i}]$. The capital requirement is binding, \[e_1^* = \alpha(\eta - i_o^*)I\] and \[e_1^* < i_o^*W.\] Interest rates satisfy \[0 < r_{do}^* < g_\alpha^* = r_A < r_{co}^*(\eta).\]

A key result is that sophisticated banks operate at the minimum level of equity also. This feature is again caused by the fact that issuing equity is more costly than raising deposits. As for simple banks, the existence of a sophisticated equilibrium requires two additional assumptions on the productivity of entrepreneurs. Condition (i) states at the deposit rate $r_{do} = r_A$, output of entrepreneur $\bar{i}$ will on average not exceed the sum of the opportunity cost of equity $W + \alpha I$ and the opportunity cost of deposits $\frac{S(r_A)}{\eta - \tilde{i}}$ per entrepreneur. Condition (ii) states that at a deposit rate $r_{do} = 0$, output of entrepreneur $\tilde{i}$ will on average be above the sum of the opportunity costs of equity and deposits.

The distinguishing feature of the sophisticated banking system is that investing entrepreneurs are paying loan interest rates that are adjusted to their default risk. The default risk of an entrepreneur will generally decrease with the quality of the production project $i$, so that loan interest rates will generally decrease with $i$. However, entrepreneurs who are able to repay the opportunity costs of the loan and the intermediation costs for all shocks, will never default and therefore will pay the same rate. This result is formulated in the following lemma.
Lemma 1

(i) The loan interest rate $r_{co}^*(i)$ is decreasing in $i \in [i_0^*, \eta]$ if
\[
q(1 + i)f < I(1 + r_{co}^*(i)),
\]
that is, if entrepreneur $i$ faces a positive default risk.

(ii) If output of high-quality entrepreneurs is sufficiently high, i.e., if
\[
q(1 + \eta)f \geq (1 + \alpha \beta + \gamma)I + \frac{K}{\eta - i_0^*}(1 + r_A),
\] (28)
then there exists $i_{NB} \in [i_0^*, \eta]$ such that all entrepreneurs $i \geq i_{NB}$ never default.
These entrepreneurs pay the same interest on loans, such that $r_{co}^*(i) = r_{co}^*(i_{NB})$ for all $i \in [i_{NB}, \eta]$.

The special case of an inelastic savings function is as follows. The marginal entrepreneur is independent of the deposit rate and hence the same as in the simple system, $i_0^* = i_*$. As the two indifference condition (24) and (10) now coincide, the marginal entrepreneur $i_0^*$ receives the same loan interest rate as in the simple system. Summarizing, we obtain the following:

Corollary 2

Under the hypotheses of Proposition 2, suppose that the savings function is inelastic, i.e., $S(r^d) = \overline{S}$ for all $r^d \geq 0$. Then $i_0^* = \eta - \frac{S - K}{(1 - \alpha + \alpha \beta + \gamma)}I$ and $r_{co}^*(i_0^*) = r_*$ with $r_*$ as given in Corollary 1. The deposit-interest rate is
\[
r_{do}^* = \frac{1}{S} \left( (\eta - i_0^*) \mathcal{R}(i_0^*, r_{co}^*(i_0^*)) - e_0^*(1 + r_A) \right) - 1.
\]

4.3 Default of the sophisticated banking system

As for simple banks, we can calculate the default probability of a sophisticated banking system. As all banks behave in the same way, this systemic default probability is equal to the default probability of a single bank. Aggregate repayments to banks in a sophisticated system are
\[
P_0^*(q) = \int_{i_0^*}^{\eta} \min \left\{ q(1 + i)f, I(1 + r_{co}^*(i)) \right\} \text{d}i.
\] (29)
Taking expectations and applying (23), expected repayments in a sophisticated equilibrium become
\[
\mathbb{E}[P_0^*] = (\eta - i_0^*) \mathcal{R}(i_0^*, r_{co}^*(i_0^*)).
\] (30)
By Proposition 2 we have $g_0^* = r_A$ which, using (22), reads
\[
\mathbb{E}[P_0^*] = e_0^*(1 + r_A) + S(r_{do}^*)(1 + r_{do}^*).
\] (31)
Inserting (31), second-period equity of the sophisticated system in equilibrium then becomes

\[ e^o_2 = G^o_*(q) := P^o_*(q) - S(r^d_*)(1 + r^d_*) \]
\[ = P^o_*(q) - \mathbb{E}[P^o_*) + e^o_1(1 + r_A). \] (32)

It follows from the indifference condition (24) that expected profit of the marginal entrepreneur \( i^o \) is positive so that in equilibrium, all producing entrepreneurs will fully repay their obligations with positive probability. Hence, as can be read off from (32), second-period equity is positive, \( e^o_2 > 0 \), with positive probability. On average, \( e^o_2 \) is equal to \( e^o_1(1 + r_A) \) and therefore on average positive.

The banking system defaults if \( G^o_*(q) < 0 \). With the assumption that shocks \( q \) sufficiently close to \( q \) occur with positive probability, the default probability \( \pi^o_{\text{default}} \) is positive if and only if

\[ P^o_*(q) < S(r^d_*)(1 + r^d_*) \]

In this case, since \( G^o \) is non-decreasing in shocks \( q \), there exists a unique critical shock \( q_{\text{crit}} \) such that the default probability of banks in the sophisticated banking system is given by

\[ \pi^o_{\text{default}} = \text{Prob}(G^o_*(q) < 0) = \int_{q_{\text{crit}}}^{q} h(q) dq. \] (33)

In Lemma 4 of the appendix, we show that \( \pi^o_{\text{default}} \) is positive whenever the lowest shock \( q \) satisfies Condition (19). Note again that (33) may be interpreted as a value-at-risk formula for the sophisticated banking system.

5 Comparison of the two Systems

In order to conduct a meaningful comparison of our two banking systems, we summarize two previously made assumptions. With regard to Propositions 1 and 2, the first one guarantees existence of equilibria in both banking systems. The second one ensures that both systems default with positive probability.

Assumption 2

1. Productivity of entrepreneurs is such that
   \[ (i) \quad (1 + \frac{i \eta}{2}) \mathbb{E}[q]f < (W + \alpha I + \frac{S(r_A)}{\eta - 1})(1 + r_A), \]
   \[ (ii) \quad (1 + \frac{7}{t}) \mathbb{E}[q]f > (W + \alpha I)(1 + r_A) + \frac{S(0)}{\eta - 1}. \]

2. Productivity shocks satisfy \( \frac{L}{W + I(1 + \alpha/\beta + \gamma) + \frac{S(r_A)}{\eta - 1}} > \frac{g}{\mathbb{E}[q]} \), such that both banking systems default with positive probability.
5.1 Interest rates and stability

The first theorem compares equilibrium deposit and loan interest rates of the two banking systems.

**Theorem 1**

Let the hypotheses of Assumptions 1 and 2 be satisfied. Then the following holds:

(i) Equilibrium deposit rates satisfy $r^{do}_* < r^d_*$.

(ii) Marginal entrepreneurs satisfy $i^o_* \geq i_*$, where the inequality is strict whenever $S'(r^d) > 0$ for all $r^d \in [0, r_A]$.

(iii) First-period equity of the two banking systems satisfy $e^{o}_{1*} \leq e_{1*}$, where the inequality is strict whenever $S'(r^d) > 0$ for all $r^d \in [0, r_A]$.

(iv) There exists an entrepreneur $i_{ER}$ with $i^o_* \leq i_{ER} < \eta$ such that

(a) $r^c_* < r^{co}_*(i)$ for all $0 \leq i < i_{ER}$,

(b) $r^c_* = r^{co}_*(i_{ER})$,

(c) $r^c_* > r^{co}_*(i)$ for all $i_{ER} < i \leq \eta$.

Theorem 1 shows that charging actuarially fair interest rates rewards high-quality borrowers $i > i_{ER}$ with lower interest rates and penalizes intermediate-quality borrowers $i < i_{ER}$ with higher rates. Since it less attractive for low-quality borrowers to apply for loans, sophisticated banks finance fewer production projects, i.e., $i^o_* \geq i_*$. As a consequence, in equilibrium sophisticated banks need less funding and therefore less equity, $e^{o}_{1*} \leq e_{1*}$, and lower deposits. This implies that equilibrium deposit rates are lower, $r^{do}_* < r^d_*$.

Since deposit rates are lower in the sophisticated system, an immediate consequence of Theorem 1 is that repayments in a simple banking system are on average higher than in a sophisticated system. Taking expectations in (17) and (32), one sees in addition that expected second-period equity of the simple banking system is larger than in the sophisticated system as $e_{1*} \geq e^{o}_{1*}$. These observations are summarized in the following proposition.
Proposition 3

Under the hypotheses of Assumptions 1 and 2, the following holds:

(i) Expected repayments to the simple banking system are strictly higher than to the sophisticated system, that is,
\[ \mathbb{E}[P_*] - \mathbb{E}[P^o_*] = (e_1 - e_1^o)(1 + r_A) + S(r^d_*)(1 + r^d_*) - S(r^d_*)(1 + r^d_*) > 0. \]

(ii) Expected second-period equity of the simple banking system is higher than in the sophisticated system, that is,
\[ \mathbb{E}[e_2] - \mathbb{E}[e^o_2] = (e_1 - e_1^o)(1 + r_A) \geq 0, \]
where the inequality is strict whenever \( S'(r^d) > 0 \) for all \( r^d \in [0, r_A] \).

For a further comparison of repayments, denote the difference in repayments by
\[ P_*(q) - P^o_*(q) = \int_{i_*}^\infty \min\{q(1 + i)f, I(1 + r^d_*)\} di + \int_{i_*}^\eta z(i, q)di, \] (34)

where
\[ z(i, q) := \min\{q(1 + i)f, I(1 + r^d_*)\} - \min\{q(1 + i)f, I(1 + r^d_*(i))\}. \] (35)

The first term on the r.h.s. of (34) describes the volume effect of the differences in repayment which accounts for the fact that a simple banking system finances more entrepreneurs. The volume effect may be small but will always be non-negative as \( i^* \geq i_* \).

The second term on the r.h.s. of (34) describes the price effect of the differences in repayments. Under the hypotheses of Theorem 1, there exists a quality level \( i_{ER} \) such that \( r^d_*(i_{ER}) = r^*_c \). For inelastic savings, \( i^* = i_* \), and the price effect is strictly positive as \( i_{ER} = i^*_o \). In this case, repayments to simple banks are always higher than repayments to sophisticated banks:
\[ P_*(q) > P^o_*(q) \quad \text{for all } q \leq \hat{q}. \] (36)

By continuity of the involved functions, equation (36) remains valid for sufficiently inelastic savings function, i.e., \( S' \geq 0 \) sufficiently small.

In the general case \( i^*_o > i_* \), there exists a critical shock \( q_{ER} > \hat{q} \) such that
\[ q_{ER}(1 + i_{ER})f = I(1 + r^c_*) = I(1 + r^d_*(i_{ER})). \] (37)

Given the shock \( q_{ER} \), all entrepreneurs \( i \geq i_{ER} \) will fully meet their repayment obligations in both systems. This implies that \( z(i, q_{ER}) > 0 \) for \( i > i_{ER} \). On the other hand, in both systems all entrepreneurs \( i < i_{ER} \) will default for the shock \( q_{ER} \). Hence, \( z(i, q_{ER}) = 0 \) for
$i < i_{ER}$. This observation is illustrated in Figure 1. The gray area of this figure describes the repayments to the simple banking system encountering the shock $q_{ER}$. As can be seen from the figure, the repayments to the sophisticated system are lower for at least all shocks $q \leq q_{ER}$, that is,

$$P_*(q) > P_*^o(q) \quad \text{for all } q \leq q_{ER}.$$ 

Figure 1: Repayments of entrepreneurs, $i_*^o > i_*^i$.

In order to compare the stability properties of the two banking systems, we compare their ability to accumulate second-period equity. Consider the case with an inelastic savings function, for which initial equity levels coincide, $e_1^* = e_1^o$. It follows from (17) and (32) that $e_2^* \geq e_2$, if and only if

$$E[P_2^*] - E[P_2^o] \geq P_2^* - P_2^o.$$ 

The following proposition shows that the sophisticated banking system will accumulate more second-period equity than the simple system for all shocks below a certain 'break-even' value $q_{BE}$. As a consequence, a sophisticated system is more able to cope with low shocks than a simple system. The reverse is true for high shocks. Here a simple system will accumulate more second-period equity.

**Proposition 4**

*Let the hypotheses of Assumptions 1 and 2 be satisfied. Then there exists a critical shock $q_{BE}$ with $\underline{q} < q_{BE} < \overline{q}$ such that*

1. $G_*(q) < G_*^o(q)$ for all $q \leq q_{BE}$,
2. $G_*(q_{BE}) = G_*^o(q_{BE})$,
3. $G_*(q) > G_*^o(q)$ for all $q_{BE} < q \leq \overline{q}$.
For an economic intuition of the result, consider the extreme case in which all entrepreneurs default in the sophisticated system. This implies that all entrepreneurs default in the simple banking system as well. Since banks earn liquidation values only, revenues in both banking systems are then identical. As a simple banking system has a higher deposit rate, their liabilities are higher. Its second-period equity is thus lower if macroeconomic shocks are below the critical level $q_{BE}$.

This relation is reversed for sufficiently positive shocks. Since $r^s(i) < r^c$ for $i > i_{ER}$, repayments to simple banks in a sufficiently favorable macroeconomic environment are higher. It follows from the respective equilibrium conditions (10) and (24) that in both systems the marginal – and thus all – entrepreneurs fully pay back their loans with positive probability. Therefore, the simple banking system accumulates more second-period equity for sufficiently positive shocks.

Our main theorem now compares the default probabilities of the two banking systems, defined in (18) and (33), demonstrating the significance of the initial level of equity.

**Theorem 2**

Let the hypotheses of Assumptions 1 and 2 be satisfied.

(i) If

$$G^s_*(q_{BE}) = G_*(q_{BE}) > 0,$$

then the default probability of the sophisticated banking system is lower than the default probability of the simple banking system, i.e., $\pi^o_{\text{default}} < \pi_{\text{default}}$.

(ii) If, on the contrary,

$$G^s_*(q_{BE}) = G_*(q_{BE}) < 0,$$

then the default probability of the sophisticated banking system is higher than the default probability of the simple banking system, i.e. $\pi^o_{\text{default}} > \pi_{\text{default}}$.

(iii) If, $G^s_*(q_{BE}) = G_*(q_{BE}) = 0$, then $\pi^o_{\text{default}} = \pi_{\text{default}}$.

The economic intuition of the theorem follows that of Proposition 4. The first part of Theorem 2 is illustrated with Figure 2(a). For adverse macroeconomic shocks, the sophisticated system benefits from the fact that it finances fewer entrepreneurs, so that potential losses are lower. As their liabilities are lower, a default of sophisticated banks is less likely. Using (17), we see that Condition (38) is satisfied if initial equity $e_{1s}$ is sufficiently high.\(^{15}\)

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\(^{15}\)Numerical Examples are available upon request.
The second part treats the case in which Condition (38) is violated and the initial level of equity $e_{1*}$ is too low. As illustrated with Figure 2(b), then banks may default if moderate adverse shocks occur. In this case, repayments from non-defaulting entrepreneurs do not suffice to cover lower liabilities of sophisticated banks. The rewards to high-quality entrepreneurs are effectively too high which is reflected by a higher default risk of sophisticated banks. The theorem demonstrates that one measure to counterbalance this effect is to increase regulatory capital.

5.2 Efficiency

To conclude the paper, this section presents some efficiency considerations. The first observation is that equilibrium aggregate output of entrepreneurial production in response to a shock $q \in [\underline{q}, \overline{q}]$ is

$$Y_s(q) = qf \int_{\underline{i}}^{\eta} (1 + i)di$$

for the simple and

$$Y_s^o(q) = qf \int_{\underline{i}}^{\eta} (1 + i)di$$

for the sophisticated banking system. Since $i_s^o \geq i_s$ by Theorem 1 (ii), we have

$$Y_s^o(q) \leq Y_s(q) \quad \text{for all } q \in [\underline{q}, \overline{q}].$$

This implies that sophisticated banks will reduce aggregate output by producing entrepreneurs. Note again, that the inequality (40) is strict, unless consumers save inelastically.

Consider now expected aggregate consumption of entrepreneurs. For an economy with a simple banking system, expected aggregate consumption of entrepreneurs is

$$E[C_E] = i_s W(1 + r_A) + \int_{\underline{i}}^{\eta} \Pi(i, r_c^*)di,$$
whereas expected aggregate consumption of entrepreneurs in an economy with a sophisticated banking system is

$$E[C^o_E] = i^o_*W(1 + r_A) + \int_{i^o_*}^{\eta} \Pi(i, r^c_*(i))di.$$  \hspace{1cm} (42)

We obtain

**Theorem 3**

*Let the hypotheses of Assumptions 1 and 2 be satisfied. Then expected aggregate consumption of entrepreneurs in an economy with a simple banking system is lower than in an economy with a sophisticated system.*

Theorem 3 shows that an ‘average’ entrepreneur will prefer an economy with a sophisticated banking system. However, while entrepreneurs \(i \in [i_{ER}, \eta]\) with high-quality projects will have higher expected profits in a sophisticated system, entrepreneurs \(i \in [i^*, i_{ER})\) with intermediate quality projects are better off with the simple system. Entrepreneurs \(i \in [0, i^*)\) with low-quality projects are indifferent and consumers will always prefer the simple system because \(r^d_* > r^d_0\). Hence refined risk assessment in banking will have income-distributional effects and may entail a trade-off between stability and aggregate consumption.

Any further an analysis regarding welfare and income-distributional consequences of refined risk assessment depends on how banks are bailed out in the case of default. Let us briefly discuss two scenarios assuming that case \((i)\) of Theorem 2 holds. Suppose first that banks are bailed out or insured by future generations as developed by Allen & Gale (1997). This scenario is also well-known in the empirical banking literature, when countries have responded to large-scale banking problems by drastically lowering future short-term interest rates. This can generate large profits for banks and may thus allow them to recover (see e.g. Hoshi & Kashyap 2004). In this scenario, agents in our model do not bear the cost of a bail-out. Expected aggregate consumption of entrepreneurs are higher with a sophisticated system, while in case of a default, future generations are worse off. A complete welfare analysis, therefore, calls for a fully fledged OLG model and is beyond the scope of this paper.

Suppose, second, that agents of your model bear the cost of a bail-out. This implies that they have to provide rescue funds for a defaulting bank. A priori, consumers in such a scenario tend to be worse off with a sophisticated banking system if they have to contribute to such rescue funds, whereas the effect on entrepreneurs is ambiguous. If bail-out schemes are anticipated by agents, their decision problems will have to be adapted before a welfare analysis can be conducted. This aspect will be left for future research.
6 Conclusion

This paper demonstrates that more a refined assessment of individual default risks will only increase banking stability if initial equity is sufficiently high. We have shown that refined risk assessment rewards high-quality entrepreneurs with lower loan rates at the expense of depositors facing lower returns. Refined risk assessment thus has distributional implications as consumers and entrepreneurs are affected differently. Our analysis suggests that regulatory policies for banking such as Basel III, which require banks to introduce more refinements in assessing the credit-worthiness of their clients, will only be beneficial if banks are sufficiently capitalized.

A number of extensions deserve further scrutiny. One example are risk-sensitive capital requirements as proposed in the Basel III regulatory framework. Such an extension tends to reinforce our conclusions, as risk-sensitive capital requirements will make the slope of the interest curve $r^c_s(i)$ steeper. Another interesting extension would be to explicitly allow for rating agencies that, at some cost, perform credit-worthiness tests and offer their services to banks.
A Technical Appendix

A.1 Proofs of Main Results

Proof of Proposition 1.

Step 1. We show that in equilibrium, (12) holds with $e_{1*} = \alpha(\eta - i_*)I$ and $e_{1*} < i_*W$. Expected return on equity $g(e_1, r^d, r^c)$ must be greater than $r^d$, otherwise banks have no equity and are not allowed to operate. By Lemma 3, Appendix A.2 below, $g(e_1, r^d, r^c)$ is then decreasing in equity $e_1$. In equilibrium, first-period equity must therefore satisfy the minimum capital requirement $e_{1*} = \alpha(\eta - i_*)I$. The balance equation (11) implies

$$e_{1*} = \frac{\alpha}{1-\alpha+\alpha\beta+\gamma}(S(r^d_* - K)$$

and $i_* = \eta - \frac{1}{(1-\alpha+\alpha\beta+\gamma)I}(S(r^d_* - K)$. (43)

It follows from (43) that $e_{1*} < i_*W$, if and only if

$$\left(\frac{\eta(1-\alpha+\alpha\beta+\gamma)I}{W+\alpha I}\right)W > S(r^d_* - K).$$

(44)

Using the fact that $S(r^d) < (1-\eta)W$ for all $r^d \geq 0$, a straightforward calculation shows that (44) is implied by $\frac{\eta(1-\alpha)}{1-\eta} \geq \frac{W}{I}$ which was stipulated in Assumption 1 (2).

Step 2. We determine the relationship between quality levels and interest rates at which an entrepreneur is indifferent between undertaking the production project and investing into the alternative asset. Since $e_{1*} < i_*W$, we have $g_* = r_A$. Consider

$$\Pi(i, r^c) = W(1 + r_A), \quad i \in [\underline{i}, \eta]$$

which contains the indifference equation (10) as a special case. Set

$$r^c(i) := \frac{\Pi(I + r_A)}{I} - 1, \quad i \in [\underline{i}, \eta],$$

(46)

where $\overline{\eta}$ denotes the highest possible shock. Then expected profit (1) of entrepreneur $i$ is zero, i.e., $\Pi(i, r^c) = 0$ for $r^c \geq r^c(i)$. It follows from Assumption 1 (4) that for each $i \in [\underline{i}, \eta]$,

$$\Pi(i, 0) \geq \Pi(\underline{i}, 0) > W(1 + r_A) > \Pi(i, r^c(i)) = 0.$$

Since for each $i \in [\underline{i}, \eta]$, $\Pi(i, r^c)$ is continuous and (strictly) decreasing in $r^c < r^c(i)$, it follows from the Intermediate Value Theorem that (45) has a unique solution $0 < h(i) < r^c(i)$, such that

$$\Pi(i, h(i)) = W(1 + r_A), \quad i \in [\underline{i}, \eta].$$

(47)

Since $\pi(i, r^c) > 0$, it is increasing with respect to $i$ and decreasing with respect to $r^c$, $h(i)$ is increasing in $i \in [\underline{i}, \eta]$. For each $r^d \geq 0$, define

$$i_E(r^d) := \eta - \frac{1}{(1-\alpha+\alpha\beta+\gamma)I}(S(r^d) - K)$$

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and set $\varphi(r^d) = h(i_E(r^d))$. Since $i_E(r^d)$ is non-increasing in $r^d$, $\varphi$ is non-increasing in $r^d$. This shows that given an equilibrium deposit rate $r^d_*$, the pair $i_* = i_E(r^d_*)$ and $r^d_* = \varphi(r^d_*)$ solves the indifference condition (10).

**Step 3. Existence and uniqueness of equilibria with intermediation.** Using (8) and (43), the equality $g_* = r_A$ takes the form

$$\frac{1}{\eta-1_*} \int_{i_*}^{\eta} \mathcal{R}(i, r^c_*) di = \alpha I(1 + r_A) + \frac{S(r^d_*)}{\eta-1_*} (1 + r^d_*) = (48).$$

Define a function $F : \mathbb{R}_+ \to \mathbb{R}$ by setting

$$F(r^d) := \frac{1}{\eta-1_E(r^d)} \int_{1_E(r^d)}^{\eta} \mathcal{R}(i', \varphi(r^d)) di' - \alpha I(1 + r_A) - \frac{S(r^d_*)}{\eta-1_E(r^d)} (1 + r^d).$$

In view of (48), we see that an equilibrium exists if and only if there exists $r^d_*$ such that $F(r^d_*) = 0$. For each $r^c \geq 0$ and each $i \in [0, \eta]$, we have

$$\Pi(i, r^c) + \mathcal{R}(i, r^c) = \mathbb{E}[q](1 + i)f.$$ (50)

Using the definition of $h$, yields

$$\mathcal{R}(i, h(i)) = \mathbb{E}[q](1 + i)f - W(1 + r_A).$$ (51)

Since $h$ is increasing, (51) and Assumption (i) imply

$$F(r_A) \leq \frac{1}{\eta-1} \int_{1}^{\eta} \mathcal{R}(i', h(i')) di' - \alpha I(1 + r_A) - \frac{S(r_A)}{\eta-1} (1 + r_A)$$

$$= \mathbb{E}[q](1 + \frac{\eta+1}{2}) f - \left(W + \alpha I + \frac{S(r_A)}{\eta-1}\right) (1 + r_A) < 0.$$ On the other hand, (51) and Assumption (ii) imply

$$F(0) \geq \mathcal{R}(\bar{h}, h(\bar{h})) - \alpha I(1 + r_A) - \frac{S(0)}{\eta-1}$$

$$= \mathbb{E}[q](1 + \bar{h}) f - (W + \alpha I)(1 + r_A) - \frac{S(0)}{\eta-1} > 0.$$ Thus $F(0) > 0 > F(r_A)$ and since $F$ is continuous, there exists $0 < r^d_* < r_A$ such that $F(r^d_*) = 0$. As $i_E(r^d)$ and $\varphi(r^d)$ are non-increasing in $r^d$, the first term in (49) is non-increasing in $r^d$, as it can be verified by taking the first derivative. The elasticity condition on $S$, Assumption 1 (2), implies that the second term in (49) is strictly decreasing in $r^d$. Therefore, $F$ is strictly decreasing for $0 < r^d < r_A$. Thus, the simple banking system admits a unique competitive equilibrium with financial intermediation $(i_*, e_1*, r^d_*, r^c_*, g_*)$, where $e_1*$ satisfies regulatory capital requirement (43) with $i_* = i_E(r^d_*)$ and $r^d_* < g_* = r_A$.  

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Step 4. We show that equilibria without financial intermediation do not exist. By Lemma 2, Appendix A.2, \( r_A > g_A(e_1, r^d_e) \) for all \( e_1 \geq e_{\text{reg}}(r^d_e) \), so that Condition (13) holds.

Finally, we show that \( r^*_e > r_A \). This follows from (48) if

\[
\alpha I(1 + r_A) + \frac{S(r^d_e)(1 + r^d_e)}{\eta - i^*_e} > I(1 + r_A).
\]

Using (43), this inequality is implied by \( S(r^d_e) > (S(r^d_e) - K)(1 + r_A) \) which in turn is implied by Assumption 1 (3).

\[
\square
\]

Proof of Proposition 2.

Step 1. It follows from a reasoning analogous to Step 1 in the proof of Proposition 1 that in equilibrium, (26) holds with \( e^o_{1*} = \alpha(\eta - i^*_e)I \) and \( e^o_{1*} < i^*_eW \). Condition (25) then implies

\[
e^o_{1*} = \frac{\alpha}{1-\alpha+\alpha \beta + \gamma}(S(r^d_o) - K) \quad \text{and} \quad i^*_e = \eta - \frac{1}{(1-\alpha+\alpha \beta + \gamma)}(S(r^d_o) - K). \quad (52)
\]

Analogously, \( e^o_{1*} < i^*_eW \) implies \( g^o_e = r_A \). Using the functions \( i_E \) and \( \varphi \) defined in Step 1 of the proof of Proposition 1, we see that given an equilibrium deposit rate \( r^d_o \), the pair \( r^o_{1*} = \varphi(r^d_o) > 0 \) and \( i^*_e = i_E(r^d_o) \) solves the indifference equation (24).

Step 2. Existence and uniqueness of an equilibrium deposit rate. Using (22), (23) and (52), equation \( g^o_e = r_A \) takes the form

\[
\mathcal{R}(i^*_e, \varphi(r^d_o)) = \alpha I(1 + r_A) + \frac{S(r^d_o)}{\eta - i^*_e}(1 + r^d_o). \quad (53)
\]

Define \( \tilde{F} : \mathbb{R}_+ \rightarrow \mathbb{R} \) by

\[
\tilde{F}(r^d) := \mathcal{R}(i_E(r^d), \varphi(r^d)) - \alpha I(1 + r_A) - \frac{S(r^d)}{\eta - i_E(r^d)}(1 + r^d). \quad (54)
\]

Inserting (50) into (54), we get

\[
\tilde{F}(r^d) = \mathbb{E}[q](1 + i_E(r^d))f - (W + \alpha I)(1 + r_A) - \frac{S(r^d)}{\eta - i_E(r^d)}(1 + r^d) \quad (55)
\]

Hence, the equilibrium deposit rate \( r^d_o \) has to satisfy \( \tilde{F}(r^d_o) = 0 \), i.e.,

\[
\mathbb{E}[q](1 + i^*_e)f = (W + \alpha I)(1 + r_A) + \frac{S(r^d_o)}{\eta - i^*_e}(1 + r^d). \quad (56)
\]

Assumptions (i) and (ii) imply \( \tilde{F}(0) > 0 > \tilde{F}(r_A) \) and by the continuity of \( \tilde{F} \), a solution \( 0 < r^d_o < r_A \) exists. Since \( i_E(r^d) \) is non-increasing, the elasticity condition on \( S \) implies that \( \tilde{F} \) is strictly decreasing such that \( r^d_o \) is uniquely determined. Hence, there exists a unique \( (i^*_e, e^o_{1*}, r^d_o, r^e_o) \) such that \( g^o_e = r_A \).
Step 3. We show existence of \( r^c_0(i) \) such that (23) holds. For each \( i \in [0, \eta] \), the function \( r^c \mapsto \mathcal{R}(i, r^c) \) is (strictly) increasing for \( r^c \leq \mathcal{R}^c(i) \) and constant for \( r^c \geq \mathcal{R}^c(i) \), where \( \mathcal{R}^c(i) \) was defined in (46). Moreover,

\[
\mathcal{R}(i, -1) = 0 \quad \text{and} \quad \mathcal{R}(i, \mathcal{R}^c(i)) = \mathbb{E}[q](1 + i)f. \tag{57}
\]

Let \( r^c_{i^o} = \varphi(i^o) > 0 \) as defined above and set \( r^c_{*0}(i^o) = r^c_{i^o} \). Since (57) holds for \( i^o \), there exists \( i_{\text{low}} \leq i^o \) such that

\[
\mathcal{R}(i^o, r^c_{i^o}) = \mathbb{E}[q](1 + i_{\text{low}})f. \tag{58}
\]

It follows from (57) that there exists a unique function \( r^c_{*0}(i) \) with \( r^c_{*0}(i^o) = r^c_{i^o} \) such that

\[
\mathcal{R}(i, r^c_{*0}(i)) = \mathcal{R}(i^o, r^c_{i^o}) \quad \text{for all} \quad i \in [i_{\text{low}}, \eta].
\]

If \( i_{\text{low}} \leq 0 \), this implies in particular that (23) holds. If \( i_{\text{low}} > 0 \), then setting \( r^c_{*0}(i) = r^c_{*0}(i_{\text{low}}) \) for all \( i \in [0, i_{\text{low}}] \), the function \( r^c_{*0}(i) \) is well-defined on \([0, \eta]\). Clearly \( i \mapsto r^c_{*0}(i) \) is continuous and non-increasing on \([0, \eta]\) because \( \mathcal{R} \) is continuous and non-decreasing in both arguments. This implies that

\[
\Pi(i, r^c_{*0}(i)) < \Pi(i^o, r^c_{*0}(i^o)) = W(1 + r_A), \quad i \in [0, i^o],
\]

showing that all entrepreneurs \( i < i^o \) either provide equity or invest into the alternative asset, whereas all entrepreneurs \( i \geq i^o \) undertake their production project.

Finally, using (23), it follows from (53) and an argument analogous to Step 4 in the proof of Proposition 1 that \( \mathcal{R}(\eta, r^c_{*0}(\eta)) > I(1 + r_A) \). Hence \( r^c_{*0}(\eta) > r_A \), so that all equilibrium loan interest rates are positive.

Step 4. The equilibrium with financial intermediation \((i^o, e^o_1, r^d^*_{do}, \{r^c_{*0}(i)\}_{i=0}^\eta, g_2^*)\) is unique as \( r^d_{do} \) is uniquely determined. It follows from Lemma 2, Appendix A.2 below, that in particular \( r_A > g_A(e_1, r^d_{do}) \) for all \( e_1 \geq e_{\text{reg}}(r^d_{do}) \), so that Condition (27) holds. Note that the lemma rules out existence of equilibria without intermediation for any \( r^d \).
Proof of Lemma 1.

(i) The equilibrium condition (23) and (53) imply
\[ R(i, r^c(i)) = \alpha I(1 + r_A) + \frac{S(r^d(i))}{\eta - i} (1 + r^d(i)), \quad i \in [i^c, \eta]. \] (59)

If \( q(1 + i)f < I(1 + r^c(i)) \) for some \( i \geq i^c \), then \( \frac{\partial R}{\partial i}(i, r^c(i)) > 0 \) and (59) implies that \( r^c(i) \) must be decreasing in \( i \).

(ii) We have
\[ R(i, r^c(i)) \geq \min \{ q(1 + i)f, I(1 + r^c(i)) \}, \quad i \in [i^c, \eta] \] (60)
and
\[ \frac{S(r^d(i))}{\eta - i} < (1 - \alpha + \alpha \beta + \gamma) I + \frac{K}{\eta - i}. \] (61)

Since \( r^d < r^c \), (28) together with (59) and (61) imply
\[ R(\eta, r^c(\eta)) < q(1 + \eta)f. \]

Thus, entrepreneur \( \eta \) never defaults. Denote by \( i_{NB} \in [i^c, \eta] \) the entrepreneur with the lowest quality level who never defaults. By (60),
\[ R(i_{NB}, r^c(i_{NB})) = I(1 + r^c(i_{NB})) \]
and (59) implies \( r^c(i) = r^c(i_{NB}) \) for all \( i \geq i_{NB} \).

Proof of Theorem 1.

(i) For each \( r^c \geq 0 \) and each \( i \in [0, \eta] \), we have
\[ \frac{1}{\eta - i} \int_{i}^{\eta} R(i', r^c)di' \geq R(i, r^c). \] (62)

Therefore, \( F \) as defined in (49) and \( \tilde{F} \) as defined in (54) satisfy \( \tilde{F}(r^d) \leq F(r^d) \) for all \( r^d \geq 0 \). This implies \( r^d \leq r^d \). Suppose \( r^d = r^d \). Then \( i^c = i = i_E(r^d) \), \( r^c = r^c = \varphi(r^d) \), and (48) and (53) imply
\[ \frac{1}{\eta - i} \int_{i}^{\eta} R(i, r^c)di = R(i^c, r^c). \]

Thus \( r^c \leq q(1 + i^c)f \), so that in equilibrium no defaults occur. By Lemma 4 below, this contradicts Assumption 2 (2). Hence \( r^d < r^d \).
(ii) This follows from (i), as \( i_* = i_E(r^d) \leq i_E(r^{do}) = i_*^o \), where the inequality is strict whenever \( S' > 0 \).

(iii) This follows directly from (43) and (52).

(iv) We have

\[
\Pi(i_*, r^c) = \Pi(i_*^o, r^{co}(i_*^o)) = W(1 + r_A)
\]

such that \( r^{co}(i_*^o) \geq r^c \) with a strict inequality \( r^{co}(i_*^o) > r^c \) holding if \( i_*^o > i_* \). By the mean value theorem for integrals, there exists \( \tilde{i} \in [i_*, \eta] \), such that

\[
\frac{1}{(\eta - i_*)} \int_{i_*}^{\eta} R(i, r^c) \, di = R(\tilde{i}, r^c).
\]

Since \( r^d > r^{do} \), we infer from Assumption 1 (2) and the two equilibrium conditions (48) and (53) that

\[
\frac{1}{(\eta - i_*)} \int_{i_*}^{\eta} R(i, r^c) \, di - R(\tilde{i}, r^c) = \frac{S(r^d)(1 + r^d)}{\eta - i_*} - \frac{S(r^{do})(1 + r^{do})}{\eta - i_*} > 0.
\]

Since \( R \) is non-decreasing in \( i \), \( R(\eta, r^c) \geq R(\tilde{i}, r^c) \). Hence it follows from (64) and (65) that

\[
R(\eta, r^c) > R(i_*^o, r^{co}(i_*^o)) = R(\eta, r^{co}(\eta))
\]

and thus \( r^c > r^{co}(\eta) \). Since \( r^{co}(i_*^o) \geq r^c \) and \( r^{co}(i) \) is continuous and non-increasing in \( i \), there exists a unique \( i_{ER} \in [i_*^o, \eta] \) such that \( r^{co}(i_{ER}) = r^c \). If \( i_*^o > i_* \), then \( r^{co}(i_*^o) > r^c \) and hence \( i_{ER} > i_*^o \).

---

**Proof of Proposition 3**

(i) It follows from (16) and (31) that

\[
\mathbb{E}[P_*] - \mathbb{E}[P_*^o] = (i_*^o - i_*) \alpha I(1 + r_A) + S(r^d)(1 + r^d) - S(r^{do})(1 + r^{do}).
\]

Since \( i_*^o \geq i_* \), \( r^d > r^{do} \), and \( S \) is non-decreasing, \( \mathbb{E}[P_*] - \mathbb{E}[P_*^o] > 0 \).

(ii) Taking expectations, this follows from (17) and (32).

---

**Proof of Proposition 4.**

Consider first the case \( S' = 0 \) so that \( i_*^o = i_* \). Using (17) and (32), \( e_2^o \geq e_2 \) if and only if

\[
P_*(q) - P_*^o(q) \leq \mathbb{E}[P_*] - \mathbb{E}[P_*^o].
\]
Since \( r^o_i < r^c_s \) for all \( i > i_s \), the l.h.s. of (67) is always non-negative and positive for sufficiently high shocks \( q \). Since \( P_\ast(q) - P^o_\ast(q) \) is non-decreasing in \( q \), there exists a unique \( q_{BE} \) such that (67) holds with equality. This implies that (67) holds if and only if \( q \leq q_{BE} \). This proves the proposition for \( S' = 0 \). The case \( S' > 0 \) follows from continuity considerations.

**Proof of Theorem 2.**
We need to show that \( q^o_\ast < q_{crit} \) in case (i) and \( q^o_\ast > q_{crit} \) in case (ii). It follows from Proposition 4 that \( G_\ast(q) < G^o_\ast(q) \) for \( q < q_{BE} \) and \( G_\ast(q) > G^o_\ast(q) \) for \( q > q_{BE} \). Since \( G_\ast(q) \) and \( G^o_\ast(q) \) are non-decreasing in \( q \) with \( G_\ast(q) < G^o_\ast(q) < 0 \), the critical values must satisfy \( q^o_\ast < q_{crit} \) in case (i) and the reverse inequality must hold in case (ii). This proves the theorem.

**Proof of Theorem 3.**
We need to show that \( E[C^o_E] - E[C_E] > 0 \). Taking integrals, equation (50) implies
\[
\int_{i_s}^\eta \Pi(i, r^c_s)di = E[q]f \int_{i_s}^\eta (1 + i)di - E[P_s] \tag{68}
\]
and
\[
\int_{i^o_s}^\eta \Pi(i, r^o_s(i))di = E[q]f \int_{i^o_s}^\eta (1 + i)di - E[P^o_s] \tag{69}
\]
Substituting (68) and (69), we see after calculating the integrals that
\[
E[C^o_E] - E[C_E] = E[P_s] - E[P^o_s] + (i^o_s - i_s) \left( W(1 + r_A) - \left( 1 + \frac{\eta - r^o_s}{\eta - r^c_s} \right) E[q]f \right).
\]
Since \( i^o_s \geq i_s \) by Theorem 1 (ii), (56) and (66) imply
\[
E[C^o_E] - E[C_E] \geq (\eta - i_s) \left[ \left( \frac{\eta - r^o_s}{\eta - r^c_s} \right) (1 + r^d_s) - \left( \frac{\eta - r^o_s}{\eta - r^c_s} \right) (1 + r^d_o) \right].
\]
The last term is positive by Assumption 1 (2).
A.2 Technical lemmas

Lemma 2
If $\frac{(1-\eta)W_{rA}}{1+r_A} < K$, then banks will always finance entrepreneurs; in particular,

$$\forall e_1 \geq e_{\text{reg}}(r^d), \quad g_A(e_1, r^d) < r_A,$$

Proof. (i) Using (9) we have 

$$1 + \frac{S(r^d)(r_A-r^d)}{1+r_A} < \beta e_1 + K.$$

The statement now follows from the fact that $S(r^d) < (1-\eta)W$.

Lemma 3
Let $0 < \beta < 1$, $\gamma > 0$ and $e \geq 0$ such that $(\eta - i^*)I(1+\gamma) + K = (1-\beta)e + S(r^d)$ for some $0 \leq i^* < \eta$. Then the following holds:

(i) For given $r^d < r^c$, expected return on equity of a simple banking system

$$g(e) = \frac{1}{e} \left( \int_{i^*}^{\eta} R(i, r^c)\,di - S(r^d)(1+r^d) \right) \quad (70)$$

is decreasing in equity $e$, whenever $g(e) \geq r^d$.

(ii) Given $r^d$ and actuarially fair interest rates $r^{co}(i)$, expected return on equity of a sophisticated banking system

$$g^o(e) = \frac{1}{e} \left( \int_{i^*}^{\eta} R(i, r^{co}(i))\,di - S(r^d)(1+r^d) \right), \quad (71)$$

is decreasing in equity $e$, whenever $g^o(e) \geq r^d$.

Proof. (i) Observe that

$$\frac{\partial}{\partial i^*} \int_{i^*}^{\eta} R(i, r^c)\,di = -R(i^*, r^c).$$

and $\frac{\partial i^*}{\partial e} = \frac{(1-\beta)}{(1+\gamma)}$. Applying the chain rule, differentiating (70) yields

$$g'(e) = \frac{1}{e} \left( \frac{1-\beta}{1+\gamma} R(i^*, r^c) - (1 + g(e)) \right).$$

If $g(e) \geq r^d$, then

$$\frac{1-\beta}{1+\gamma} R(i^*, r^c) \leq \frac{1-\beta}{\eta - r^c} \int_{i^*}^{\eta} R(i, r^c) \leq \frac{(1-\beta)[e + S(r^d)]}{(1-\beta)e + S(r^d) - K} (1 + g(e, r^d)). \quad (72)$$

The second inequality in (72) implies that $g'(e) < 0$ as $\beta S(r^d) > K$. 

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(ii) Analogously to (i), differentiating (71) yields
\[ g''(e) = \frac{1}{e} \left( \frac{1-\beta}{\gamma+1} \mathcal{R}(i^*, r^{co}(i^*)) - (1 + g'(e)) \right). \]

The rest of the proof is analogous to Part (i), noting that
\[ (\eta - i^s)\mathcal{R}(i^s, r^{co}(i)) = \int_{i^s}^{\eta} \mathcal{R}(i, r^{co}(i))di. \]

\[ \square \]

Lemma 4

In both banking systems, bankruptcies of producing entrepreneurs occur with positive probability if
\[ \frac{I}{W + I(1 + \alpha \beta + \gamma) + \frac{K}{\eta - 1}} > \frac{q}{\mathbb{E}[q]]. \]

**Proof.** Using (55) and (61), the equilibrium condition (56) implies
\[ \mathbb{E}[q](1 + i^*_o) f < \left( W + I(1 + \alpha \beta + \gamma) + \frac{K}{\eta - 1} \right) (1 + r_A). \]
Thus
\[ 1 + i_* \leq 1 + i^*_o < \frac{W + I(1 + \alpha \beta + \gamma) + \frac{K}{\eta - 1}}{\mathbb{E}[q] f} (1 + r_A). \] (73)

Entrepreneur \( i \) defaults with positive probability if and only if \( (1 + r^c)I > q(1 + i)f \). Since \( r^{coo}(i) \geq r^c > r_A \), the marginal entrepreneurs \( i^*_o \) and \( i_* \) default with positive probability if
\[ \frac{I(1 + r_A)}{q} > 1 + i^*_o. \]

Using the upper bound (73), a sufficient condition for both \( i_* \) and \( i^*_o \) to default with positive probability is
\[ \frac{I(1 + r_A)}{q f} > \frac{W + I(1 + \alpha \beta + \gamma) + \frac{K}{\eta - 1}}{\mathbb{E}[q] f} (1 + r_A) \]
or, equivalently, if
\[ \frac{I}{W + I(1 + \alpha \beta + \gamma) + \frac{K}{\eta - 1}} > \frac{q}{\mathbb{E}[q]}. \]
B Uniformly Distributed Shocks

The purpose of this appendix is to illustrate our model with a more tractable parameterization yielding explicit loan-interest rates. We assume that the macroeconomic productivity shocks are uniformly distributed on \([q, \overline{q}]\) and that the savings function is inelastic. The following lemma gives explicit loan-interest rates for the simple banking system.

**Lemma 5**

Under the hypotheses of Proposition 1, assume that savings are inelastic, \(S(r^d) = \overline{S}\), for all \(r^d \in [0, r_A]\), and that macroeconomic shocks are uniformly distributed, i.e., \(h(q) := \frac{1}{q - \overline{q}}\). Then the competitive loan-interest rate takes the form

\[
1 + r_c^* = \frac{(1 + i^*_c)f}{f - \sqrt{\frac{2(\overline{q} - q)W}{(1 + i^*_c)f}(1 + r_A)}}.
\]  

(74)

where \(i^*_c = \eta - \frac{S - K(1 - \alpha + \alpha\beta + \gamma)}{I}\).

**Proof.** Setting \(r(i) = (1 + i)qf/I - 1\) and \(\overline{r}(i) = (1 + i)\overline{q}f/I - 1\), the expected profit of entrepreneur \(i\) given an interest rate \(r_c^*\) is

\[
\Pi(i, r_c^*) = \begin{cases} 
(1 + i)(qf + q^2 - I(1 + r_c^*)) & \text{if } r_c^* < r(i), \\
\frac{(1 + i)f}{2(q - \overline{q})} \left(q - \frac{I(1 + r_c^*)}{(1 + i)f}\right)^2 & \text{if } r(i) \leq r_c^* \leq \overline{r}(i), \\
0 & \text{if } \overline{r}(i) < r_c^*.
\end{cases}
\]

Then (74) follows from condition (10) by solving for \(r_c^*\). \(\square\)

The expected repayment from entrepreneur \(i\), given an interest rate \(r_c^*\), is

\[
R(i, r_c^*) = \begin{cases} 
I(1 + r_c^*) & \text{if } r_c^* < r(i), \\
\left(\frac{1}{q - \overline{q}} + \frac{1}{I(1 + r_c^*) - \frac{I^2(1 + r_c^*)^2}{2(1 + i)f}} - \frac{1}{2q^2}(1 + i)f\right) & \text{if } r(i) \leq r_c^* \leq \overline{r}(i), \\
\frac{\overline{r}(i) - i^*_c}{2} (1 + i^*_c)f & \text{if } \overline{r}(i) < r_c^*.
\end{cases}
\]

(75)

where \(r(i)\) and \(\overline{r}(i)\) as defined above. Inserting the loan-interest rate (74) into (75), we obtain

\[
R(i^*_c, r_c^*) = \frac{\overline{r}(i) - i^*_c}{2} (1 + i^*_c)f - W(1 + r_A).
\]

By Corollary 2, loan-interest rates of both banking systems coincide for the marginal entrepreneur \(i^*_c = i^*_c\). We can therefore solve condition (23) for \(r_c^*(i)\) to obtain the following lemma:
Lemma 6

Under the hypotheses of Proposition 2, assume that the macroeconomic shocks are uniformly distributed, \( h(q) := \frac{1}{\pi/2} \). Then the deposit-interest rate takes the form

\[
1 + r^{do}_* = \frac{\eta - i_*}{\gamma} \left( \frac{\pi + q}{2} (1 + i_*) f - (W + \alpha I)(1 + r_A) \right).
\]

The loan-interest rates for producing entrepreneurs in a sophisticated banking equilibrium are given by

\[
1 + r^{co}_*(i) = \begin{cases} 
\frac{(1+i)f}{I} \left[ q - \sqrt{2(q - q) \left( \frac{\pi + q}{2} - \frac{R(i, r^*_c)}{(1+i)f} \right)} \right] & \text{if } i \in [i_*, i_{NB}), \\
\frac{R(i, r^*_c)}{I} & \text{if } i \in [i_{NB}, \eta],
\end{cases}
\]

where the quality level \( i_{NB} \) above which no entrepreneur defaults is given by

\[
i_{NB} = \min \left\{ \frac{R(i_*, r^*_c)}{\eta q} - 1, \eta \right\}.
\]

Observe that \( r^{co}_*(i) \) exhibits the typical pattern. The loan interest rate is decreasing in the quality level \( i \) for as long as \( i \in [i_*, i_{NB}] \), while it is constant for \( i \geq i_{NB} \).
References


